Cereal trade in developing countries: stochastic spatial equilibrium models

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Abstract:

Since the introduction of the Structural Adjustment Programmes in West Africa, the role of the national governments has changed considerably. Prices are no longer controlled by the state and governments do no longer intervene as major marketing agents. It remains to be seen whether the free market system leads indeed to an efficient food allocation, especially in remote and less endowed regions. In this report a quantitative analysis is made of arbitrage in time and space. We pursue two objectives. First, a model is developed to simulate the interaction between the various agents on the market: producers, traders and consumers. Particular attention is given to 1) differences between perfect and monopolistic markets; 2) farmers' supply behaviour in various seasons, and 3) optimal traders' strategies. A stochastic, spatial equilibrium model is set up to analyse price formation and optimal supply, demand, transport and storage strategies by the market actors.

Secondly, the model is used to analyse the direct impact of transport and storage costs on the distribution of cereals in space and time in Burkina Faso, in West Africa. In particular, it is analysed how changes in these costs influence cereal prices, consumption, sales, transport and storage in all regions of the country and during all periods of the year. An important question is to what extent the most vulnerable regions are affected by these changes. In the literature on the functioning of food markets in West Africa transport costs are often perceived as a major constraint for food marketing and rural development in general. The results, however, indicate that the direct impact of these costs on prices and cereal distribution is only marginal. This is mainly due to the inability of farmers to increase production, and the inability of consumers to increase purchases. The paper concludes with a discussion on the usefulness of these models as instruments for policy analysis.

Table of contents:

1. Introduction 1
2. Food allocation by the market: an overview of persistent imperfections 5
   2.1 Seasonal and spatial arbitrage with imperfect information
   2.2 Thin markets
   2.3 Missing or incomplete markets
   2.4 Markets and famines
   2.5 Alternative institutions to improve the food situation
   2.6 Final remarks
3. Non-linear programming revisited 14
   3.1 Non-negativity constraints
   3.2 Equality constraints
   3.3 Inequality constraints
   3.4 Equality and inequality constraints
   3.5 Stochastic programming
4. Supply functions, demand functions and equilibrium 33
   4.1 Supply functions
   4.2 Demand functions
   4.3 Seperability of supply and demand decisions
   4.4 Equilibrium
5. Spatial equilibrium on *n* markets; one period model 51
   5.1 Strategies of producers, consumers and traders
   5.2 Maximization of welfare; perfect competition between traders
   5.3 Monopolistic behaviour of traders
6. Spatial equilibrium on *n* markets; multi-period model 69
   6.1 Strategies of producers, consumers and traders
   6.2 Maximization of welfare; perfect competition between traders
   6.3 Monopolistic behaviour of traders
7. Spatial equilibrium on *n* markets in *T* periods: stochastic future prices 83
   7.1 Strategies of producers, consumers and traders

7.2 Maximizing welfare; stochastic future prices
7.3 Monopolistic behaviour of traders
8. Cereal markets in Burkina Faso 120
8.1 Empirical evidence of supply and demand
8.1.1 Rural and urban population
8.1.2 Cereal production
8.1.3 Cereal sales
8.1.4 Cereal purchases
8.1.5 Revenues and expenditures
8.1.6 Agricultural prices
8.2 Trading costs
8.2.1 Transport costs
8.2.2 Storage and other trading costs
9. Estimation of cereal demand and supply functions for the case of Burkina Faso 153
9.1 Cereal demand functions
9.1.1 Linear expenditure system
9.1.2 Estimating cereal demand function for Burkina Faso
9.1.3 Cereal demand functions per period
9.2 Cereal supply functions
9.2.1 Cereal supply model
9.2.2 Estimating cereal supply functions for Burkina Faso
9.3 Review of the stochastic, multi-period, spatial equilibrium model
10 Discussion of model results 202
11 Final discussion 212

References 217
Appendix 1: Proofs of Properties and Theorems in the Chapters 5, 6 and 7 225
Appendix 2: Deriving the results of the supply model 256
Appendix 3: Parameter estimation 268
Appendix 4: Trading costs 330

1 Introduction

This report deals with trade on cereal markets in semi-arid West Africa, and the distribution of cereals in particular. A quantitative analysis will be made of arbitrage in space and time. In many West African countries trade costs, i.e. transport, storage and transaction costs, are said to be high, induced by an inefficient market system. In the literature on the functioning of food markets in West Africa, these costs are often perceived as a major constraint for food trade and rural development in general. Since the introduction of Structural Adjustment Programmes in Africa the role of national governments in the food market has been reduced considerably. Prices are no longer controlled by the state but have been liberalized, and governments do no longer intervene as major marketing agents, because markets have been privatized. A lively debate is taking place on the effects of these programmes on poverty alleviation (see e.g. Sahn et al., 1997, Thorbecke, 2000). Despite some improvements, it is still an open question whether the free market system leads indeed to a greater food security in West Africa due to a more efficient market system, especially in remote and less endowed regions. In this report this question is addressed.[1]

We pursue two objectives. First, an instrument will be developed to analyse the interaction between the various actors on the market: producers, traders and consumers. Spatial equilibrium models (see e.g. Samuelson, 1952; Takayama and Judge, 1971; Judge and Takayama, 1973; Martin, 1981; Florian and Los, 1982; Labys et al., 1989; Guvenen et al., 1990; Roehner, 1995; Van den Berg et al., 1995) are used as instruments of analysis. They describe arbitrage in space and time. In the first part of this paper the theory of these models is discussed. In three respects the spatial

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[1] This research is a component of a joint research programme on food security in West Africa, in which researchers of the University of Ouagadougou in Burkina Faso, of the Institute of the Environment and Agricultural Research (INERA) in Burkina Faso and of the Centre for Development Studies of the University of Groningen participate. Some of the research deals with modelling the behaviour of various agents on cereal markets and of interregional cereal flows between markets (see e.g. Yonli, 1997, Sirpé, 2000, Bassolet, 2000, Maatman, 1996, 2000, Lutz and Bassolet, 1999).

equilibrium models as developed in this paper differ from standard theory. First, equilibrium models are set up for both perfectly competitive and monopolistic models. Secondly, the farmers' supply of cereals in various periods of the year depends on supply decisions in previous periods and on *uncertain prices* in later periods. Thirdly, the traders' optimal strategies of buying from the producers and selling to the consumers are explicitly taken into account.

The second objective is the application of these models to the cereal market in Burkina Faso. It will in particular be analysed what the direct impact is of transport and storage costs on the distribution of cereals in space and time in Burkina Faso. It is analysed how changes in these costs influence cereal prices, consumption, sales, transport and storage in all regions of the country and during all periods of the year. An important question is to what extent the most vulnerable regions and trade are affected by these changes during the lean season. Marketed cereal flows between surplus and shortage regions in the various periods of the year are calculated as functions of farmers' supply, consumers' demand and traders' strategies of purchasing, selling, storage and transport. Key parameters in the models will be estimated on the basis of an extensive exploration of many resources. [2]

First, in Chapter 2, some characteristics of food markets in developing countries are reviewed. Some persistent imperfections of the food market in many developing countries are discussed. These imperfections determine to a large extent the functioning of the food market, and are used as a background to the model .

The Chapters 3 and 4 are also of an introductory nature. A review is given of some basic elements of optimization theory, stochastic programming and of micro-economics, which will be used in later chapters. In Chapter 3, some elements

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[2] See for example studies of the University of Michigan and the University of Wisconsin (McCorkle, 1987; Szarleta, 1987; Sherman et al., 1987), of CILSS (Pieroni, 1990), of ICRISAT (Reardon et al., 1987, 1988a, 1988b, 1989, 1992), of Yonli (1997), of Broekhuyse (1988, 1998), of INSD (1995a, 1995b, 1996a, 1996b, 1998), of the Ministry of Agriculture and Animal Resource (1984-1996), and data provided by SIM/SONAGESS.

of non-linear programming will be discussed, in particular necessary conditions (Lagrange and Kuhn-Tucker conditions) for optimality. These conditions play a key role in the interpretation of results of applying spatial equilibrium models. The review of the non-linear programming is set up step by step; we start with simple non-linear programming problems with only non-negativity constraints and finish with complicated problems with general non-linear equality and inequality constraints. Furthermore, the theory of stochastic programming is briefly discussed in Section 3.5, in order to analyse in Chapter 7 decision making under uncertainty. Chapter 4 reviews some basic concepts from micro-economics. Attention is focused on supply and demand functions and their properties and some basic concepts of equilibrium. These introductory chapters are included, because the present paper is intended to be used as well as teaching material for university students in developing countries, who not always have easily access to the proper literature. Readers who are already familiar with the contents, may skip these chapters.

Chapters 5, 6 and 7 deal with the theory of spatial equilibrium models. All models deal with only one commodity, cereals. In the Chapters 5 and 6 a distinction is made between equilibrium models for a perfect market system where a large number of competitive producers, traders and consumers operate who are all price takers and for a monopolistic market system where the traders can set the prices to some extent. The spatial equilibrium models of Chapter 5 deal with n markets and one period of time. No storage is involved. In Chapter 6 multi-period spatial equilibrium models are discussed. Here storage is a key factor. For the model of Chapter 6, future prices are assumed to be known. In Chapter 7 multi-period spatial equilibrium models are discussed for a situation with *uncertain* future prices. Supply, demand and storage decisions are based on what is observed on the market, and on what is expected to happen in the future. In the Chapters 5, 6 and 7 much attention is given to the interpretation of results and properties of the solutions, and to the optimality of the individual strategies of the agents: producers, traders and consumers.

In Chapter 8 empirical evidence of the market behaviour of the different actors is discussed for the case of Burkina Faso. On the basis of a large number of surveys performed in the past, supply and demand behaviour of cereal producers and consumers is discussed, as well as the costs involved in cereal trade. In Chapter 9 supply and demand functions, key elements of the stochastic, multi-period, spatial equilibrium models, are presented for Burkina Faso. On the basis of the evidence presented in Chapter 8 cereal demand is estimated per period as a function of cereal prices. Furthermore, the distribution of cereal supply over the year as a function of cereal production and cereal prices is estimated. In Section 9.3, the stochastic, multi-period, spatial equilibrium model discussed in Chapter 7 is shortly summarized.

In Chapter 10, results of the stochastic, multi-period, spatial equilibrium model presented in Chapter 7 are discussed. It is a case study of regional transport of cereals in Burkina Faso. The paper concludes with some reflections on the results and on the use of these models.

2 Food allocation by the market: an overview of persistent imperfections

The functioning of food markets is a major policy issue in many developing countries. The reason of its importance is twofold. Firstly, availability of food is a precondition for survival and socio-economic stability and, secondly, many regions regularly face climatic hazards (supply shocks). Food markets play an important role in food distribution. Their performance is the result of a complex set of institutions (rules) which regulates exchange and initiatives undertaken by individuals (traders, farmers) and governmental and non-governmental organizations (cereal banks, co-operatives).

In the commonly used neo-classical perfect market theory strong assumptions are made to simplify this complex set of institutions:

- Farmers and traders are price takers, because their large numbers preclude any influence on prices.
- No uncertainty or risk exists, as information on market conditions is perfect.
- No entry or exit barriers constrain the behaviour of potential competitors.
- The commodity is homogeneous: quality and variety do not influence prices.

Rural food markets in Africa differ from this ideal market type. This section presents some of these features, which do not correspond with the 'perfect conditions'.

In the debates on the food policy in the semi-arid tropics policy-makers and researchers have tended to view sedentary rural households as dependent almost exclusively on their own cereal production to ensure household food security.[3] Rural

[3] Indeed, farmers have taken new and promising initiatives to master the food situation. These activities include among others:
- activities on the farm-household level: improvement of strategies to reduce risks of low yields by careful choice of different varieties and of intercropping and rotation patterns, and by timely land-preparation and sowing; adoption of low external input methods to restore soil fertility and water management methods to improve hydrological capacities of soils; use of animal draught power for land

markets were seen as primary markets that should simply drain surpluses to urban deficit markets. Various recent research results have undermined this view and show that many farm households are net buyers of substantial food quantities (see Reardon et al., 1989 and 1992). Revenues from livestock and nonfarm activities provide an important part of the necessary food entitlements for the rural population. This implies that trade flows within a country are much more complex than the simple 'model' of rural areas that provision urban centres. The rural economies are increasingly monetized and nowadays food markets play a crucial role in food distribution. Petty trade and processing activities are an important income source for many of the poor.

Properly functioning markets will serve both the producers at the one end of the marketing chain and the consumers at the other end; market failures will affect opportunities for producers, as well as food availability for consumers. Views on the performance of food markets in developing countries have shifted in the course of time. During the 1960s the debate stressed the existence of market failures. For example:

- Due to a lack of competition traders were alleged to abuse their market power.
- A lack of capital and credit constituted an entry barrier for small traders.
- Due to a lack of information, market integration was deficient.

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preparation and weeding; agroforestry and the integration of animal husbandry and crop production; investments in non-farming activities (trade, processing);
- 'collective activities' by farmers' groups: village cooperatives working together on the construction of small water-reservoirs, anti-erosive measures and horticulture; exchange of information between farmer groups; education and information activities; establishment of cereal banks with the aim of building up reserve stocks to strengthen food security in the village and to improve the local distribution and marketing system.
They have taken up the twofold challenge: survival in the lean season and the transformation towards a more sustainable agrarian system. Some of these initiatives are almost entirely based on strategies of 'self-reliance' in food production. However, others do rely directly or indirectly on market-exchanges. These initiatives can be individual or collective; the latter, often structured by 'new' forms of agrarian institutions, aim to improve access to product- and factor-markets (in particular food, finance and inputs) for some group of relatively 'isolated' farmers.

In line with the desire of the newly independent African states to plan economic development, interventionist policies were developed to correct for these failures. However, the 1970s have shown that many of the so-called ‘market failures’ were only replaced by ‘government failures’. Ellis (1992) summarises the government failures as follows:

- Information failures. It appeared almost always wrong to assume that state officials have any clearer idea, of the supply and demand conditions in the market than private sector operators. This resulted in serious misallocation and the coexistence of a network of formal and informal parallel markets.
- Complex side effects. Interventions have secondary effects in an economy, e.g. policies striving for low consumer prices may lower farm-gate prices or increase government budget-deficits.
- Implementation and motivation failures. Most of the developing countries are ‘soft states’ with ‘soft bureaucracies’, making the implementation of market policies all over the country's territory a difficult task. Moreover, low salaries affect the motivation of the civil servants in charge.
- Rent-seeking. Under the above-mentioned conditions state action may easily lead to bribery and malpractice.

As a result of the experiences in the 1970s, structural adjustment policies in the 1980s and 1990s advocated market liberalization. These have put to an end the interventionist policies of many governments. The new market policies foster the functioning of the market. Despite the liberalization, several market imperfections (market failures) persist.

2.1 Seasonal and spatial arbitrage with imperfect information

Food production is not synchrone with food consumption. For example, in the semi-arid areas of West Africa, producers have only one harvest a year, while consumption is continuous. Moreover, harvests are regularly threatened by climatic hazards: yields are volatile and the start of the harvest (end of the lean season) differs between the years. This seasonal aspect may cause substantial price fluctuations, as storage costs (due to storage losses and capital needs to finance the cereals) are important and information on local supply and demand conditions is imperfect. Indeed, prices in the cereal market can be volatile: during the harvest the value of old stocks depreciates quickly (30 to 50% in a few weeks) and, on the contrary, prices may be sky-high at the end of the lean season as traders are hesitant to run the price risk and keep only minimum amounts of cereals in stock. Under these conditions traders may realize high speculative profits or losses, dependent on the accurateness of their market price expectations. Moreover, the lack of access to credit seriously hampers the functioning of seasonal arbitrage. Most traders operate with very small funds and most farmers have little withholding capacity (they need money to settle debts and household expenses), while credit, insurance or futures markets are imperfect or missing.

In the same vein, we observe that the place of food production usually does not correspond to the place of food consumption. In particular, after a bad year arbitrage over long distances may be necessary to provision consumers. The food chain is complex as many food producers are constrained by variable seasonal agro-ecological conditions and appear to be net food buyers: local supply and demand conditions vary between years and within years. This implies that adequate information on local market conditions (prices, quantities, local market rules) is a prerequisite for successful traders. In most of the African countries this information is difficult to obtain as the telecommunication infrastructure is imperfect and market rules are non-transparent. In many cases information depends on personal networks of individual traders.

On a perfect market, prices convey information from households to firms concerning what consumers want, and from firms to households about the production costs (see Stiglitz, 1994:8). However, one of the major constraints, which hamper the functioning of the rural markets, is imperfect information on the potential market opportunities. In order to safeguard their existing trade relations, traders are reluctant to share their information with competitors. Some information simply does not exist for instance information on uncertainty in the production process. Other sources of information may exist but are not always accessible for all traders and farmers. Moreover, in many countries official regulations are not transparent and their implementation arbitrary. The existence of oligopolistic markets often seems to be based on the possibility for certain wholesalers to detain specific information. In practice we observe that traders stick to their individual marketing networks which are nested in particular geographical regions. This restricts competition, as a lack of information constitutes an entry barrier.

2.2 Thin markets

Most producers are peasants who are to a high degree self-sufficient with regard to cereals and are incidentally buying/selling their deficit/surplus in the market. The grain stock is perceived as a liquid source that may be used for urgently needed household necessities. The problem for the market is that most of these transactions concern small and highly variable quantities, scattered all over the country's territory. This fragmented structure inflates transaction costs: the assembly and distribution of cereals becomes a labour-intensive and costly activity. An example may explain this argument. In Benin, the average retailers' turnover per market day is often less than 100 kg. If we assume an average price of 50 Fcfa per kg and a normal average income per day of 500 Fcfa, then a net margin of at least 10% is necessary to remunerate the retailer's labour time, who is only one of the intermediaries in the market chain. If the turnover doubles the margin for labour remuneration can be lowered significantly. Despite the somewhat higher turnover of wholesalers, the same argument applies for their activities.

The development of a personal network of trade agents and clients (farmers and consumers) may provide traders the necessary information on supply and demand. These networks may reduce the number of intermediairies in the market chain, as well as the transaction costs. However, the elaboration of such a network presupposes the availability of sufficient working capital (the agent has to be pre-financed) and takes time. This constitutes an entry barrier for potential competitors. Moreover, small marketable surpluses also restrict competition among traders (in particular wholesalers), as only a limited number of traders are sufficient to drain the surplus.

Thin markets increase market imperfections (e.g. lack of competition) and high transaction costs make markets even thinner or may result in missing markets. In order to evade the high transaction costs, farmers may increase the number of non-market transactions. Cereals can be exchanged within the family and some services and goods can be paid in kind. Matthews (1986) formulated this issue as follows: 'Family production tends to make for high production costs because it restricts exploitation of scale economies and may create mismatches between talents and occupation. On the other hand it tends to reduce transaction costs, because if instead you have a lot of dealing with strangers you have to devote more resources to checking up on their personal characteristics and safeguarding yourself against opportunism'. If transaction costs are high, it will decrease the competitiveness of farmers and, consequently, they may decide to withdraw from the market (see de Janvry et al., 1991). However, the disadvantage of this strategy is that food security of farmers, who have no other food entitlements, will be at stake if production falls short. Market exchange makes it possible to specialize, or to exploit comparative advantages and to spread production risks (production of cash and food crops, insurance against crop failures). If the transaction costs are high, these markets may be missing (or imperfect) and, consequently, these opportunities will not be available (or not interesting).

2.3 Missing or Incomplete markets

In most developing countries, the set of commodity and service markets is highly incomplete. Imperfections in three related markets, providing essential services for cereal trade, hamper the functioning of the food market and increase the transaction costs:

- Transport services are only available to a limited extent. A small group of large-scale wholesalers have their own transport facilities, but the majority of small-scale traders depend on public transport facilities, which are mainly oriented toward the urban centres. During rainy seasons large rural areas may even become inaccessible. Consequently, the transport of commodities is less flexible than required for optimal trade flows.

- Credit facilities constrain the commercial activities of traders and farmers, in particular the storage function. The formal financial sector does not provide credit for trade activities and even if credit facilities do exist, most traders and farmers lack the necessary collateral (see Zeller et al., 1997).
- Finally, an insurance (harvest failures) and futures (hedging) market, accessible to individual traders and farmers, does not exist. Hedging against price fluctuations is impossible. Only recently some experiences are noted (see below). However, the institutional structure necessary to guarantee the enforcement of these contracts between individuals is weak, often resulting in the non-existence of this market.

2.4 Markets and Famines

Agricultural production in developing countries is highly dependent on climatic circumstances. Climatic hazards may provoke serious supply shocks, leading to food deficits. Various authors have studied food insecurity and hunger situations and particularly discussed the relationship between famines and markets (Ravallion, 1987; Drèze and Sen, 1989). They have documented situations where market failures, thin markets and missing food markets have made hunger and famines more severe. Markets work badly during famines when panic buying and excess hoarding exacerbates scarcities. The food insecurity is aggravated by the seasonality of food production, which makes that food demand is highest during the lean season, whereas the availability of food stocks is at its lowest level. Consequently, governments should be alert and guarantee sufficient supply in drought prone areas. Adequate policies are necessary to attenuate the problem of transitory food-insecurity.

2.5 Alternative institutions to improve the food situation

Cereal Banks

Cereal banks are a type of organisation that may challenge the existing market structure (Saul, 1987; Yonli, 1997). They concern a communal village organisation that co-ordinates the marketing and storage of cereals. In general, cereals are bought in harvest time and sold during the lean season to members of the community. The idea behind this structure is that farmers in the rural areas are obliged to sell a part of their production just after the harvest in order to settle debts and other financial obligations. The same farmers have to buy during the lean season to supplement the cereal deficit. Put differently, they sell low and buy high. The difference between these prices may be considerable in a situation of remote semi-arid regions. In such regions cereals may have to be imported over large distances. Rural population density is low, meaning that the market is thin. Large-scale traders are not interested in provisioning these regions, and supply may even be lacking. Under these circumstances a farmers' organisation (cereal bank) may be useful; there are opportunities to beat the market.

Cereal banks substitute to a certain extent for market-exchanges, but at the same time they may play a key-role in improving access of farmers to rural markets. The cereal bank may provide farmers' access to rural group credit schemes to finance cereal stocks. The organization can also be helpful to develop new market strategies: buying directly in surplus markets (rural centres), or selling directly in deficit markets (urban centres). However, it should be noted that many cereal banks, established during the last decade, failed. Often, the objectives were too ambitious and organisational problems were frequent.

Cereal auction market (futures market)

A more recent initiative in Burkina Faso is quite interesting: the development of a cereal auction market. In 1991 the auction started as an experiment, with the aim to facilitate the exchange between farmers' organisations, in particular cereal banks. Nowadays also private traders are participating in this market. Yonli (1997) indicates that the auction facilitates the functioning of cereal banks as it may provide the structure to link directly surplus and deficit cereal banks and, consequently, limit transaction costs. Moreover, the auction may introduce a futures cereal market as contracts can be concluded for delivery at a certain time, which may result in an effective instrument to protect farmers against price changes.

2.6 Final remarks

The presentation of persistent market failures is not meant to be a plea for government intervention. Market institutions are complex and experiences with interventions in the past have shown that also governments can fail. Nevertheless, the challenge for market policies is still to foster improvements in market institutions that decrease transaction costs and improve food-security.

The objective of this introduction was to enumerate some important imperfections that characterize the functioning of food markets in developing countries. The models presented in the following chapters are based on severe restrictions and do not take into account all these imperfections. Mainly the problems of non-synchrone food production and consumption (Section 2.1), and of thin markets (low supply and demand by rural households; Section 2.2), are dealt with in the next chapters. The models discussed in the Chapters 4, 5 and 6 pre-suppose perfect markets: atomistic supply and demand, perfect information, perfect mobility (no entry or exit barriers), homogeneity of commodities and, last but not least, the existence of a set of related markets, such as transport, credit and insurance/futures markets. In Chapter 7 also the problem of imperfect price information is discussed. In practice many of the perfect market conditions are not fulfilled. This should be kept in mind when the results of simplified models are interpreted.

3 Non-linear programming revisited

In the models to be described in the next chapters the behaviour of various agents - producers, consumers and traders - is formulated in such a way that their decisions on quantities to be produced, consumed or traded are the 'best ones'. Of course, it will not be easy to give a proper definition of the 'best decisions', in particular in a situation where interests of producers, consumers and traders can be different, even conflicting. This definition will be a key issue in the next chapters. The structure of models describing the behaviour of the various agents is as follows: the maximum value of a certain 'objective function' is to be found, where the decision variables (e.g. quantities to be produced, consumed or traded) are determined in such a way that certain conditions are to be satisfied (e.g. equilibrium conditions). Such models belong to the class of optimization models, known as non-linear programming models. In this chapter some basic elements of such models are reviewed. Furthermore, in Section 3.5 the theory of stochastic programming is briefly discussed.

Let x be a n-dimensional vector with elements x_j, $j = 1,2,...,n$. Here the problem of determining the global or a local maximum of a non-linear function $F(x)$ is dealt with. The variables x may have to satisfy non-linear equality and inequality constraints. The review is presented as follows. Stepwise, four maximization problems, (i) - (iv), will be discussed. First in problem (i) the maximum of $F(x)$ has to be found, where some of the variables x have to satisfy (only) non-negativity constraints. Then, in problem (ii), x has to satisfy only one (non-)linear equality constraint, in (iii) one (non-)linear inequality constraint. Finally, in problem (iv) more (non-)linear equality and inequality constraints are included. In this chapter a persistent distinction is made between non-linear equality and inequality constraints on one hand and non-negativity constraints on the other hand. The number n_1 with $n_1 \leq n$ refers to the number of non-negativity constraints to be taken into account. The number m_1 refers to the number of (non-)linear equality constraints and m to the total number of (non-)linear constraints - both equality and inequality constraints with the exception of non-negativity constraints. Let $g(x)$ and $g_i(x)$, $i = 1, 2, \ldots, m$ be (non-)linear functions of x. In the next four sections the following maximization problems will be discussed:

(i) max $\{F(x) \mid x_j \geq 0, j = 1,2,\dots,n_1\}$

(ii) max $\{ F(x) \mid g(x) = 0 \}$

(iii) max $\{ F(x) \mid g(x) \geq 0 \}$

(iv) max $\{F(x) \mid g_i(x) = 0, i=1,2,\dots,m_1; g_i(x) \geq 0, i = m_1+1, m_1+2,\dots,m;$
$x_j \geq 0, j = 1,2,\dots,n_1\}$

In this paper all functions $F(x)$, $g(x)$ and $g_i(x)$, i = 1, 2, ..., m are assumed to be differentiable. A function $F(x)$ is called concave, if for all x^* and x and the scalar λ with $0 \leq \lambda \leq 1$ holds $F(\lambda x + (1-\lambda)x^*) \geq \lambda F(x) + (1-\lambda)F(x^*)$; strictly concave if in this expression $\geq$ may be replaced by >. The function $F(x)$ is (strictly) convex, if $-F(x)$ is (strictly) concave. A linear function is both concave and convex. A differentiable function is concave if and only if for all x^* and x holds

$$(3.1) \qquad F\left(x^*\right) - F(x) \leq \sum_{j=1}^{n} \left(x_j^* - x_j\right) \frac{\partial F}{\partial x_j}(x)$$

If $F(x)$ is twice differentiable, then $F(x)$ is (strictly) concave if and only if the $n \times n$ Hessian matrix consisting of the elements $\frac{\partial^2 F}{\partial x_i \partial x_j}$, i = 1, 2, .., n; j = 1, 2, ..., n is negative (semi-)definite (see e.g. Bazaraa et al., 1993: p. 91).

3.1 Non-negativity constraints

It can easily be seen that the solution x of (i) has to satisfy:

$$(3.2) \qquad x_j \frac{\partial F}{\partial x_j} = 0, j = 1,2,...,n_1$$

(3.3) $\dfrac{\partial F}{\partial x_j} \leq 0$, j = 1,2,...,n_1

(3.4) $\dfrac{\partial F}{\partial x_j} = 0$, j = n_1 +1, n_1 +2, ..., n.

For a function in one variable x the conditions (3.2) and (3.3) are illustrated in Figure 3.1.

If $F(x)$ is concave, the conditions (3.2) - (3.4) imply that the function $F(x)$ is in the point x the global maximum. This can be shown as follows. Consider any point $x^* \neq x$ satisfying $x_j^* \geq 0$, j=1,2,...,n_1. Then it may be written - by making use of property (3.1) and the conditions (3.2) - (3.4) - that $F(x^*) \leq F(x)$. So $F(x)$ is the global maximum indeed. If $F(x)$ is strictly concave, the point x is the only point where the maximum is attained.

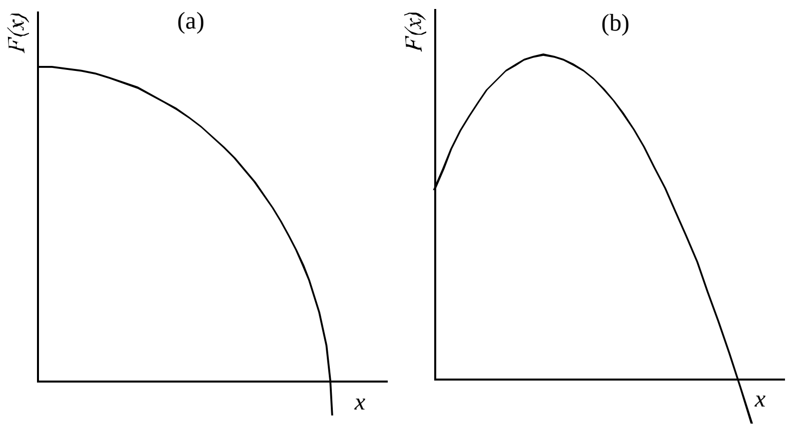

Figure 3.1: Illustration of the conditions (2) and (3) for a function *F(x)* in one variable *x*. In situation (a) a maximum exists for *x*=0, in situation (b) for *x*>0.

3.2 Equality constraint

The derivation of necessary optimality conditions for optimization problems where *equality constraints* have to be satisfied is greatly due to Lagrange (1736 - 1813). Assume that from $g(x) = 0$ one of the variables, say x_1, can be expressed in terms of the other variables $x_2,...,x_n$ and we can write

(3.5) $\quad x_1 = \varphi(x_2, x_3,..., x_n).$

The function φ is assumed to be differentiable with respect to $x_2,\ x_3,\ \ldots,\ x_n$. The maximization problem (ii) is equivalent to:

$$\text{Max } \{ F(x_1, x_2,\ldots,x_n) \mid x_1 = \varphi(x_2,\ldots,x_n) \},$$

which is a maximization problem in the n-1 variables $x_2,\ x_3,\ \ldots,\ x_n$. Substituting $x_1 = \varphi(x_2,\ldots,x_n)$ in $F(x)$, necessary conditions for $x_2,\ \ldots,\ x_n$ to be optimal are:

(3.6) $$\frac{\partial F}{\partial x_1}\frac{\partial \varphi}{\partial x_j} + \frac{\partial F}{\partial x_j} = 0, \ j = 2, ...,n.$$

If φ is known, then (3.6) is a set of n-1 equations, from which the values of the n-1 variables $x_2,\ \ldots,\ x_n$ have to be determined. If φ is not known, then we can proceed as follows. Since

$$g(\varphi(x_2, x_3,..., x_n), x_2,\ldots,x_n) = 0$$

for all values of $x_2, ..., x_n$, it may be written

(3.7) $$\frac{\partial g}{\partial x_1}\frac{\partial \varphi}{\partial x_j} + \frac{\partial g}{\partial x_j} = 0, \ j = 2, ...,n.$$

Assume that in the solution of the maximization problem (ii) $\frac{\partial g}{\partial x_1} \neq 0$ (otherwise in the solution would hold, see (3.7), that all $\frac{\partial g}{\partial x_j} = 0$). So, it follows that

$$\frac{\partial \varphi}{\partial x_j} = -\frac{\partial g}{\partial x_j} \bigg/ \frac{\partial g}{\partial x_1}$$

So (3.6) may be written as:

$$(3.8) \qquad -\frac{\partial F}{\partial x_1}\frac{\partial g}{\partial x_j} \bigg/ \frac{\partial g}{\partial x_1} + \frac{\partial F}{\partial x_j} = 0,\ j = 2,\ldots,n.$$

All terms in (3.8) are functions of the n variables $x_1, \ldots, x_n$. The solution of (ii) can be found by solving the (n-1) equations (3.8) and the constraint $g(x)=0$.

Define

$$(3.9) \qquad \lambda = -\frac{\partial F}{\partial x_1} \bigg/ \frac{\partial g}{\partial x_1}$$

then the necessary conditions for the solution of (ii) may be rewritten as - see (3.8) and (3.9):

$$(3.10) \qquad \frac{\partial F}{\partial x_j} + \lambda \frac{\partial g}{\partial x_j} = 0,\ j = 1,2,\ldots,\ n \text{ and}$$

$$(3.11) \qquad g(x) = 0$$

Solving ($x_1, x_2, \ldots, x_n$) from (3.8) and (3.11) is equivalent to solving ($x_1, x_2, \ldots,$ x_n) and λ from (3.10) and (3.11). Lagrange has shown that the conditions (3.10) and

(3.11) can also be written in a different way. He introduced a function, which became later know as the Lagrangean function defined by:

(3.12) $L(x,\lambda) = F(x) + \lambda g(x)$.

Note that the function $L(x,\lambda)$ is a function of both the variables $x = x_1, x_2, \ldots, x_n$ and λ.

The conditions (3.10) and (3.11) may be written as

(3.13) $\dfrac{\partial L}{\partial x_j} = 0,\ j = 1,2,\ldots,n$

and

(3.14) $\dfrac{\partial L}{\partial \lambda} = 0$

So the conditions for optimality of the solution of (ii) are the same, if in (ii) the function $F(x)$ is replaced by the 'Lagrangean' function given by (3.12) and the new optimization problem is considered as a problem in the variables $x_1, x_2, \ldots, x_n$ and λ. The coefficient λ is called the multiplier of Lagrange.

For all feasible points x (i.e. satisfying $g(x)=0$) the value of the Lagrangean function (3.12) equals the value of $F(x)$.

If in the maximization problem (ii) the function $F(x)$ is concave, then the conditions (3.10) and (3.11) or (3.13) and (3.14) do not necessarily imply that a global maximum is found. This can be illustrated by an example. Let $F(x) = -x_1^2 - x_2^2$ be maximized given the equality constraint $g(x) = -20x_2 - (5x_1 - 6)^2 + 36 = 0$. The function $F(x)$ is concave. The point $(2,1)$ satisfies the conditions (3.10) and (3.11) as easily can be verified. However, the global maximum is found for the point $(0,0)$. Note that concavity or convexity of the constraint function $g(x)$ does not matter. Let $F(x)$ be concave and $g(x)$ a linear function, so $g(x) = a_0 + \sum_{j=1}^{n} a_j x_j^*$. In a point x satisfying (3.10) and (3.11) $F(x)$ takes a global maximum. This follows from the following reasoning. Let x^* be any point satisfying $g(x^*) = a_0 + \sum_{j=1}^{n} a_j x_j^*$. Due to property (3.1) and conditions (3.10) and (3.11) it may be written $F(x^*) - F(x) \leq -\lambda \sum_{j=1}^{n} \left(x_j^* - x_j\right) a_j$. So $F(x)$ is a global maximum indeed. If $F(x)$ is strictly concave and $g(x)$ is *linear*, then x satisfying (3.10) and (3.11) is the only point where $F(x)$ takes its global maximum.

3.3 Inequality constraints

We pass now to maximization problem (iii). By introducing a slack variable s, the maximization problem (iii) is equivalent to:

(3.15) max: { $F(x) \mid g(x) - s = 0, s \geq 0$ }

It will again be assumed that x_1 can be expressed in terms of $x_2, \ldots, x_n$ and s, so

(3.16) $x_1 = \tilde{\varphi}(x_2, ..., x_n, s)$

and that $\tilde{\varphi}$ is differentiable with regard to x_j, $j = 2, 3, \ldots, n$. Analogous to (3.7) it may be written:

(3.17) $\dfrac{\partial g}{\partial x_1}\dfrac{\partial \tilde{\varphi}}{\partial x_j} + \dfrac{\partial g}{\partial x_j} = 0, \; j = 2,3, ...,n.$

(3.18) $\dfrac{\partial g}{\partial x_1}\dfrac{\partial \tilde{\varphi}}{\partial s} - 1 = 0$

Defining $\tilde{F}(x_2, x_3, ..., x_n, s) = F(\tilde{\varphi}(x_2, ..., x_n, s), x_2, ..., x_n)$ we may replace (3.15) by

(3.19) $\max \{ \tilde{F}(x_2, x_3, ..., x_n, s) \mid s \geq 0 \}$

Referring to (3.2) - (3.4), necessary conditions of optimality are:

(3.20) $\dfrac{\partial \tilde{F}}{\partial x_j} = \dfrac{\partial F}{\partial x_1} \dfrac{\partial \tilde{\varphi}}{\partial x_j} + \dfrac{\partial F}{\partial x_j} = 0, \quad j = 2, \ldots, n$

(3.21) $\dfrac{\partial \tilde{F}}{\partial s} s = \dfrac{\partial F}{\partial x_1} \dfrac{\partial \tilde{\varphi}}{\partial s} s = 0$

(3.22) $\dfrac{\partial \tilde{F}}{\partial s} = \dfrac{\partial F}{\partial x_1} \dfrac{\partial \tilde{\varphi}}{\partial s} \leq 0$

(3.23) $s \geq 0$

Making use of (3.17) and (3.18) and assuming that $\frac{\partial g}{\partial x_1} \neq 0$ these conditions may be written as:

$$-\frac{\partial F}{\partial x_1} \Big/ \frac{\partial g}{\partial x_1} \cdot \frac{\partial g}{\partial x_j} + \frac{\partial F}{\partial x_j} = 0, \quad j = 2, \ldots, n$$

$$s \cdot \frac{\partial F}{\partial x_1} \Big/ \frac{\partial g}{\partial x_1} = 0$$

$$\frac{\partial F}{\partial x_1} \Big/ \frac{\partial g}{\partial x_1} \leq 0$$

$$s \geq 0.$$

Defining λ by (3.9) and because $s = g(x)$, these conditions may be written as:

(3.24) $\dfrac{\partial F}{\partial x_j} + \lambda \dfrac{\partial g}{\partial x_j} = 0, \quad j = 1,2,\ldots,n$

(3.25) $g(x)\lambda = 0$

(3.26) $\lambda \geq 0$

(3.27) $g(x) \geq 0$

Making use of the Lagrangean (3.12) the conditions (3.24) - (3.27) may be rewritten as:

(3.28) $\dfrac{\partial L}{\partial x_j} = 0\,, \; j = 1,2,\ldots,n$

(3.29) $\lambda \dfrac{\partial L}{\partial \lambda} = 0$

(3.30) $\lambda \geq 0$

(3.31) $\dfrac{\partial L}{\partial \lambda} \geq 0$

These conditions are usually referred to as the *Kuhn-Tucker* conditions.

These conditions correspond to the necessary conditions of optimality of the Lagrangean function as function of both x and λ. In the optimal solution of (iii) the value of the Lagrangean function equals the maximal value of $F(x)$, due to (3.25). It follows from (3.9), (3.18) and (3.22) that

(3.32) $\lambda = -\dfrac{\partial \tilde{F}}{\partial s}$

is the decrease of the value of the objective function, if s increases with one unit. So λ corresponds to the *opportunity costs* and -λ to the *shadow price*. If $s = g(x) > 0$ then $\lambda = 0$ due to (3.25).

If $F(x)$ is concave and $g(x)$ is concave, then the conditions (3.24) - (3.27) imply that the function $F(x)$ takes in x its global maximum value. Consider any x^* satisfying $g(x^*) \geq 0$ then due to property 3.1 and the conditions (3.24) - (3.27) we may write

$$F(x^*) - F(x) \leq \sum_{j=1}^{n} (x_j^* - x_j) \cdot \frac{\partial F}{\partial x_j} = -\sum_{j=1}^{n} \lambda \cdot (x_j^* - x_j) \cdot \frac{\partial g}{\partial x_j} \leq \lambda \cdot (g(x) - g(x^*)) \leq 0$$

So in point x satisfying (3.24) - (3.27) $F(x)$ takes its global maximum. If $F(x)$ and $g(x)$ are strictly concave the solution x is unique.

3.4 Equality and inequality constraints

For the optimal solution of the general non-linear programming problem (iv):

(iv) max $\{F(x) \mid g_i(x) = 0,\ i = 1,2,\ldots,m_1;\ g_i(x) \geq 0,\ i = m_1+1, m_1+2,\ldots,m;$
$x_j \geq 0,\ j = 1,2,\ldots,n_1\}$

the necessary conditions can be formulated as follows. Introduce the vector λ consisting of m multipliers of Lagrange, $\lambda_1,\ \lambda_2,\ \ldots,\ \lambda_m$. The function of Lagrange $L(x,\lambda)$ is defined as:

(3.33) $$L(x,\lambda) = F(x) + \sum_{i=1}^{m} \lambda_i g_i(x)$$

Referring to the previous sections and to many handbooks of non-linear programming, see e.g. Hazell and Norton, 1986, Bazaraa et al, 1993, the necessary conditions can be formulated as:

(3.34) $\dfrac{\partial L}{\partial \lambda_i} = 0\,,$ $\qquad i = 1, 2, ..., m_1$

(3.35) $\lambda_i \dfrac{\partial L}{\partial \lambda_i} = 0,$ $\qquad i = m_1+1,\ m_1+2, ..., m$

(3.36) $\lambda_i \geq 0\,,$ $\qquad i = m_1+1,\ m_1+2, ..., m$

(3.37) $\dfrac{\partial L}{\partial \lambda_i} \geq 0\,,$ $\qquad i = m_1+1,\ m_1+2, ..., m$

(3.38) $x_j \dfrac{\partial L}{\partial x_j} = 0,$ $\qquad j = 1,2,\ldots,n_1$

(3.39) $\dfrac{\partial L}{\partial x_j} \leq 0\,,$ $\qquad j = 1,2,\ldots,n_1$

(3.40) $x_j \geq 0\,,$ $\qquad j = 1,2,\ldots,n_1$

(3.41) $\dfrac{\partial L}{\partial x_j} = 0\,,$ $\qquad j = n_1+1, n_1+2,\ldots,n$

In the optimal solution of (iv) the value of the Lagrangean function equals the value of F due to (3.34) and (3.35). For the inequality constraints λ_i refers to the opportunity costs of constraint i.

Next to the non-negativity constraint $x_j \geq 0$, in the next chapters also the constraints $x_j \leq a_j$ will play an important role. These constraints can be taken into account as new inequality constraints. In that case $g_j(x_j) \geq 0$ in (iv) is written as $g_j(x_j) = a_j - x_j \geq 0$. A Lagrange multiplier λ_j can be introduced for this constraint, and the optimal solution has to satisfy the necessary conditions (3.35) - (3.37). It is, however, simpler not to introduce a new inequality constraint and a Lagrange multiplier λ_j, but to deal with the lower and upper bounds: $0 \leq x_j \leq a_j$, by replacing (3.38) - (3.40) by:

(3.42) if $0 < x_j < a_j$ then $\dfrac{\partial L}{\partial x_j} = 0,$ $\qquad j = 1,2,\ldots,n_1$

(3.43)   if $x_j = 0$ then $\dfrac{\partial L}{\partial x_j} \le 0$, $j = 1,2,\dots,n_1$

(3.44)   if $x_j = a_j$ then $\dfrac{\partial L}{\partial x_j} \ge 0$, $j = 1,2,\dots,n_1$

Without the upper bound $x_j \le a_j$, (3.42) and (3.43) follow from (3.38) - (3.40). With $x_j \le a_j$, (3.42) and (3.44) follow from (3.38) - (3.40), by writing $\xi_j = a_j - x_j$ and replacing in (3.38) - (3.40) x_j by ξ_j.

We return to (iv). Let $F(x)$ be concave, $g_i(x)$, $i=1,2,\dots,m_1$, linear functions and $g_i(x)$, $i = m_1+1, m_1+2,\dots,m$ concave functions. If a point $x=x_1,x_2,\dots,x_n$ and λ satisfy the conditions (3.34) - (3.41) then the function $F(x)$ takes in point x its global maximum. This follows in a similar way as derived in the previous sections from the following reasoning. Let x and λ satisfy (3.34) - (3.41). Consider any feasible point $x^* \neq x$ with $g_i(x^*)=0$, $i=1,2,\dots,m_1$; $g_i(x^*)\ge 0$, $i=m_1+1,m_1+2, \dots,m$; $x_j^* \ge 0$, $j=1,2,\dots,n_1$. Making use of property (3.1) and of (3.33) it may be written:

$$F\left(x^*\right) - F(x) \le \sum_{j=1}^{n}\left(x_j^* - x_j\right)\frac{\partial F}{\partial x_j} = \sum_{j=1}^{n}\left(x_j^* - x_j\right)\left(\frac{\partial L}{\partial x_j} - \sum_{i=1}^{m}\lambda_i \frac{\partial g_i}{\partial x_j}\right)$$

Due to (3.38) and (3.41), to the linearity of the $g_i(x)$, $i=1,2,\dots,m_1$ and to the concavity of the functions $g_i(x)$, $i=m_1+1,m_1+2,\dots,m$, see also (3.1), it follows that

$$F\left(x^*\right) - F(x) \le \sum_{j=1}^{n} x_j^* \frac{\partial L}{\partial x_j} + \sum_{i=m_1+1}^{m}\lambda_i\left(g_i(x) - g_i\left(x^*\right)\right)$$

Due to $x_j^* \ge 0$, $j=1,2,\dots,n_1$, (3.39) and (3.41), (3.35), (3.36) and $g_i(x^*)\ge 0$, $i=m_1+1,m_1+2,\dots,m$, it follows that $F(x^*) - F(x) \le 0$.

So the function $F(x)$ takes its global maximum in the point x satisfying (3.34) - (3.41). If $g_i(x_i)$, $i = 1,2,\ldots,m_1$, are linear functions, $F(x)$ is strictly concave, and $g_i(x)$, $i = m_1+1, m_1+2, \ldots, m$ are concave, the point x, where $F(x)$ takes its global maximum, is unique.

3.5 Stochastic programming

In Chapter 7 we will set up some stochastic programming models, in which market actors decide sequentially on their optimal strategies, taking into account the uncertain character of future prices. The sequential decision process is modelled using so-called recourse models. In these recourse models the objective functions contain the expected costs and revenues of future decisions which depend on random future prices. Furthermore, the right hand side values of the constraints depend on realisations of the random future prices. Probability distributions of random future prices are assumed to be known. In this section the structure of these models is briefly discussed.

Consider a time horizon of T periods, and introduce for the periods $t \in \{1,\ldots,T\}$ the following vectors:

(3.45) x_t vector of decision variables, corresponding to the decisions taken in period t

(3.46) P_t vector of random variables, corresponding to the uncertain prices in period t

The vector x_0 contains as parameters the initial values of the decision variables. In each period $t \in \{1,\ldots,T\}$, optimal values of x_t are determined. They depend on the decisions x_{t-1} taken in the previous period,[4] on the observed realisations of P_t, written

[4] Without loss of generality, in this introductory section it is assumed that x_t does only depend on x_{t-1} rather than on $x_0, x_1,\ldots,x_{t-1}$. In many recourse models – as in Chapter 7 – x_t depends on a ‘state variable’ which is a function of $x_0, x_1,\ldots, x_{t-1}$. A stock level is a typical example of such a state variable.

as p_t, and on the expected future revenues which depend on the distribution of the random future prices $P_{t+1},\ldots,\ P_T$. Simultaneously, for each possible realisation of $P_{t+1},\ P_{t+2},\ldots,P_T$, optimal values of $x_{t+1},\ x_{t+2},\ldots,x_T$ are determined as well. Write this as $x_{t+1}(P_{t+1}),\ x_{t+2}(P_{t+2}),\ldots,x_T(P_T)$. These decisions on future strategies, which are expected to be optimal, are of a preliminary nature. They can be revised in period t+1, when the realisations of P_{t+1}, i.e. p_{t+1}, are observed, and new information comes available on the probability distribution of P_{t+2}. The decision structure and the deterministic and stochastic elements for the decision on x_1 are illustrated in Figure 3.2.

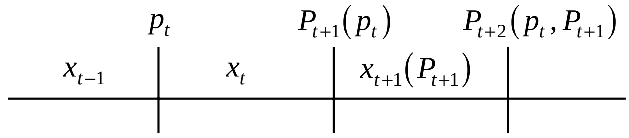

Figure 3.2: Recourse model: illustration of decisions taken in period 1: x_1 depending on x_0 and observed price p_1, preliminary decisions on $x_2,\ldots,x_T$ on random prices $P_2,\ldots,P_T$.

It is assumed that the random variables of P_t, for $t \in \{1,\ldots,T\}$, have a finite discrete probability distribution. Assume without loss of generality that $P_1,\ P_2,\ldots,\ P_T$, are independent random variables. In Chapter 9, the stochastic programming models will be reformulated for conditional probability distributions. Introduce for all $t \in \{1,\ldots,T\}$ the set K_t, containing the number of possible realisations of P_t. Define for $t \in \{1,\ldots,T\}$ the vector p_t^k, as the vector of possible outcomes of P_t for a $k \in K_t$. Define for each $t \in \{1,\ldots,T\}$:

(3.47) $\quad \Pr\left(P_t = p_t^k\right) = f_t^k, \qquad\qquad k \in K_t.$

with probabilities f_t^k satisfying $f_t^k \geq 0$ and $\sum_{k\in K_t} f_t^k = 1$.

For each period t, the values of the decision variables x_t and the preliminary decision variables $x_{t+1}(P_{t+1})$, $x_{t+2}(P_{t+2}),\ldots,x_T(P_T)$ are based on the maximization of an objective function, which consists of the net revenues in period t and the expected net revenues in future periods. The value of the objective function depends on the decisions taken in the previous period, x_{t-1}, and the observed value of p_t. In the theory of recourse models, the value of the objective function as a function of p_t and x_{t-1} is called the *value function*. Define $z_t(x_{t-1},p_t)$ the value function of the decision problem of period t. The decision problems discussed in Chapter 7 can in short be written as, for period $t \in \{1,\ldots,T\}$:

$$\text{(3.48)} \quad z_t(x_{t-1},p_t) = \underset{x_t}{Max}\left\{c_t(p_t,x_t) + Ez_{t+1}(x_t,P_{t+1}) \,\middle|\, W_t x_t + T_t x_{t-1} = h_t(p_t),\ x_t \geq 0\right\}$$

where $Ez_{t+1}(\cdot)$ refers to the expectation of $z_{t+1}(\cdot)$ with respect to P_{t+1}, and $z_{t+1}(\cdot)$ is the value function of the decision problem for period t+1.[5] We assume that $z_{T+1}(\cdot) = 0$. W_t and T_t are matrices, $h_t(p_t)$ is a vector depending on p_t, and $c_t(p_t,x_t)$ is the net revenue in period t. It is assumed that $c_t(p_t,x_t)$ is a function in x_t, some parameters in the function depend on p_t. The vector x_t contains the necessary slack variables, so that the constraints can be written as equalities. Define x_{t+1}^k the vector of preliminary decision variables in period t+1 for a price realisation p_{t+1}^k of P_{t+1}, for $k \in K_{t+1}$. For period t, a given value of decision variable x_t, and realisations p_{t+1}^k, $k \in K_{t+1}$, it may be written:

$$\begin{aligned} Ez_{t+1}(x_t,P_{t+1}) &= \sum_{k\in K_{t+1}} f_{t+1}^k z_{t+1}\left(x_t,p_{t+1}^k\right) \\ &= \sum_{k\in K_{t+1}} f_{t+1}^k \Bigg[\underset{x_{t+1}^k}{Max}\Big\{ c_{t+1}\left(p_{t+1}^k,x_{t+1}^k\right) + Ez_{t+2}\left(x_{t+1}^k,P_{t+2}\right) \\ &\qquad \Big| W_{t+1}x_{t+1}^k + T_{t+1}x_t = h_{t+1}\left(p_{t+1}^k\right),\ x_{t+1}^k \geq 0 \Big\} \Bigg] \end{aligned}$$

[5] $z_{t+1}(\cdot)$ refers to the corresponding expression $z_{t+1}(x_t, P_{t+1})$. $(\cdot)$ is introduced to simplify the notation, when no misunderstanding is possible.

$$(3.49) \qquad = \underset{x_{t+1}^k}{Max}\left\{ \sum_{k\in K_{t+1}} f_{t+1}^k \left[c_{t+1}\left(p_{t+1}^k, x_{t+1}^k\right) + Ez_{t+2}\left(x_{t+1}^k, P_{t+2}\right)\right] \middle| W_{t+1}x_{t+1}^k + T_{t+1}x_t = h_{t+1}\left(p_{t+1}^k\right),\ x_{t+1}^k \geq 0 \right\}$$

In (3.49) $Ez_{t+2}(\cdot)$ refers to the expectation of $z_{t+2}(\cdot)$ with respect to P_{t+2}, and $z_{t+2}(\cdot)$ is the value function of the decision problem for period t+2. It follows that the recourse problem (3.48) - (3.49) is equivalent to the following model:

$$(3.50) \qquad z_t\left(x_{t-1}, p_t\right) = \underset{x_t, x_{t+1}^k}{Max}\left\{ c_t\left(p_t, x_t\right) + \sum_{k\in K_{t+1}} f_{t+1}^k \left[c_{t+1}\left(p_{t+1}^k, x_{t+1}^k\right) + Ez_{t+2}\left(x_{t+1}^k, P_{t+2}\right)\right] \middle| W_t x_t + T_t x_{t-1} = h_t\left(p_t\right),\ W_{t+1}x_{t+1}^k + T_{t+1}x_t = h_{t+1}\left(p_{t+1}^k\right),\ x_t, x_{t+1}^k \geq 0 \right\}$$

For realisations p_{t+2}^l of P_{t+2}, for $l \in K_{t+2}$, and period t+1 decision x_{t+1}^k, $Ez_{t+2}(x_{t+1}^k, P_{t+2})$ can be written analogous to model (3.49).

As an illustration of the structure of the decision problems if a short time horizon of three periods is considered, i.e. T = 3, we write the three decision problems for the periods 3, 2, and 1:

$$(3.51) \qquad z_3\left(x_2, p_3\right) = \underset{x_3}{Max}\left\{ c_3\left(p_3, x_3\right) \middle| W_3 x_3 + T_3 x_2 = h_3\left(p_3\right),\ x_3 \geq 0 \right\}$$

$$(3.52) \qquad z_2\left(x_1, p_2\right) = \underset{x_2, x_3^k}{Max}\left\{ c_2\left(p_2, x_2\right) + \sum_{k\in K_3} f_3^k c_3\left(p_3^k, x_3^k\right) \middle| W_2 x_2 + T_2 x_1 = h_2\left(p_2\right),\ W_3 x_3^k + T_3 x_2 = h_3\left(p_3^k\right),\ x_2, x_3^k \geq 0,\ k \in K_3 \right\}$$

$$z_1(x_0, p_1) = \underset{x_1, x_2^k, x_3^{lk}}{Max} \left\{ c_1(p_1, x_1) + \sum_{k \in K_2} f_2^k \left(c_2(p_2^k, x_2^k) + \sum_{l \in K_3} f_3^l c_3(p_3^l, x_3^{lk}) \right) \right.$$

$$(3.53) \qquad \left| W_1 x_1 + T_1 x_0 = h_1(p_1),\ \ W_2 x_2^k + T_2 x_1 = h_2(p_2^k), \right.$$

$$\left. W_3 x_3^{lk} + T_3 x_2^k = h_3(p_3^l),\ x_1, x_2^k, x_3^{lk} \geq 0,\ k \in K_2, l \in K_3 \right\}$$

In (3.53) x_3^{lk} represents the preliminary decision variable x_3 in period 3 for price realizations p_2^k and p_3^l in period 2 and 3.

If, for t = 1,2,3, x_t is a n-dimensional vector, W_t and T_t $m \times n$-dimensional matrices, $h_t(p_t)$ an m-dimensional vector depending on p_t, and the set K_t contains k_t elements, then model (3.53) is a model with $n(1+k_2(1+k_3))$ decision variables and $m(1+k_2(1+k_3))$ constraints. These models are in fact large scale mathematical programming models of the form:

$$(3.54) \qquad \underset{y}{Max} \left\{ c(y) \middle| Wy = h,\ y \geq 0 \right\}$$

with y a vector of decision variables, $c(y)$ a function in y, W a matrix, and h a vector.

Optimal solutions of the recourse models

To derive some properties of the optimal solution of these models, the same methods can be used as discussed in the previous sections. As an illustration, we discuss the Kuhn-Tucker conditions for model (3.52). For the other models, the approach is similar. Introduce the vectors λ_1, λ_2^k, for $k \in K_2$, consisting of the Lagrange multipliers of the constraints of model (3.52). Define $L\left(x_1, x_2^k, \lambda_1, \lambda_2^k \middle| k \in K_2\right)$ the Lagrange function of (3.52) as a function of $x_1, x_2^k, \lambda_1, \lambda_2^k$ for all $k \in K_2$:

$$
(3.55)\quad \begin{aligned} L\left(x_1, x_2^k, \lambda_1, \lambda_2^k \,\middle|\, k \in K_2\right) &= c_1(p_1, x_1) + \sum_{k \in K_2} f_2^k c_2\left(p_2^k, x_2^k\right) + \\ &+ \lambda_1{}^T\left[W_1 x_1 + T_1 x_0 - h_1(p_1)\right] + \sum_{k \in K_2} \lambda_2^k{}^T\left[W_2 x_2^k + T_2 x_1 - h_2\left(p_2^k\right)\right] \end{aligned}
$$

Recall that, for $t \in \{1,2\}$ and $k \in K_2$, x_1, x_2^k are n-dimensional vectors, the matrices W_t and T_t are of dimension $m \times n$, and h_t are m-dimensional vectors. The multipliers λ_1 and λ_2^k are m-dimensional. Define x_{j1} the jth element of the vector x_1, for $j \in \{1,\ldots,n\}$. x_{j2}^k and $\lambda_{i1}, \lambda_{i2}^k$, for $i \in \{1,\ldots,m\}$, $j \in \{1,\ldots,n\}$ are defined analogously. W_{ij1} is defined as the element on the ith row and jth column of the matrix W_1, for $i \in \{1,\ldots,m\}$, $j \in \{1,\ldots,n\}$. W_{ij2}, T_{ij1}, and T_{ij2} are defined analogously. Furthermore, define W_{j1} as the jth column of the matrix W_1, for $j \in \{1,\ldots,n\}$. W_{j1} is a m-dimensional vector. W_{j2}, T_{j1}, and T_{j2} are defined analogously. Referring to (3.38) – (3.40), the necessary conditions of optimality can be written as:

$$
(3.56)\quad x_{j1}\frac{\partial L}{\partial x_{j1}} = 0, \qquad \frac{\partial L}{\partial x_{j1}} \le 0, \qquad x_{j1} \ge 0 \qquad j \in \{1,\ldots,n\}
$$

$$
(3.57)\quad x_{j2}^k\frac{\partial L}{\partial x_{j2}^k} = 0, \qquad \frac{\partial L}{\partial x_{j2}^k} \le 0, \qquad x_{j2}^k \ge 0 \qquad j \in \{1,\ldots,n\}, k \in K_2
$$

It follows that for $j \in \{1,\ldots,n\}$, $k \in K_2$:

$$
(3.58)\quad \begin{cases} \text{if } x_{j1} = 0 & \text{then } \dfrac{\partial c_1}{\partial x_{j1}}(p_1, x_1) + \lambda_1{}^T W_{j1} + \displaystyle\sum_{k \in K_2} \lambda_2^k{}^T T_{j2} \le 0 \\ \text{if } x_{j1} > 0 & \text{then } \dfrac{\partial c_1}{\partial x_{j1}}(p_1, x_1) + \lambda_1{}^T W_{j1} + \displaystyle\sum_{k \in K_2} \lambda_2^k{}^T T_{j2} = 0 \end{cases}
$$

$$
(3.59) \quad \begin{cases} \text{if } x_{j2}^k = 0 & \text{then } f_2^k \dfrac{\partial c_2}{\partial x_{j2}^k}\left(p_2^k, x_2^k\right) + \lambda_2^{k\,T} W_{j2} \leq 0 \\ \text{if } x_{j2}^k > 0 & \text{then } f_2^k \dfrac{\partial c_2}{\partial x_{j2}^k}\left(p_2^k, x_2^k\right) + \lambda_2^{k\,T} W_{j2} = 0 \end{cases}
$$

In the models discussed in Section 7.1, the function $c_t(p_t,x_t)$ is a linear function $c_t(p_t,x_t) = d_t(p_t)\cdot x_t$, with $d_t(p_t)$ a vector which is, without loss of generality, linear in p_t. In the models discussed in Section 7.2, the function $c_t(p_t,x_t)$ is a non-linear function. The functions $c_t(p_t,x_t)$ and matrices W_t and T_t will be such that λ_2^k, following from (3.59), can easily be substituted in (3.58). This results in a number of elegant properties indicating the influence of the expected future prices on the current optimal strategies (see Section 7.2).

4 Supply functions, demand functions and equilibrium

One of the objectives of building a spatial equilibrium model is to analyse the functioning of the agricultural market system and the rationale for government intervention on agricultural markets. The standard analysis of agricultural markets is based on the microeconomic analysis of the behaviour of agricultural producers and consumers. Producers are supposed to maximize profits and consumers to maximize utility. From these assumptions demand and supply can be derived as a function of prices. If markets are perfectly competitive, supply and demand will be in equilibrium and equilibrium prices and quantities can be generated using supply and demand functions. In standard economic theory it is usually assumed that producers sell all production. In developing countries, however, many farmers consume on-farm a large part of their own production. Therefore, production and consumption decisions are interrelated, and can not always be analysed separately. Household models can be applied to determine simultaneously production, consumption, sales and purchases of agricultural households.

This chapter deals in particular with supply and demand functions. Their derivation and properties will be shortly reviewed. Furthermore, the need to analyse simultaneously supply and demand decisions will be shortly discussed. Finally, some basic concepts of a market equilibrium will be discussed. For further reading on these subjects see e.g. Varian (1992) and Nicholson (1995).

4.1 Supply functions

In standard economic theory it is supposed that goods are produced by firms which maximize net profits, i.e. the difference between revenues received from selling the produce and production costs incurred. Consider a firm producing one good. Let p be the *given* price per unit, x the quantity to be produced and sold by the firm and $c(x)$ the costs of producing x. The question how much the firm should supply corresponds to determining x by solving:

(4.1) $\qquad W(p) = \underset{x}{Max}\left\{px - c(x) \mid x \geq 0\right\}$

$W(p)$ gives the firm's optimal profit as a function of prices, and is called the profit function. Usually, it is assumed that $c(x)$ is twice differentiable, and that it costs more to produce more, so $c'(x) > 0$, and that the costs to produce one unit extra are higher the more is produced, so

(4.2) $\qquad c''(x) > 0$

This last condition excludes economies of scale, so costs per unit can not be reduced if more is produced. If the cost function is differentiable and satifies $c'(x) > 0$ and $c''(x) > 0$, then the assumption of profit maximization induces that the profit function, $W(p)$, is non-decreasing, convex and continuous in output prices. Let x be the optimal production level. Given prices p, the firm can easily derive the optimal production x by solving (4.1). Call $F(x)=px - c(x)$, then necessarily holds, see (3.2) and (3.3):

$$x\frac{dF}{dx}(x) = 0, \quad \frac{dF}{dx}(x) \leq 0$$

So,

(4.3) $\qquad$ if $x = 0$, then $F'(0) = p - c'(0) \leq 0$

(4.4) $\qquad$ if $x > 0$, then $F'(x) = p - c'(x) = 0$

The solution $x = 0$, i.e. zero production, may be excluded by postulating, see (4.3):

(4.5) $\qquad p > c'(0)$

So the (interior) solution x satisfies, see (4.4):

(4.6) $p - c'(x) = 0$

This shows that in the optimum, marginal revenues equal marginal costs. Stated otherwise, profit is optimal if the revenues provided by the last unit sold, equal the costs of the last unit produced. The marginal revenues equal the product price. The optimization problem (4.1) gives for each price p a different optimal supply, x. x as a function of p can now be interpreted as the supply function. This function gives the firm's most profitable production plan x as a function of price p. Writing the supply function as $x(p)$, it follows from (4.6) that

$$p - c'(x(p)) = 0$$

Since $c(x)$ is differentiable, $x(p)$ is differentiable in p. It then follows from differentiating to p that:

$$1 - c''(x)\frac{dx}{dp} = 0$$

implying that supply on food markets increases when prices increase, i.e.

(4.7) $\frac{dx}{dp} = \frac{1}{c''(x)} > 0$, due to (4.2)

In economic analysis one often uses a measure for the responsiveness of supply to price changes. The *price elasticity of supply* measures the percentage change in supplied quantity as a result of a percentage change in the goods' price:

$$\varepsilon_p^s = \frac{dx}{dp}\frac{p}{x}$$

Due to (4.7), $\varepsilon_p^s > 0$. Property (4.7) is not at all evident. Farmers in developing countries who consume a large part of their production on-farm, are often obliged to sell a part of their harvest in order to repay debts or to pay for daily important expenses, even if they are in a food shortage situation. If a farmer needs a certain amount of money *m*, he may sell a quantity $x = m/p$, so $\frac{dx}{dp} < 0$, and $\varepsilon_p^s < 0$. This result differs from (4.7), since the objective function of such a farm household is different from the profit maximizing objective of the firm discussed in this section. Their objectives will be more concerned with satisfying household food security or maximizing household utility. In section 4.3 some short notes will be made on modeling household behaviour.

Example: Consider a quadratic cost function: $c(x) = ax + ½bx^2$, with $a>0,\ b>0$. If (4.5) is satisfied, $p>a$. For given price, *p*, profit can be written as: $F(x) = px - ax$ $½bx^2$, and the supply function can be derived by (4.6): $x = -a/b + p/b$.

4.2 Demand functions

In a similar way the demand function of an individual consumer consuming a number of goods is determined. In the analysis of consumer behaviour, it is studied how a consumer chooses what to consume if (s)he can choose between various goods with different prices and if (s)he is confronted with a limited income. Consumers have preferences on the consumption of different goods. Consider a situation with *k* different goods. Introduce the vector of consumed goods, $y = (y_1 \ldots y_k)$, with y_i the consumption of good *i*, $i=1,\ldots,k$. To the consumption of each bundle of goods, *y*, a level of satisfaction is associated, called utility. A continuous utility function, $u(y)$, can be defined, which orders the consumers' preferences. For each possible bundle of goods, *y*, consumers get a certain level of utility $u(y)$. In micro-economics it is usually supposed that a consumer always chooses the most preferred bundle of goods from the set of afordable alternatives. These alternatives depend on the available budget. Expenses to the purchase of bundle *y*, may not exceed the available budget *m*.

Let π be the vector of prices the consumer has to pay when he purchases the goods on the market. This is given for the consumer. Now the consumer problem of preference maximization can be defined as:

(4.8) $$v(\pi,m) = \underset{y}{Max}\left\{ u(y) \mid \pi y \leq m, y \geq 0 \right\}$$

where $v(\pi,m)$ is the indirect utility function. This function gives the maximum utility as a function of price π and income m. Usually it is assumed that $u'(y) > 0$ and $u''(y) < 0$. This means that utility increases if more is consumed, and that the increase of utility by consuming one extra unit decreases if consumption increases. The Lagrangian for the consumer problem can be written:

(4.9) $$L = u(y) + \lambda(m - \pi y),$$

with λ the Lagrange multiplier. Write the price of good i as π_i, $i = 1, ..., k$. Let y_i be the optimal demand of good i, $i = 1,\ldots,k$, and $y = (y_1,\ldots,y_k)$ be the vector of optimal demanded goods. If the utility function is differentiable, then the optimal solution of (4.8), y, has to satisfy the optimality conditions - see (3.34) - (3.41) :

$$y_i \frac{\partial L}{\partial y_i}(y) = 0\,, \quad \frac{\partial L}{\partial y_i}(y) \leq 0\,, \qquad y_i \geq 0,\ i = 1,\ldots,k$$

$$\lambda \frac{\partial L}{\partial \lambda}(y) = 0\,, \quad \frac{\partial L}{\partial \lambda}(y) \geq 0\,, \qquad \lambda \geq 0$$

Write $\frac{\partial u}{\partial y}(y) = u_i'(y)$. Then the above optimality conditions imply,

(4.10) $$\sum_{i=1}^{k} \pi_i y_i \leq m$$

(4.11) if $y_i = 0$, then necessarily $u_i'(0) - \lambda \pi_i \leq 0$, $i = 1,2,\ldots,k$

(4.12) if $y_i > 0$, then necessarily $u_i'(y) - \lambda\pi_i = 0$, $i = 1, 2, \ldots, k$

Assume that in the optimum of (4.8) $y > 0$ and $\pi\ y = m$, so that (4.10) and (4.12) have to be satisfied. Multiply (4.12) with y_i, sum over the number of goods, and fill in $\pi{\cdot}y = m$ to get the inverse demand function (i.e. the price as a function of demand and income):

$$\pi_i(y,m) = \frac{m u_i'(y)}{\sum_{j=1}^{k} u_j'(y) y_j} \tag{4.13}$$

Using the indirect utility function (4.8) it can also be shown that (see Varian, 1992:106, 149):

$$y_i(\pi,m) = \frac{\dfrac{\partial v(\pi,m)}{\partial \pi_i}}{\sum_{j=1}^{k} \dfrac{\partial v(\pi,m)}{\partial \pi_j}\pi_j} = -\frac{\dfrac{\partial v(\pi,m)}{\partial \pi_i}}{\dfrac{\partial v(\pi,m)}{\partial m}} \tag{4.14}$$

A type of utility function that is often used in applied economics is the *quasilinear utility function*. With this utility function, simple demand functions can be derived. A utility function, is quasilinear if it is linear in one of the goods, i.e. if it can be written as:

$$\tilde{u}(y_1, y_2, \ldots, y_k) = y_1 + u(y_2, \ldots, y_k)$$

Consider for simplicity a situation with 2 goods, y_0 and y, where the variable y_0 is the amount of 'money', and the variable y is the amount of cereals consumed. Suppose π is the (given) price for cereals. Note that y and π are not vectors in this example.

Suppose that the 'price' for money is 1, $\pi_0 = 1$. The consumer problem (4.8) can now be written as:

(4.15) $$\underset{y_0,y}{Max}\left\{y_0 + u(y) \mid \pi y + y_0 \leq m, y \geq 0, y_0 \geq 0\right\}$$

In the optimum, all income will be spent on cereals and money, $\pi y + y_0 = m$. If income is large, the consumer will consume good y until marginal utility of consuming y is smaller then marginal utility of consuming y_0, i.e. until $u'(y) < 1$. The remainder of income will be spent on consuming y_0. In this case the constraint may be substituted in the objective function. The problem may now be reduced to the maximisation problem:

(4.16) $$\underset{y}{Max}\left\{u(y) - \pi y \mid y \geq 0\right\}$$

In that case, the solution will be independent of m. If the problem is written in this way, it can be given a special interpretation which resembles the producer problem in section 3.1. Utility $u(y)$ may be interpreted as the 'revenues of consuming y' and $\pi \cdot y$ as the 'costs of consumption'. So, (4.16) conveys a situation in which 'revenues' minus 'costs' are maximized. Analogous to section 2.1, we find two classes of solutions, depending on whether the optimal demand $y > 0$ or $y = 0$. The solution of (4.15) has a convenient form, if we find an interior solution, $y > 0$:

(4.17) $$u'(y) = \pi$$

which simply says that the marginal utility of consumption is equal to the price of the good. This utility function, thus, results in a simple demand structure, and simplifies the analysis of market equilibrium. Note, however, that this only holds for large enough levels of income. If income is too low such that all income will be spent on consuming y, and y_0 is zero, (4.17) is not valid. Another feature of the quasilinear utility function is that the indirect utility function (4.8) can be written as (Varian, 1992: 154):

(4.18) $v(\pi,m) = v(\pi) + m$

This is a special case of the so-called Gorman form. In section 4.4 this will be further discussed. The demand function (4.14) for good y can now be written in the following convenient form:

(4.19) $$y^*(\pi,m) = \frac{\partial v(\pi)}{\partial \pi}$$

Example: Linear expenditure system

As an example of how demand functions can be derived, consider the often used utility function of the form:

$$u(y) = \sum_{i=1}^{k} a_i \ln(y_i - \gamma_i)$$

where $y_i > \gamma_i$. In this utility function k goods are considered and γ_i is the minimum consumption requirement of good i. The utility maximisation problem is:

$$v(\pi,m) = Max\ \ u(y) \qquad s.t.\ \pi y = m.$$

Solving this problem, see Section 3.2, gives the following demand function:

$$y_i = \gamma_i + a_i \frac{m - \sum_{i=1}^{k} \pi_i \gamma_i}{\pi_i}$$

(see Varian 1992, p. 212). This demand system is often used in applied economics. A drawback is, however, that it implies a linear relation between demand and income (linear Engel functions) and that it can at best be true over a short range of variation (Sadoulet and De Janvry, 1995). □

Next to a measure for the price responsiveness of supply, economists also use a measure for the responsiveness of demand to price or income changes. Analogous to the price elasticity of supply, the *price elasticity of demand* for a good *i* measures the percentage change of demand, y_i^*, after a percentage change of the goods price, π_i:

$$(4.20) \qquad \varepsilon_i^d = \frac{\partial y_i}{\partial \pi_i}\frac{\pi_i}{y_i}$$

Demand of a good *i* depends not only on its own price, but also on the price of the other goods *j*. So, the demand function has to be written as: $y_i = y_i(\pi_1,\ldots,\pi_i,\ldots,\pi_k)$, if *k* goods are considered. The cross-price elasticity of demand, ε_{ij}^d, measures the responsiveness of demand of good *i* after a price change of good *j*. Also the income elasticity, η_i, is often calculated. This measures the responsiveness of demand of good *i*, if income changes.

$$(4.21) \qquad \varepsilon_{ij}^d = \frac{\partial y_i}{\partial \pi_j}\frac{\pi_j}{y_i}$$

$$(4.22) \qquad \eta_i = \frac{\partial y_i}{\partial m}\frac{m}{y_i}$$

Goods can be categorized according to the signs and magnitudes of the elasticities. A good is a *normal good* if the goods' price elasticity of demand is negative. Demand is said to be *elastic* if $\varepsilon_i^d < -1$. This means that demand decreases more then proportionally if the price increases. If demand decreases less then proportionally after a price increase, i.e. if $-1 < \varepsilon_i^d < 0$, then demand is said to be *inelastic*. Most food crops have inelastic demand. If demand for a good decreases if its price decreases, i.e. $\varepsilon_i^d > 0$, we call the good a *Giffen good*. An example are potatoes in the Netherlands, or any other staple food crop which serves as base ingredient in daily meals. If the price of the basic food decreases, people have to spend less money on their basic expenditures, and consequently their purchasing power increases. Accordingly, they shift their consumption pattern to the consumption of less basic and more luxury foods, which are more appreciated (see Heijman et al., 1991). Two goods *i* and *j* are *gross complements* if demand of good *i* decreases if the price of good *j* increases, i.e. if $\varepsilon_{ij}^d < 0$. An example are tobacco and cigarette paper. Goods are *gross substitutes* of each other if demand of a good increases if the price of the other good increases, i.e. if $\varepsilon_{ij}^d > 0$. An example of two substitutes are maize and rice in West Africa. If the price of maize increases, maize consumption will be substituted by rice consumption. Furthermore, goods are called *normal goods* if the income elasticity, η_i is positive. The good is a *necessary* normal good if $0 < \eta_i < 1$, and a *luxury* normal good if $\eta_i > 1$. A good is an *inferior good* if $\eta_i < 0$. Examples of inferior goods are basic food crops such as potatoes in the Netherlands and millet and sorghum in some regions in West Africa. If income increases, people will shift their consumption to more luxury goods (Varian, 1992).

4.3 Seperability of supply and demand decisions

In the previous discussion it was supposed that producers sell all their produce, and consumers have to purchase all goods they consume. In developing countries, farmers often save a part of their harvest, to be consumed on-farm, and only a small part of production is sold. They integrate in the household decisions regarding production and consumption. For that reason, we can not allways estimate supply and demand functions the way we did above, but they have to be determined on an integrated way in a household model.

Such an integrated analysis is not necessary if consumer prices and producer prices are the same, and the markets function well. Then production and consumption decisions can be analysed separately. In that case households are indifferent between consuming their own produce or selling it to rebuy what they need for their own consumption. The value of the consumed goods will in both cases be the same. In the first case, if households consume their own produce, they still have to sell a part of the harvest to pay for the production costs. Suppose the household produces a quantity x^* of a consumption good, which costs $c(x^*)$. Let the price be p. To cover the production costs, the household has to sell a quantity $c(x^*)/p$, and they can consume a quantity $x^* - c(x^*)/p$. In the second case, if the household sells all produce and rebuys the consumption, the income from agriculture will be $m = px^* - c(x^*)$. Suppose no income is earned from other sources, then a quantity $m/p = x^* - c(x^*)/p$ can be purchased. This shows that consumption will be the same in both cases. In this case, a supply function, $x^*(p)$, can be derived analogous to section 4.1. Using this supply function income can be calculated, $m(p) = px^*(p) - c(x^*(p))$, which is an input in the consumer problem to determine demand.

However, if some markets fail, or if transaction costs exist, production, supply and consumption decisions are no longer separable. Define p and π to be producer and consumer prices of a good, respectively, with $p < \pi$. If producers consume, in this case, a part of their produce on farm, they can consume a quantity $x^* - c(x^*)/p$. On the other hand, if producers would sell all produce and rebuy their consumption, they could only consume $m/\pi = px^*/\pi - c(x^*)/\pi$. So, in order to reflect reality, production, supply, demand and consumption decisions have to be analysed simultaneously, which can be done in a household model. In such models it is supposed that a household optimizes utility. Consumed quantities may be partly purchased and partly self produced. Decisions are constrained by a money income constraint, labour time constraints and a production function that calculates production as a function of inputs (see e.g. Ellis, 1993; Sadoulet and de Janvry, 1995). The household allocates labour time of the members of the household between home production, wage work, and leisure. The money income constraint is determined by the time allocated to wage work, and revenues from own production. Many household models have been built. Maatman et al. (1996) recently built one for a representative household on the Central Plateau in Burkina Faso. Their linear programming model evaluated production, consumption, trade and storage decisions of one houshold. They analyzed, in a very detailed manner, the different production possibilities of a household, and included the household's consumption patterns. The model clearly showed the interdependance between production and consumption decisions for subsistence farmers in West Africa.

In spite of this, we suppose in the next chapters that supply and demand decisions can be taken seperately. This is defensible since: 1) Subsistence households in many developing countries safeguard a part of their harvest for own consumption; only a small part may be sold on the market (see also Section 8.1.3); 2) In many developing countries, supply and demand decisions in an extended family are taken by different persons, with each their own, sometimes conflicting, objectives (see e.g. Maatman et al., 1996).

4.4 Equilibrium

In section 4.1 and 4.2 supply and demand functions for a single good have been derived, that represent the producer's and consumer's reaction on prices. Individual consumers and producers have no influence on prices, but the total supply and demand of all consumers and producers together certainly influence prices. Consider a situation with k consumers and n producers. Suppose that markets are perfectly competitive, and that supply and demand decisions can be analyzed separately. The functions $x_i(p)$ and $y_j(\pi, m_j)$ render the optimal supply and demand of for example cereals, for a producer i and consumer j, for a certain producer price p, consumer price π, and income level m_j, for $i = 1, ..., n$, $j = 1, ..., k$. Total supply on the market is the sum of all supplies. Define the market supply function $x(p) = \sum_{i=1}^{n} x_i(p)$.[6] Since each producer chooses a production level such that marginal costs equal the price (see

[6] Note the different denotation of the variables x, x_i, y, and y_j in this section and the Sections 4.1 and 4.2.

(4.6)), each producer must have the same marginal costs at price p, $c_i'(x_i^*) = p$. The market demand function can be defined as $y(\pi, m_1, ..., m_k) = \sum_{j=1}^{k} y_j(\pi, m_j)$. In general this demand function does not have properties like (4.14). It can be proven that the aggregate demand function has the same properties as the individual demand functions, if the indirect utlity function is of the *Gorman form*. This means that it can be written as: $v_j(\pi, m_j) = a_j(\pi) + b(\pi)m_j$, and aggregate indirect utility is: $v(\pi, m) = \sum_{j=1}^{k} a_j(\pi) + b(\pi)m_j$, with $m = \sum_{j=1}^{k} m_j$. The quasilinear utility function is a special case of the Gorman form, for which $b(\pi) = 1$ (see (4.18)).

Producers and consumers can sell and purchase all supply and demand only if the market is in equilibrium. A market equilibrium in a closed economy can be defined as follows: prices, p and π, and levels of supply and demand, x and y, exist for which market demand equals market supply, so $x(p) = y(\pi)$, such that no producer or consumer does have the tendency to change his decisions. This implies that the producer will supply $x_i(p)$ and the consumer will demand $y_j(\pi, m_j)$. One condition for such an equilibrium to exist is that exit from or entrance to the market are free.

We consider two situations:

(i) Producers and consumers sell and buy on one market. There is one market price, so $p = \pi$. This postulate together with the equilibrium condition determines the market price p, which follows from $x(p) = y(p)$. Note that no trader is explicitly introduced, and that no trading costs are involved.

(ii) A trader buys from the producers at a producer price p, and sells to the consumers at the consumer price π. What he buys is also sold. The consumer price is assumed to be an amount γ higher than the producer price, so $\pi = p + \gamma$. The equilibrium condition reads $x(p) = y(\pi)$. Both conditions together determine prices and produced and consumed quantities: $x(p) = y(p+\gamma)$.

Both conditions are illustrated in figure 4.1.

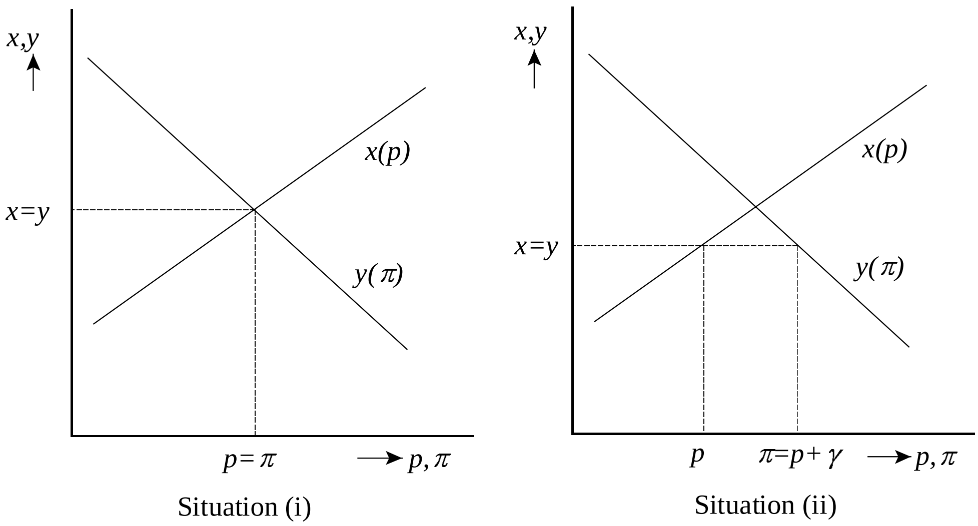

Figure 4.1: Illustration of equilibrium prices and produced and consumed quantities for situation (i) and (ii) as desribed in the text.

In stead of postulating $p = \pi$ in situation (i), we can also rewrite the consumer and producer problem. Suppose that producer i can produce a quantity of cereals x_i with costs $c_i(x_i)$, i = 1, ..., n. Consumer j can choose between consuming a quantity y_{0j} of good 0 (money) and a quantity of cereals y_j, j = 1, ..., k. Preferences for consumer j are ordered by a quasilinear utility function (see (4.15)):

$$u_j(y_j) + y_{0j}, \quad j = 1,\ldots,k$$

In stead of postulating market equilibrium to determine output, we maximize a *welfare function* which measures total consumer utility, subject to a money balance and a production balance:

$$
\text{(4.23)} \qquad \begin{aligned}
&\max \quad \sum_{j=1}^{k} u_j(y_j) + \sum_{j=1}^{k} y_{0j} \\
&s.t. \quad \sum_{i=1}^{n} c_i(x_i) = \sum_{j=1}^{k} m_j - \sum_{j=1}^{k} y_{0j} \\
&\qquad \sum_{i=1}^{n} x_i = \sum_{j=1}^{k} y_j \\
&\qquad x_i \geq 0, y_j \geq 0, y_{0j} \geq 0, \quad i = 1,\dots,n;\ j = 1,\dots,k
\end{aligned}
$$

If each m_j is supposed to be large, this problem may be rewritten as:

$$
\text{(4.24)} \qquad \begin{aligned}
&\max \quad \sum_{j=1}^{k} u_j(y_j) + \sum_{j=1}^{k} m_j - \sum_{i=1}^{n} c_i(x_i) \\
&s.t. \quad \sum_{i=1}^{n} x_i = \sum_{j=1}^{k} y_j \\
&\qquad x_i \geq 0, y_j \geq 0, \quad i = 1,\dots,n;\ j = 1,\dots,k
\end{aligned}
$$

If λ is the lagrange multiplier on the constraint, the optimal solution x_i, y_j of (4.24) has to satisfy, see also (3.12) and (3.13):

$$
\text{(4.25)} \qquad u_j'(y_j) = \lambda = c_i'(x_i), \ \ \forall i, j
$$

In section 4.1 and 4.2 we have seen that producers supply a quantity such that marginal costs equal the producer price, $c_i'(x_i) = p$, and consumers demand a quantity such that marginal utility equals the consumer price, $u_j'(y_j) = \pi$. So, (4.25) shows that welfare is optimal if $p = \pi$.

It is possible to rewrite the welfare function in (4.24) in a form which is common in economics. First we rewrite the first term in (4.24), the summation of individual utility. (4.17) showed that $u_j'\left(y_j\right)$ = π. Because the utility function is differentiable, we can introduce the inverse demand function for consumer j, $\pi_j(y_j)$. Suppose $u_j(0)$ = 0, then:

$$(4.26) \qquad u_j(y_j) = \int_0^{y_j} \pi_j(\xi)d\xi$$

By making use of $\pi = \pi(y_j)$ and of partial integration, the last term may be written as:

$$\int_0^{y_j} \pi_j(\xi)d\xi = \pi y_j - \int_0^{y_j} \xi\pi_j'(\xi)d\xi = \pi y_j + \int_\pi^\infty y_j(\vartheta)d\vartheta$$

Total utility from consuming cereals can be written as:

$$(4.27) \qquad \begin{aligned} &\sum_{j=1}^{k} u_j\left(y_j\right) = \sum_{j=1}^{k}\int_0^{y_j} \pi_j(\xi)d\xi = \sum_{j=1}^{k}\left[\pi y_j + \int_\pi^\infty y_j(\vartheta)d\vartheta\right] = \\ &\left[\pi y + \int_\pi^\infty \sum_{j=1}^{k} y_j(\vartheta)d\vartheta\right] = \left[\pi y + \int_\pi^\infty y(\vartheta)d\vartheta\right] = \int_0^{y} \pi(\xi)d\xi \end{aligned}$$

Furthermore, rewrite the last term in the objective function of (4.24), the summation of production costs. Suppose $c_i(0)=0$, and introduce the inverse supply function for producer i, $p_i(x_i)$. Given that $c_i'\left(x_i\right)$ = p - see (4.6) - for all $i=1,2,\ldots,n$, we can write:

$$(4.28) \qquad c_i(x_i) = \int_0^{x_i} p_i(\zeta)d\zeta$$

Since $x(p) = \sum_{i=1}^{n} x_i(p)$ we can write total production costs:

$$\sum_{i=1}^{n} c_i(x_i) = \sum_{i=1}^{n} \int_0^{x_i} p_i(\zeta) d\zeta = \sum_{i=1}^{n} \left[p x_i(p) + \int_0^{p} x_i(\eta) d\eta \right]$$

(4.29)

$$\left[px + \int_0^{p} \sum_{i=1}^{n} x_i(\eta) d\eta \right] = \left[px + \int_0^{p} x(\eta) d\eta \right] = \int_0^{x} p(\zeta) d\zeta$$

Because the summation of individual income, $\sum_{j=1}^{m} m_j$, has no influence on the solution of (4.24) it may be skipped from the formulation. Using (4.27) and (4.29), problem (4.24) can be rewritten as:

(4.30)

$$\underset{y_j, x_i}{Max} \left\{ \sum_{j=1}^{k} u_j(y_j) - \sum_{i=1}^{n} c_i(x_i) \;\middle|\; \sum_{j=1}^{k} y_j = \sum_{i=1}^{n} x_i,\, y_j \geq 0, x_i \geq 0 \right\} =$$

$$\underset{y,x}{Max} \left\{ \int_0^{y} \pi(\xi) d\xi - \int_0^{x} p(\zeta) d\zeta \mid y = x,\, y \geq 0, x \geq 0 \right\}$$

This gives the well known problem of maximising consumer plus producer surplus. Consumer surplus is defined as:

$$\int_0^{y} \pi(\xi) d\xi - \pi y$$

In fact, consumer surplus measures the difference between the optimal benefits from consuming *y* and the expenditures to purchase it. Producer surplus is defined as:

$$px - \int_0^{x} p(\zeta) d\zeta$$

This surplus reflects, in fact, the optimal profits from producing x. Since in the optimum $y = x$ and $p = \pi$, the terms $\pi \cdot y$ and $p \cdot x$ are the same, and the sum of consumer and producer surplus gives exactly (4.30). If producer plus consumer surplus is optimized, the solution is the same as the solution of problem (4.23). Samuelson (1952) was the first who showed that the surplus concept was relevant in converting the market equilibrium problem into an optimization problem (Van den Bergh et al., 1995b). The objective function of this problem is also called a *semi-welfare function*.

Note that this derivation only holds for a quasilinear utility function and when all m_j are large. If welfare is defined as aggregate utility, consumer plus producer surplus gives an exact measure of welfare only if the utility function is quasilinear. In other cases, it will not be an exact measure. However, it often gives a reasonable approximation to more precise but also more complicated measures. In the next chapters the semi-welfare function will play an important role.

5 Spatial equilibrium on *n* markets; one period model

In Chapter 4, we discussed one of the basic concepts of equilibrium models. That is, the market clears at a certain market price, which means that producer supply and consumer demand are in equilibrium. This price, and the corresponding supply and demand levels, are called equilibrium price and equilibrium quantities. In Section 4.4 we saw that a market equilibrium problem can be solved by writing it as an optimization problem, in which consumer plus producer surplus are maximised. This surplus is interpreted as ‘semi-welfare’. In the Chapters 5 and 6, this method is extended, to be able to take into account several regions (Chapter 5) and several periods (Chapter 6). The equilibrium models for competitive markets presented in the Chapters 5 and 6, correspond with the methods first formulated by Samuelson (1952), and discussed extensively and extended by Takayama and Judge (1971). Ever since, these methods have been applied frequently, especially for agricultural, energy and mineral resources problems (see e.g. Takayama and Judge, 1971; Judge and Takayama, 1973; Labys et al., 1989; Guvenen et al., 1990; Roehner, 1995; Van den Berg et al., 1995). In multi-region and multi-period-models, the concept of (price) equilibrium requires special attention.

Dealing with equilibrium on spatially seperated, competitive markets, Takayama and Judge use the term ‘Spatial Price Equilibrium’ (SPE) for a situation where prices and quantities satisfy the following properties: 1) in each region, there is only one market producer and one market consumer prices (p and π, see Chapter 4; i.e in any region prices are homogeneous and unique); 2) there is no excess demand or supply in any of the regions; and 3) commodities purchased in one region will only be transported to another region, to be sold there, if the difference between the consumer price in the importing region and the producer price in the exporting region is at least equal to transport costs (Van den Bergh et al., 1985, p.50, see also Takayama and Judge, 1971, p.34). An intertemporal SPE for multi-period equilibrium models, satisfies as well the following property (see Takayama and Judge, 1971, p. 378): 4) commodities purchased in a certain region, will only be stored, to be sold later in the same region, if the difference between the consumer price in the selling period and the producer price in the purchase period is at least equal to storage costs. For the multi-period situation, excess supply is possible in every period. The surplus will be stored, and sold in later periods. If a finite time horizon is considered, it is usually assumed that excess demand and supply are zero in the last period. This means that no stock remains after the last period. Takayama and Judge optimize in their equilibrium models ‘semi-welfare’, subject to supply-demand equilibrium on the market. The welfare optimal prices and quantities, satisfy the properties of a SPE. Takayama and Judge conclude from this that the models are suitable for analysing price formation on competitive markets. Why price formation on a competitive market can be decribed accurately by a SPE, usually receives little attention. Takayama and Judge only consider the behaviour of producers and consumers. Other market actors playing a role in market price formation, like traders, are not taken into account explicitly. The way prices are established on a market with traders, can, however, not be fully understood if only producers and consumers are considered. The process of price formation can be made more transparent, if traders are taken into account explicitly. Traders purchase goods from producers and sell to consumers. A market will clear, i.e. will be in equilibrium, because traders do only purchase from the producers the quantities they can sell to the consumers or store for sales in later periods. Furthermore, traders do only purchase from the producers, transport between the regions, store, and sell to the consumers, if prices are such that they make no losses.

In the next chapters, we will show that the economic foundations of the SPE and the equilibrium models of Takayama and Judge can be better comprehended, if also the behaviour of traders is considered explicitly. We will show that the results of the equilibrium models satisfy the profit maximizing behaviour of traders. The approach to deal explicitly with traders’ behaviour will in particular be useful in Chapter 7, in which the uncertain character of future prices is taken into account. In a situation of uncertain prices, the definition of price equilibrium as presented by Takayama and Judge is difficult to apply. By taking into account explicitly the behaviour of traders, we are able to analyse such situations.

In this chapter we discuss an extension of the method of Takayama and Judge (1971), to analyse cereal price formation and trade flows in a country where cereals are sold by producers, distributed by traders over a number of regions, and purchased by consumers. The quantities supplied, demanded and transported by individual producers, consumers and traders are a function of producer and consumer prices. In Section 5.1 we discuss the optimal strategies of the agents operating on the cereal market, if producer and consumer prices are known. In Section 5.2 we set up a spatial equilibrium model, which results in welfare optimal supply, demand and transport plans. We will show that the welfare optimal quantites are equal to the aggregate optimal sales, purchases and transport flows of the individual market agents, at market equilibrium prices. In Section 5.3, a different market situation is considered, in which a monopolistic trader determines market prices. A model is discussed to analyse this situation.

5.1 Strategies of producers, consumers and traders

Consider a situation in which an area of land (e.g. a country) is divided into n regions, which are numbered $i = 1, 2, .., n$. In each region is one market, numbered $i = 1, 2, ..., n$ as well. If a farmer produces cereals, part of it may be stored for home consumption, the rest is sold on the market. Farmers of region i sell only to traders at market i, not at other market places and not directly to consumers. At marketplace i, farmers get a kg-price p_i, called the *producer price* of region i. The total quantity of cereals sold by the producers of region i is called the *producer supply* (of cereals) in region i. Consumers in region i buy from traders at market i. They have to pay a kg-price π_i, called the *consumer price*. The quantity bought by the consumers of region i is called the *consumer demand* in region i. Traders in region i purchase the producer supply in this region, may transport cereals to regions $j = 1,...,n$, $j \neq i$, where they sell the consumer demand to the consumers.

Strategies of producers and consumers:

We define for $i = 1,...,n$:

	x_i	producer supply in region i
(5.1)	y_i	consumer demand in region i
	p_i	producer price in region i
	π_i	consumer price in region i

In analogy with Chapter 4 it is assumed that producer and consumer strategies are reflected by (aggregate) market supply and demand functions, which are given by $x_i(p_i)$ and $y_i(\pi_i)$. These functions give the producers' profit maximizing cereal supply at price p_i, and the consumers' utility optimizing demand at price π_i. We prefer to use here the inverse supply and demand functions $p_i(x_i)$ and $\pi_i(y_i)$ rather than $x_i(p_i)$ and $y_i(p_i)$. For each region supply and demand functions are assumed to be *known*. It is recalled that - see (4.26) and (4.28):

(5.2) $$u_i(y_i) = \int_0^{y_i} \pi_i(\xi) d\xi, \quad i=1,2,\ldots,n$$

(5.3) $$c_i(x_i) = \int_0^{x_i} p_i(\zeta) d\zeta, \quad i=1,2,\ldots,n$$

with

(5.4)	$u_i(y_i)$	utility of consumption of y_i by the consumers of region i
	$c_i(x_i)$	costs of producing x_i by the producers of region i

As in chapter 4 it is assumed that $\pi_i(y_i) > 0$ and $p_i(x_i) > 0$, due to the assumptions that $u_i'(y_i) > 0$ and $c_i'(x_i) > 0$, that the derivatives of $\pi_i(y_i)$ and $p_i(x_i)$ exist and that:

(5.5) $$\begin{aligned} &\pi_i'(y_i) < 0 \quad \text{due to the assumption that } u''(y_i) < 0 \\ &p_i'(x_i) > 0 \quad \text{due to the assumption that } c''(x_i) > 0 \end{aligned}$$

Some characteristics of traders' strategies

In the sections 5.1 and 5.2 it is assumed that the traders operate on a competitive market. They are all price followers who can not influence prices. The traders together are called here the aggregated trader, who operates on all n markets. Introduce the following variables for $i,j=1,...,n,\ i \neq j$:

	q_i	total quantity of produce purchased by the aggregated trader from the producers in region i
(5.6)	r_i	total quantity of produce sold by the aggregated trader to the consumers in region i
	q_{ij}	total amount of produce transported by the aggregated trader from region i to region j.

Assume that the (aggregated) trader does not want to have a stock left over, but that he wants to sell the entire purchase. This means that the quantity he purchases on a market i plus the quantity transported to this market, has to be equal to the quantity he sells on market i plus the quantity transported to other markets to be sold there. We call this the traders' equilibrium condition for region i:

$$(5.7)\qquad q_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ji} = r_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ij}$$

Knowing producer and consumer price levels, p_i and π_i, also producer supply and consumer demand levels are known, $x_i = x_i(p_i)$ and $y_i = y_i(\pi_i)$. The quantities the trader can purchase and sell on the market are bound by these supply and demand levels, $q_i \leq x_i$ and $r_i \leq y_i$. We define the parameter:

(5.8) τ_{ij} costs of transfer of one kg from market i to market j, $i,j=1,...,n,\ i \neq j$.

The costs of transfer, which obviously satisfy $\tau_{ij} > 0$, refer to transaction costs including transport costs, costs of insurance, contracts, taxes, information collection etc.. We define:

(5.9)
τ^* all transaction costs per unit of weight excluding transportation costs
τ_{ij}^* costs of transporting one unit of weight from market i to market j

and write:

(5.10) $\tau_{ij} = \tau^* + \tau_{ij}^*$

In (5.9) τ^* is assumed not to depend on i or j. Without loss of generality it is assumed here that $\tau^*=0$. So τ_{ij} refer to transportation costs only. The value of τ_{ij} depends on the mode of transport, for road transport on the size of trucks, on the distance between market i and j, conditions of roads, etc.. Estimates of τ_{ij} for a practical situation will be discussed in Chapter 8.2. The definition of τ_{ij} deserves further specification. τ_{ij} is often defined as the costs for the shortest route between market i and j. Here a different definition is adopted. τ_{ij} is defined as the minimum costs of transport between market i and market j, i.e. if the *cheapest* way of transport is chosen. In transport models, especially in industrialized countries, it is often assumed that transport costs $\tau_{ij} = c \cdot d_{ij}$ where d_{ij} is the distance between towns i and j and c the costs per km. Then the cheapest τ_{ij} corresponds to the shortest distance between town i and j. In developing countries the situation can be different. Taking the road with the shortest distance between markets i and j is not necessarily the cheapest way of transport, for instance if the direct road between market i and j is a dirt road in bad condition and costs can be reduced by taking a longer tarmac road. In this chapter the definition of τ_{ij} as costs of transport for the cheapest way of transport will play an important role. It follows from this definition that for any three different markets, i, j and s - see Figure 5.1 - minimum costs of transport of one unit of weight between market i and market j can never exceed the costs of transport if the route is taken from market i via market s to market j. So it may be written: $\tau_{ij} \leq \tau_{is} + \tau_{sj}$, $i \neq j$, $i \neq s$, $j \neq s$.

If for the regions i,j and s, $i \neq j$, $i \neq s$, $j \neq s$, $\tau_{ij} = \tau_{is} + \tau_{sj}$, a trader is indifferent between transporting directly from region i to j, or to transport first from region i to s and later from region s to j. The costs will for both possibilities be the same. In order to avoid this situation and to simplify the mathematical reasoning and proofs later in this chapter, it will be assumed that the $\leq$ in the triangle equation above may be replaced by $<$, so:

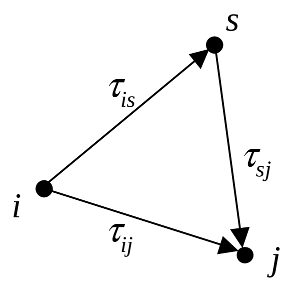

Figure 5.1: Schematic representation of three markets i, j and s with the corresponding costs of transportation. Two situations are illustrated: direct transportation from market i to market j and transport via market s.

(5.11) $\quad \tau_{ij} < \tau_{is} + \tau_{sj}, \quad i \neq j, i \neq s, j \neq s.$

The trader's objective is to maximize profits from cereal purchases, transport and sales. We are interested in the optimal levels of r_i, q_i, and q_{ij}, if p_i, π_i, x_i and y_i, are *known*. To show how a trader's decisions depend on producer and consumer prices, consider the following decision problem, in which he maximizes his profits subject to equilibrium conditions and upper bounds:

$$(5.12) \qquad \underset{r_i, q_i, q_{ij}}{Max} \left\{ \sum_{i=1}^{n} \left(\pi_i r_i - p_i q_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ij} q_{ij} \right) \middle| \; q_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ji} = r_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ij}; \right.$$
$$\left. 0 \leq q_i \leq x_i;\ 0 \leq r_i \leq y_i\ ;\ q_{ij} \geq 0;\ i, j = 1, \ldots, n,\ j \neq i \right\}$$

Introduce λ_i the Lagrange multiplier of the equilibrium condition (5.7) in model (5.12) – see also (3.33). The optimal quantities q_i, r_i, and q_{ij}, have to satisfy equilibrium condition (5.7) and the following conditions– see also (3.42) - (3.44):

$$\begin{array}{llll} & \text{if } q_i = 0 \text{ then } \lambda_i \le p_i; & \text{if } 0 < q_i < x_i \text{ then } \lambda_i = p_i; & \text{if } q_i = x_i \text{ then } \lambda_i \ge p_i; \\ (5.13) & \text{if } r_i = 0 \text{ then } \lambda_i \ge \pi_i; & \text{if } 0 < r_i < y_i \text{ then } \lambda_i = \pi_i; & \text{if } r_i = y_i \text{ then } \lambda_i \le \pi_i; \\ & \text{if } q_{ij} = 0 \text{ then } \lambda_j \le \lambda_i + \tau_{ij}; & \text{if } q_{ij} > 0 \text{ then } \lambda_j = \lambda_i + \tau_{ij} & \end{array}$$

From this, we can derive the following properties, which show the influence of the difference between producer and consumer price levels, on purchased, sold and transported quantities.

Trader property 5.1: For each region $i \in \{1,...,n\}$:

a) If $\pi_i < p_i$, then any optimal solution of (5.12) satisfies: $q_i = 0$ or $r_i = 0$.

b) If $\pi_i \ge p_i$, then an optimal solution of (5.12) exists which satisfies the condition $q_i = x_i$ or $r_i = y_i$. Nota bene: for $\pi_i > p_i$, any optimal solution of (5.12) has to satisfy this condition; for $\pi_i = p_i$, other optimal solutions may exist not satisfying this condition.

Proof: see Appendix 1.

Trader property 5.1 can be well understood. If $\pi_i < p_i$, the trader will certainly not purchase and sell in the same region, since he would only make losses out of this transaction. If $\pi_i > p_i$, it is obviously profitable for the trader to buy and sell in region i. In that case he will buy the maximum possible quantity, x_i, *or* sell the maximum possible quantity, y_i, in region i. We can not say that he will buy as much as possible from the producers in region i to sell to the consumers in the same region. This depends on producer and consumer prices in the other regions. It may be more profitable to sell in another region j. We come back to this issue after *Trader property* 5.3 below. If $\pi_i = p_i$, the only thing we can say, is that the trader would not loose if he would buy and sell in the same region.

Trader property 5.2: Let q_i, r_j, q_{ij}, $j \neq i$, $i,j = 1,\ldots,n$, be an optimal solution of (5.12). Let a trader transport from a region i to a region j, so $q_{ij} > 0$, for $i,j \in \{1,\ldots,n\}$, $i \neq j$, then:

a) no goods are transported from a region $s = 1,\ldots,n$, $s \neq i$, to region i, $q_{si} = 0$

b) no goods are transported from region j to a region $s = 1,\ldots,n$, $s \neq j$, $q_{js} = 0$.

c) purchases in region i are positive, $q_i > 0$

d) sales in region j are positive, $r_j > 0$.

Proof: see Appendix 1.

q_{ij} was defined – see (5.6) – as the amount transported from region i to region j. Trader property 5.2 implies that the quantity q_{ij} is purchased in region i and sold in region j.

Trader property 5.3: For the regions i and j, $i,j \in \{1,...,n\}$, $i \neq j$:

a) If $\pi_j < p_i + \tau_{ij}$, then any optimal solution of (5.12) satisfies $q_{ij} = 0$.

b) If $\pi_j \geq p_i + \tau_{ij}$ and $q_{ij} > 0$, then an optimal solution of (5.12) exists satisfying $q_i = x_i$ or $r_j = y_j$; for $\pi_j = p_i + \tau_{ij}$ and $q_{ij} > 0$ an optimal solution of (5.12) is not necessarily unique.

Proof: see Appendix 1.

Also *Trader property* 5.3 can be well understood. If $\pi_j < p_i + \tau_{ij}$, the trader can only make losses from transporting between region i and j. If $\pi_j = p_i + \tau_{ij}$, he would make neither losses nor profits if he would transport between region i and j. If $\pi_j > p_i + \tau_{ij}$, transport between i and j will be profitable. As a consequence, he will buy as much as possible in region i, x_i, or sell as much as possible in region j, y_j. Note that it is possible that $q_{ij} = 0$ if $\pi_j > p_i + \tau_{ij}$. If for example, $\pi_i - p_i > \pi_j - p_i - \tau_{ij} > 0$, selling in region i will be more profitable than selling in region j.

These three *Trader properties* will play an important role in the next section: solutions of the equilibrium models to be developed should not violate these properties, otherwise the found solutions would not be acceptable for the traders.

5.2 Maximization of welfare; perfect competition between traders

In this section we extend equilibrium model (4.30), to take into account transport between the different regions. We first discuss the set-up and results of the spatial equilibrium model, in which semi-welfare is optimized for all agents together. In this spatial equilibrium model optimal values of the following variables are determined for all regions $i = 1,\dots,n$: producer and consumer prices (p_i and π_i), producer supply (x_i, total quantity sold by the producers), consumer demand (y_i, total quantity purchased by consumers) and total transported quantities to the various regions. We define for $j = 1,\dots,n$ and $i \neq j$:

(5.14) $\quad x_{ij} \qquad$ total amount of produce transported from region i to region j.

This optimum is called the market equilibrium solution. Secondly, we show that this solution is sustained by the individual market agents. This means that the market equilibrium supplied, demanded and transported quantities, are equal to, respectively, the aggregate optimal sales of the individual producers, the aggregate optimal purchases of the individual consumers and the aggregate optimal transport flows of the traders, at the market equilibrium prices. These individual strategies have been discussed in the previous section.

In analogy with (4.30), in this section we maximize the sum of total utility minus all costs made, which has been defined as semi-welfare. The following maximization problem is solved:

$$(5.15) \quad \underset{y_i, x_i, x_{ij}}{Max} \underbrace{\sum_{i=1}^{n} u_i(y_i)}_{\text{Total utility}} - \underbrace{\sum_{i=1}^{n} c_i(x_i)}_{\text{Total producer costs}} - \underbrace{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ij} x_{ij}}_{\text{Total transportation costs}}$$

where the variables x_i, y_i, x_{ij}, i,j = 1,2, ..., n; $j \neq i$ have to satisfy the market equilibrium conditions -see also (5.7):

$$(5.16) \quad x_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ji} = y_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ij} \text{, for } i = 1,...,n$$

$$(5.17) \quad x_i \geq 0, y_i \geq 0, x_{ij} \geq 0, \qquad i, j = 1,2, ...,n; j \neq i.$$

The utility and cost functions $u_i(y_i)$ and $c_i(x_i)$ are given in (5.2) and (5.3). In principle, semi-welfare can also be defined as the sum of the 'net revenues' of the consumers, producer, and traders, with 'net consumer revenues' defined as the utility from consuming y_i (i.e. $u_i(y_i)$) minus the costs from purchasing y_i (i.e. $\pi_i \cdot y_i$). This definition of semi-welfare seems to be more appropriate than the definition in (5.15). In other words, semi-welfare is:

$$(5.18) \quad \underbrace{\sum_{i=1}^{n} \left(u_i(y_i) - \pi_i y_i\right)}_{\text{Net consumer revenues}} + \underbrace{\sum_{i=1}^{n} \left(p_i x_i - c_i(x_i)\right)}_{\text{Net producer revenues}} + \underbrace{\sum_{i=1}^{n} \left(\pi_i y_i - p_i x_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ij} x_{ij} \right)}_{\text{Net trader revenues}}$$

(5.18) is equal to (5.15). Because of the properties of utility and production costs, the objective function (5.15) can be replaced by the integral of the inverse demand function minus the integral of the inverse supply function minus transport costs – see (5.2) and (5.3). We arrive at the following maximization problem to be solved:

$$(5.19)\quad \underset{y_i,x_i,x_{ij}}{Max}\left\{\sum_{i=1}^{n}\left[\int_0^{y_i}\pi_i(\xi)d\xi-\int_0^{x_i}p_i(\zeta)d\zeta-\sum_{\substack{j=1\\j\neq i}}^{n}\tau_{ij}x_{ij}\right]\;\middle|\; x_i+\sum_{\substack{j=1\\j\neq i}}^{n}x_{ji} = y_i+\sum_{\substack{j=1\\j\neq i}}^{n}x_{ij};x_i,y_i,x_{ij}\geq 0,i,j=1,\ldots,n,j\neq i\right\}$$

Since the objective function in (5.15) is a linear combination with positive coefficients of concave functions - see (5.5) - the objective function is concave. Referring to the discussion at the end of section 3.4 it can easily be shown that a global maximum is found and that the optimal values of x_i, $p_i(x_i)$, y_i and $\pi_i(y_i)$ in the solution are unique. The values of x_{ij} are not necessarily unique. Introduce λ_i, for $i \in \{1,2, ...,n\}$, the Lagrange multipliers for the constraints (5.16). The Lagrangean function may be written as, see (3.33):

$$(5.20)\quad L\left(x_i,y_i,x_{ij},\lambda_i\middle|\, i,j\in\{1,\ldots,n\},j\neq i\right)= \sum_{i=1}^{n}\left[\int_0^{y_i}\pi_i(\xi)d\xi-\int_0^{x_i}p_i(\zeta)d\zeta-\sum_{\substack{j=1\\j\neq i}}^{n}\tau_{ij}x_{ij}\right]+\sum_{i=1}^{n}\lambda_i\left[x_i+\sum_{\substack{j=1\\j\neq i}}^{n}x_{ji}-y_i-\sum_{\substack{j=1\\j\neq i}}^{n}x_{ij}\right]$$

$L(x_i,y_i,x_{ij},\lambda_i|\ i,j \in \{1,\ldots,n\}, j \neq i)$ signifies the Lagrangian function as a function of x_i, y_i, x_{ij}, and λ_i, for all $i, j \in \{1,2, ...,n\}$, $j \neq i$. Let x_i, y_i, x_{ij}, $i, j \in \{1,2, ...,n\}$, $j \neq i$, be a solution of (5.19). From the Kuhn-Tucker optimality conditions, see (3.33) and (3.38) - (3.40), follows that:

(5.21) if $x_i > 0$ then $\dfrac{\partial L}{\partial x_i} = -p_i(x_i) + \lambda_i = 0$

(5.22) if $x_i = 0$ then $\dfrac{\partial L}{\partial x_i} = -p_i(0) + \lambda_i \leq 0$

(5.23) if $y_i > 0$ then $\frac{\partial L}{\partial y_i} = \pi_i(y_i) - \lambda_i = 0$

(5.24) if $y_i = 0$ then $\frac{\partial L}{\partial y_i} = \pi_i(0) - \lambda_i \leq 0$

(5.25) if $x_{ij} > 0$ then $\frac{\partial L}{\partial x_{ij}} = -\tau_{ij} + \lambda_j - \lambda_i = 0$

(5.26) if $x_{ij} = 0$ then $\frac{\partial L}{\partial x_{ij}} = -\tau_{ij} + \lambda_j - \lambda_i \leq 0$

Using these conditions we can derive some properties of a solution of (5.19).

Equilibrium property 5.1: For region $i \in \{1,\ldots,n\}$:

a) In the optimal solution of (5.19), $\pi_i(y_i) \leq p_i(x_i)$.

b) If in the optimal solution of (5.19), $\pi_i(y_i) < p_i(x_i)$, then $x_i = 0$ or $y_i = 0$.

c) If in the optimal solution of (5.19), supply and demand in region i are both positive, so $x_i > 0$ and $y_i > 0$, then necessarily $p_i(x_i) = \pi_i(y_i)$.

Proof: see Appendix 1.

Equilibrium property 5.2: In the optimal solution of (5.19), let transport take place from market i to market j, i.e. $x_{ij} > 0$, with $i, j \in \{1,2, ...,n\}$, $j \neq i$,, then:

a) no cereals are transferred from other regions into market i, i.e. $x_{si} = 0$, for all $s \neq i$

b) no cereals are transported from market j to other regions, i.e. $x_{js} = 0$, for all $s \neq j$

c) the producer supply x_i in region i satisfies $x_i > 0$,

d) the consumer demand y_j in region j satisfies $y_j > 0$,

Proof*:* see Appendix 1.

Equilibrium property 5.3: For region i and j, $i,j \in \{1,\ldots,n\}$, $i \neq j$:

a) In the optimal solution of (5.19), $\pi_j(y_j) \leq p_i(x_i) + \tau_{ij}$.

b) If in the optimal solution of (5.19), $\pi_j(y_j) < p_i(x_i) + \tau_{ij}$, then $x_{ij} = 0$.
c) If in the optimal solution of (5.19), transport between region i and j is positive, $x_{ij} > 0$, then the optimal prices satisfy necessarily $\pi_j(y_j) = p_i(x_i) + \tau_{ij}$.

Proof: see Appendix 1.

For a situation in which the optimal solution results in supply and demand in a certain region i, so that $x_i > 0$ and $y_i > 0$, the producer price and consumer price are the same, $\pi_i(y_i) = p_i(x_i)$. If transport takes place between region i and j, so that $x_{ij} > 0$, $i \neq j$, then $\pi_j(y_j) = p_i(x_i) + \tau_{ij}$. If no commodities are supplied or demanded in region i, so $x_i = 0$ or $y_i = 0$, then $\pi_i(y_i) \leq p_i(x_i)$. Likewise, if no commodities are transported from region i to region j, so $x_{ij} = 0$, $i \neq j$, then $\pi_j(y_j) \leq p_i(x_i) + \tau_{ij}$. One may wonder whether traders are interested to buy x_i from the producers, transport x_{ij}, and sell y_j to the consumers. This follows from the following theorem.

Theorem 5.1:
Let x_i, y_i, x_{ij}, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of the equilibrium model (5.19). Let $\pi_i = \pi_i(y_i)$, $p_i = p_i(x_i)$. The solution:

$$(5.27) \quad q_i = x_i \;\; ; \;\; r_i = y_i \;\; ; \;\; q_{ij} = x_{ij} \qquad\qquad \text{for } i,j \in \{1,\ldots,n\},\ i \neq j$$

is an optimal solution of trader decision problem (5.12). The value of the objective function is equal to 0, meaning that the trader makes no profits or losses.

Proof: see Appendix 1.

It follows from Theorem 5.1, that it is optimal for the traders to buy, sell, and transport the equilibrium quantities. They will make no losses from these transactions. The result that $\pi_i(y_i) = p_i(x_i)$ on a competitive market on which $y_i > 0$ and $x_i > 0$, is a well known result. The reasoning is as follows. Suppose that in region i, $\pi_i(y_i) > p_i(x_i)$. Then a trader could acquire all supply in region i, still make profits, and price all his competitors out of the market, by offering a price just above the producer price $p_i(x_i)$. In that case x_i would increase, due to (5.5). In order to sell this extra quantity he would have to decrease the consumer price $\pi_i(y_i)$, see (5.5). Other traders would do the same, in this way increasing the producer price and lowering the consumer price, until $\pi_i(y_i) = p_i(x_i)$. Similarly, it is not possible on a competitive market that $\pi_j(y_j) > p_i(x_i) + \tau_{ij}$.

Takayama and Judge (1971, p112) conclude that the optimal quantities of equilibrium model (5.19) will indeed be transacted on a competitive market, because the “solution satsifies the conditions for a spatial price equilibrium (SPE)” – see the introduction of Chapter 5 for the definition of a SPE. We come to the same conclusion, but based on other arguments. The optimal quantities of equilibrium model (5.19) will be transacted on a competitive market, because they are equal to the aggregate quantities which are optimal for each individual producer, consumer and trader. This implies that each agent reaches optimal profits or utility if the equilibrium quantities are transacted, and that the traders’ purchases and sales are in equilibrium. This argument is more convincing than the argument that the solution satisfies a (debatable) definition.

The result that price differences equal transport costs – see *Equilibrium property* 5.3 - is usually argued by assuming perfect competition between traders. As was discussed in Chapter 2, this mechanism of perfect competition is often not satisfied on food markets in developing countries. In the next section the behaviour of a monopolistic trader will be investigated.

5.3 Monopolistic behaviour of traders

In this section the trader is not a price taker, but a monopolist who can set prices. To what extent do price formation on the market and the flows between the various regions change, if not the semi-welfare function (5.15) or (5.19) would be maximized, but the traders' net profits? The model is based on the following assumptions, for i,j = $1,\ldots,n$, $i \neq j$:

- The producers of region i sell an amount $x_i = x_i(p_i)$ to the monopolistic trader - see (5.1) - at a producer price p_i.
- If the monopolist buys from a producer from region i, he may sell (part of) the purchases to the consumers of region i at price π_i or transport it to an other market j to be sold to the consumers there at a price π_j.
- If an amount x_{ij} is transported between market i and market j, the transport costs are $\tau_{ij} \cdot x_{ij}$.
- Consumers of region i buy a quantity $y_i = y_i(\pi_i)$, at consumer price π_i at market i from the trader.

Using the inverse supply and demand functions, the monopolists profit maximization problem may be written as – compare (5.12):

(5.28)
$$\underset{y_i,x_i,x_{ij}}{Max}\left\{\sum_{i=1}^{n}\left(\pi_i(y_i)y_i - p_i(x_i)x_i - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ij}x_{ij}\right)\right.$$
$$\left|\, x_i + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ji} = y_i + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij};\ x_i, y_i, x_{ij} \geq 0; i, j = 1,\ldots,n \right\}$$

The corresponding Lagrangean function is given by, see (3.33) and (5.20):

(5.29)
$$L\left(y_i, x_i, x_{ij}, \lambda_i \,\middle|\, i,j \in \{1,\ldots,n\}, j \neq i\right) =$$
$$\sum_{i=1}^{n}\left[\pi_i(y_i)y_i - p_i(x_i)x_i - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ij}x_{ij}\right] + \sum_{i=1}^{n}\lambda_i\left[x_i + \sum_{\substack{j=1\\ j\neq i}}^{n}x_{ji} - y_i - \sum_{\substack{j=1\\ j\neq i}}^{n}x_{ij}\right]$$

The corresponding conditions (5.21) - (5.26) result into:

(5.30) if $x_i > 0$ then $\frac{\partial L}{\partial x_i} = -p_i(x_i) - x_i p_i'(x_i) + \lambda_i = 0$

(5.31) if $x_i = 0$ then $\frac{\partial L}{\partial x_i} = -p_i(x_i) - x_i p_i'(x_i) + \lambda_i \leq 0$

(5.32) if $y_i > 0$ then $\frac{\partial L}{\partial y_i} = \pi_i(y_i) + y_i \pi_i'(y_i) - \lambda_i = 0$

(5.33) if $y_i = 0$ then $\frac{\partial L}{\partial y_i} = \pi_i(y_i) + y_i \pi_i'(y_i) - \lambda_i \leq 0$

(5.34) if $x_{ij} > 0$ then $\frac{\partial L}{\partial x_{ij}} = -\tau_{ij} + \lambda_j - \lambda_i = 0$

(5.35) if $x_{ij} = 0$ then $\frac{\partial L}{\partial x_{ij}} = -\tau_{ij} + \lambda_j - \lambda_i \leq 0$

The following properties can easily be derived from (5.30) - (5.35)

Monopoly property 5.1: If $x_i > 0$ and $y_i > 0$, then: $p_i + x_i p_i'(x_i) = \pi_i + y_i \pi_i'(y_i)$.

This condition follows immediately from (5.30) and (5.32), and says that if a monopolist purchases and sells in the same region, his marginal revenue equals his marginal cost. It follows due to (5.5) that in that case: $\pi_i(y_i) - p_i(x_i) \geq 0$.

Monopoly property 5.2: In the solution, let transport take place from market i to market j, i.e. $x_{ij} > 0$, with $i, j \in \{1,2, ...,n\}$, $j \neq i$, then (compare *Equilibrium property* 5.2):

a) no cereals are transferred from other regions into market i, i.e. $x_{si} = 0$, for all $s \neq i$

b) no cereals are transported from market j to other regions, i.e. $x_{js} = 0$, for all $s \neq j$

c) the producer supply x_i in region i satisfies $x_i > 0$

d) the consumer demand y_j in region j satisfies $y_j > 0$.

<u>*Monopoly property* 5.3:</u> If $x_{ij} > 0$, then necessarily - see (5.30) - (5.35):

$$\tau_{ij} = y_j \pi'_j(y_j) + \pi_j - x_i p'_i(x_i) - p_i,.$$

It follows, due to (5.5), that: $\pi_j - p_i = \tau_{ij} - y_j \pi'_j(y_j) + x_i p'_i(x_i) \geq \tau_{ij}$

Monopoly properties 5.1 and 5.3 differ from the *Equilibrium properties* 5.1 and 5.3, in which it is not possible that in the solution $\pi_i(y_i) - p_i(x_i) > 0$ or $\pi_j(y_j) - p_i(x_i) > \tau_{ij}$. This shows that on a monopolistic market, traders will make positive profits, whereas traders play even on a competitive market. Note that we can not say on beforehand, that producers and consumer are worse off on a monopolistic market. For example, assume producer prices on a monopolistic market are lower than producer prices on a competitive market. In that case, producer supply on the monopolistic market is lower than on the competitive market. Consequently, due to (5.5), the costs a producer has to make on a monopolistic market are lower than his costs on a competitive market. In total, the producers’ net revenues may still be higher on the monopolistic market than on the competitive market.

6 Spatial equilibrium on *n* markets; multi-period model

In this chapter the market situation of Chapter 5 is extended. A year, for instance from one harvest to the next harvest, is divided in T periods of time, which we number $t = 1,2, ...,T$.[7] Now, cereals can not only be distributed over all n regions of a country, but can also be stored for one or more periods. In Section 6.1 we discuss the strategies of the agents operating on the market, if the producer and consumer prices are known. In Section 6.2 and 6.3 we discuss methods to analyse cereal price formation and trade flows for a competitive and a monopolistic market, respectively. The two models of Chapter 5 - one for perfect competition between traders, one for monopolistic behaviour of traders - are extended to multi-period models. Again, in Section 6.2 we will show that the optimal quantities of the equilibrium model are equal to the aggregate optimal sold, purchased, transported and stored quantities of the individual market agents, at market equilibrium prices.

6.1 Strategies of producers, consumers and traders

Strategies of producers and consumers:

In analogy to (5.1) we define, for $i = 1,...,n$, $t = 1,...,T$:

	x_{it}	producer supply in region i during period t
(6.1)	y_{it}	consumer demand in region i during period t
	p_{it}	producer price in region i during period t
	π_{it}	consumer price in region i during period t

Suppose again that consumer demand and producer supply strategies are reflected by demand and supply functions. Suppose, furthermore, that demand and supply in period t depend only on current prices, and not on prices in the other periods, so $y_{it} = y_{it}(\pi_{it})$ and $x_{it} = x_{it}(p_{it})$. In the next chapter we will consider a situation in which

7 In the semi-arid countries of West Africa, there is only one growing season.

supply depends also on prices in other periods. In analogy with (5.2) and (5.3) it is assumed that:

$$(6.2)\qquad u_{it}(y_{it}) = \int_0^{y_{it}} \pi_{it}(\xi)d\xi\,, \qquad i = 1,2,\dots,n,\ t = 1,\dots,T$$

$$(6.3)\qquad c_{it}(x_{it}) = \int_0^{x_{it}} p(\zeta)d\zeta\,, \qquad i = 1,2,\dots,n,\ t = 1,\dots,T$$

with:

(6.4) $u_{it}\ (y_{it})$ utility of consumption of y_{it} by consumer i during period t
$c_{it}(x_{it})$ costs of producing x_{it} by producer i during period t.

Again, see (5.5) it is assumed that:

$$(6.5)\qquad \begin{aligned} &\pi'_{it}(y_{it}) < 0, \quad i = 1,\dots,n;\ t = 1,\dots,T \\ &p'_{it}(x_{it}) > 0, \quad i = 1,\dots,n;\ t = 1,\dots,T \end{aligned}$$

Some characteristics of traders' strategies:

To describe the optimal strategies of the traders operating on a competitive market, we introduce in analogy with (5.6) the following variables for the aggregated trader, for $i,j=1,\dots,n$, $j\neq i$, $t=1,\dots,$T:

	r_{it}	quantity sold by the (aggregated) trader to the consumers in region i in period t
	q_{it}	quantity purchased by the trader from the producers in region i in period t
(6.6)	q_{ijt}	total quantity of produce transported by the trader from region i to region j in period t
	v_{it}	the quantity of produce in store by the trader in region i at the end of period t.

We suppose that traders have perfect foresight, i.e. that they know in advance, or can predict with certainty, the producer and consumer prices for all T periods, p_{it} and π_{it}. Knowing prices, also producer supply, $x_{it} = x_{it}(p_{it})$, and consumer demand, $y_{it} = y_{it}(\pi_{it})$, are known to the trader. Traders can not buy more than the producers supply ($q_{it} \leq x_{it}$), and they can not sell more than the consumers demand ($r_{it} \leq y_{it}$). Introduce the following parameters for $i,j=1,...,n$, $i \neq j$, $t=1,...,$T.

	v_{i0}	initial stock of the trader in region i
(6.7)	τ_{ijt}	costs of transfer of one unit of weight from market i to market j during period t
	k_{it}	costs of storage of one unit of weight in region i during period t

Analogous to property (5.11), we assume that:

(6.8) $$\tau_{ijt} < \tau_{ist} + \tau_{sjt}, \quad i \neq j,\ i \neq s,\ j \neq s,\ t = 1,\ldots,T.$$

Storage costs k_{it} are the costs which will be paid in period t, to store one unit of weight from the moment of storage in period t, until the moment when they will be taken from the stock in period t+1. If no storage losses are taken into account, the quantity in stock at the end of period t in region i, can be written as

(6.9) $$v_{it} = \left(q_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{jit} + v_{i,t-1} \right) - \left(r_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ijt} \right), \qquad t = 1,\ldots,T.$$

The traders' objective is to maximize profits, i.e. the revenues from sales minus the costs from purchases, transports and storage. To show how the (aggregated) trader's decisions depend on consumer and producer prices, we analyse the following decision problem – compare (5.12):

$$
\underset{q_{it}, r_{it}, q_{ijt}, v_{it}}{Max} \left\{ \sum_{t=1}^{T} \sum_{i=1}^{n} \left[\pi_{it} r_{it} - p_{it} q_{it} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ijt} q_{ijt} - k_{it} v_{it} \right] \right.
$$

$$
(6.10) \qquad \left| \; q_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{jit} + v_{i,t-1} = r_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ijt} + v_{it}, \quad 0 \leq q_{it} \leq x_{it}; \right.
$$

$$
\left. 0 \leq r_{it} \leq y_{it};\ q_{ijt}, v_{it} \geq 0, \ \ i, j = 1, \ldots, n, j \neq i,\ t = 1, \ldots, T \right\}
$$

Introduce the Lagrangian multipliers of the equilibrium constraints, λ_{it}, for $i \in \{1,\ldots,n\}$, $t \in \{1,\ldots,T\}$. The optimal solution of model (6.10), q_{it}, y_{it}, q_{ijt}, v_{it} for $i,j = 1,\ldots,n, j \neq i, t = 1,\ldots,T,$ has to satisfy the following Kuhn-Tucker conditions – see also (3.42) - (3.44):

$$
(6.11) \quad
\begin{array}{ll}
\text{if } q_{it} = 0 \text{ then } \lambda_{it} \leq p_{it} & \text{if } r_{it} = 0 \text{ then } \lambda_{it} \geq \pi_{it} \\
\text{if } 0 < q_{it} < x_{it} \text{ then } \lambda_{it} = p_{it} & \text{if } 0 < r_{it} < y_{it} \text{ then } \lambda_{it} = \pi_{it} \\
\text{if } q_{it} = x_{it} \text{ then } \lambda_{it} \geq p_{it} & \text{if } r_{it} = y_{it} \text{ then } \lambda_{it} \leq \pi_{it} \\
 & \\
\text{if } q_{ijt} = 0 \text{ then } \lambda_{jt} \leq \lambda_{it} + \tau_{ijt} & \text{if } v_{it} = 0 \text{ then } \lambda_{i,t+1} \leq \lambda_{it} + k_{it},\ t = 1,\ldots,T\text{-}1 \\
\text{if } q_{ijt} > 0 \text{ then } \lambda_{jt} = \lambda_{it} + \tau_{ijt} & \text{if } v_{it} > 0 \text{ then } \lambda_{i,t+1} = \lambda_{it} + k_{it},\ t = 1,\ldots,T\text{-}1
\end{array}
$$

It follows immediately that $v_{iT} = 0$.[8] From (6.11), we can derive some properties which show the influence of the difference between producer and consumer price levels, on the traders' optimal purchases, sales, transports and storage:

---

[8] For period T, the Kuhn-Tucker conditions (3.38) - (3.40) show that v_{iT} $(-k_{iT} - \lambda_{iT}) = 0$, $v_{iT} \geq 0$, and $-k_{iT} - \lambda_{iT} \leq 0$. If $v_{iT} > 0$, then $\lambda_{iT} = -k_{iT} < 0$, which is impossible due to (6.11), from which follows that $\lambda_{iT} \geq 0$. Consequently, $v_{iT} = 0$.

Trader property 6.1: For region $i \in \{1,\dots,n\}$, and period $t \in \{1,\dots,T\}$:

a) If $\pi_{it} < p_{it}$, then any optimal solution of (6.11) satisfies $q_{it} = 0$ or $r_{it} = 0$

b) If $\pi_{it} \geq p_{it}$, then an optimal solution of (6.11) exists satisfying the condition $q_{it} = x_{it}$ or $r_{it} = y_{it}$; for $\pi_{it} = p_{it}$, other optimal solutions of (6.11) may exist, not satisfying this condition.

Proof: See Appendix 1.

Trader property 6.2: Let q_{it}, r_{jt}, q_{ijt}, v_{it}, $j \neq i$, $i,j = 1,\dots,n$, $t = 1,\dots,T$, be an optimal solution of (6.11). Let a trader transport in a period t from a region i to a region j, so $q_{ijt} > 0$, for $i,j \in \{1,\dots,n\}$, $j \neq i$, $t \in \{1,\dots,T\}$, then:

a) in period t no goods are transported from region $s = 1,\dots,n$, $s \neq i$, to region i, $q_{sit} = 0$

b) in period t no goods are transported from region j to region $s = 1,\dots,n$, $s \neq j$, $q_{jst} = 0$.

c) in period t, purchases in region i are positive, $q_{it} > 0$, or the stock remaining from the previous period is positive, $v_{i,t-1} > 0$.

d) in period t, sales in region j are positive, $r_{jt} > 0$, or the stock at the end of period t in region j is positive, $v_{jt} > 0$.

Proof: See Appendix 1.

Trader property 6.3: For region $i,j \in \{1,\dots,n\}$, $j \neq i$, and period $t \in \{1,\dots,T\}$:

a) If $\pi_{jt} < p_{it} + \tau_{ijt}$, then any optimal solution of (6.11) has to satisfy $q_{it} = 0$ or $q_{ijt} = 0$ or $r_{jt} = 0$.

b) If $\pi_{jt} \geq p_{it} + \tau_{ijt}$, and $q_{it} > 0$, $q_{ijt} > 0$ and $r_{jt} > 0$, then an optimal solution of (6.11) exists satisfying $q_{it} = x_{it}$ or $r_{jt} = y_{jt}$; for $\pi_{jt} = p_{it} + \tau_{ijt}$, an optimal solution of (6.11) is not unique.

Proof: See Appendix 1.

Define the costs of storage of one unit of weight in region i, from the moment of storage in period t to the moment when it will be taken from the stock in period τ, for $t = 1,\ldots,T$-1, $\tau = t$+1,…,T – see (6.7):

$$(6.12) \quad \kappa_{it\tau} = \sum_{l=t}^{\tau-1} k_{il} \ ,$$

<u>*Trader property* 6.4:</u> For region $i \in \{1,\ldots,n\}$, $j \neq i$, period $t \in \{1,\ldots,T\text{-}1\}$:

a) If $\pi_{i,t+1} < p_{it} + k_{it}$, then any optimal solution of (6.11) has to satisfy $q_{it} = 0$ or $v_{it} = 0$ or $r_{i,t+1} = 0$.

b) Analogously for $\tau \in \{t+1,\ldots,T\}$: if $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$, then any optimal solution of (6.11) has to satisfy $q_{it} = 0$ or $v_{it} = 0$, or …, or $v_{i,\tau-1} = 0$, or $r_{i\tau} = 0$.

c) If $\pi_{i,t+1} \geq p_{it} + k_{it}$, and $q_{it} > 0$, $v_{it} > 0$ and $r_{i,t+1} > 0$, then an optimal solution of (6.11) exists which satisfies the condition $q_{it} = x_{it}$ or $r_{i,t+1} = y_{i,t+1}$. Nota bene: for $\pi_{i,t+1} > p_{it} + k_{it}$, any optimal solution of (6.11) has to satisfy this condition; for $\pi_{i,t+1} = p_{it} + k_{it}$, an optimal solution is not unique.

d) Analogously for $\tau \in \{t+1,\ldots,T\}$: if $\pi_{i\tau} \geq p_{it} + \kappa_{it\tau}$, and $q_{it} > 0$, $v_{it} > 0,\ldots,$ $v_{i,\tau-1} > 0$ and $r_{i\tau} > 0$, then an optimal solution of (6.11) exists satisfying the condition $q_{it} = x_{it}$ or $r_{i\tau} = y_{i\tau}$. For $\pi_{i\tau} = p_{it} + \kappa_{it\tau}$, an optimal solution of (6.11) is not unique.

<u>Proof:</u> See Appendix 1.

The interpretation of the first and second property is the same as the interpretation of *Trader property* 5.1 and 5.2 in Section 5.1. *Trader property* 6.3 describes for which price levels a trader will purchase from producers in region i, and transport to region j, where he sells to consumers. This property is almost similar to *Trader property* 5.3. The difference is that it is possible that goods are transported from region i to j, even if $\pi_{jt} < p_{it} + \tau_{ijt}$. In that case, it is not possible that a trader sells in region j the commodities he purchased in region i in the same period. But the goods transported to region j have to be taken from the stock from the previous period $v_{i,t\text{-}1}$, or they have to be put in stock in region j, v_{jt}. *Trader property* 6.4 describes the possibility of the trader to purchase in period t, store till period τ, and sell to the consumers in period τ, for $\tau = t+1,\ldots,T$. A trader will not sell in period τ the goods purchased in period t, if he would make a loss out of it, i.e. if $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$. For $\pi_{i\tau} = p_{it} + \kappa_{it\tau}$, storage will give losses nor profits. Finally, for $\pi_{i\tau} > p_{it} + \kappa_{it\tau}$, selling in τ the commodities purchased in t, will be profitable. The trader will purchase the maximum possible quantity, x_{it}, in period t, or sell the maximum possible quantity, $y_{i\tau}$, in period τ.

Note that it is possible that goods are transported from region i to j, even if $\pi_{jt} < p_{it} + \tau_{ijt}$, or that goods are stored in period t, even if $\pi_{i,t+1} < p_{it} + k_{it}$, as show the following examples:

- If $\pi_{jt} < p_{it} + \tau_{ijt}$ but $\pi_{jt} > p_{i,t-1} + k_{i,t-1} + \tau_{ijt}$, it will be profitable to purchase commodities in region i in period t-1, store it until period t, and then transport it to region j, where it is sold to the consumers. In that case it is possible that $q_{i,t-1} > 0$, $v_{i,t-1} > 0$, $q_{it} = 0$, $q_{ijt} > 0$ and $r_{jt} > 0$.
- If $\pi_{i,t+1} < p_{it} + k_{it}$, but $\pi_{i,t+2} > p_{it} + k_{it} + k_{i,t+1}$, then it will be profitable to purchase in period t, store until period t+2, when it is sold the the consumers. So, possibly $q_{it} > 0$, $v_{it} > 0$, $r_{it} = 0$, $v_{i,t+1} > 0$, $r_{i,t+1} = 0$ and $r_{i,t+2} > 0$.
- If $\pi_{i,t+1} < p_{it} + k_{it}$, but $\pi_{i,t+1} > p_{jt} + \tau_{jit} + k_{it}$, it will be profitable to purchase in period t in region j, transport it to region i, then store it until period t+1, whereupon it is sold to the consumers. For those prices, it is possible that $q_{jt} > 0$, $q_{it} = 0$, $q_{jit} > 0$, $v_{it} > 0$ and $r_{i,t+1} > 0$.

The above properties will be used in the next section to verify whether the optimal solutions of the market equilibrium model to be developed, will satisfy the properties of the traders' optimal behaviour. The equilibrium solutions will only be acceptable for the traders if they are also optimal for them individually.

6.2 Maximization of welfare; perfect competition between traders

In this section we extend equilibrium model (5.19), to take storage into account. We set up a multi-period, spatial equilibrium model in which the optimal values of the following variables are determined for all regions $i = 1,\ldots,n$, and all periods $t = 1,\ldots,T$: producer and consumer prices (p_{it} and π_{it}), producer supply (x_{it}), consumer demand (y_{it}), total transported quantities to the various regions, and the quantity put in store in period t. We define for $i,j = 1,\ldots,n$, $i \neq j$, $t = 1,..,T$:

(6.13)	x_{ijt}	total produce transported from region i to region j in period t
	s_{it}	the quantity of produce in stock in region i at the end of period t.

Without loss of generality it may be assumed that no goods are in stock at the beginning of period 1, $s_{i0} = 0$. In analogy with Section 5.2, we first discuss the set-up and results of the multi-period spatial, equilibrium model. Secondly, we show that the welfare optimizing supplies, demand, transported and stored quantities, are equal to, respectively, the aggregate optimal sales of the individual producers, the aggregate optimal purchases of the individual consumers, the aggregate optimal transport flows of the traders, and the aggregate optimal stock levels of the traders, at the market equilibrium prices.

Semi-welfare in the multi-period, spatial equilibrium model may be written as total consumer utility minus all costs made, which include producer, transport and storage costs – see (4.30) and (5.15). As we may write consumer plus producer surplus as the integral of the inverse demand function minus the integral of the inverse supply funtion – see (6.2) and (6.3), semi-welfare can be written as:

$$(6.14) \qquad \underset{y_{it},x_{it},x_{ijt},s_{it}}{Max} \sum_{t=1}^{T}\sum_{i=1}^{n}\left\{\int_0^{y_{it}} \pi_{it}(\xi)d\xi - \int_0^{x_{it}} p_{it}(\zeta)d\zeta - \sum_{\substack{j=1\\ j\neq i}}^{n} \tau_{ijt}x_{ijt} - k_{it}s_{it}\right\}$$

where $\pi_{it}(y_{it})$ and $p_{it}(x_{it})$ are given demand and supply functions satisfying (6.5), the parameters τ_{ijt} and k_{it} have been defined in (6.7), and the variables x_{it}, y_{it}, s_{it} and x_{ijt} have to satisfy the equilibrium conditions and non-negativity conditions

(6.15) $$x_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{jit} + s_{i,t-1} = y_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ijt} + s_{it}$$

(6.16) $x_{it} \geq 0$, $y_{it} \geq 0$, $s_{it} \geq 0$, $x_{ijt} \geq 0$, $i,j = 1, 2, ..., n;\ j \neq i;\ \ t = 1,...,T.$

Let x_{it}, y_{it}, s_{it} and x_{ijt}, $i,j = 1, 2, ..., n;\ j \neq i;\ \ t = 1,...,T$, be the optimal solution of (6.14) - (6.16). The Kuhn-Tucker optimality conditions show that – see (3.33) and (3.38) - (3.40), see also (5.21) - (5.26):

(6.17) if $x_{it} > 0$ then $-p_{it}(x_{it}) + \lambda_{it} = 0$

(6.18) if $x_{it} = 0$ then $-p_{it}(0) + \lambda_{it} \leq 0$

(6.19) if $y_{it} > 0$ then $\pi_{it}(y_{it}) - \lambda_{it} = 0$

(6.20) if $y_{it} = 0$ then $\pi_{it}(0) - \lambda_{it} \leq 0$

(6.21) if $x_{ijt} > 0$ then $-\tau_{ijt} - \lambda_{it} + \lambda_{jt} = 0$

(6.22) if $x_{ijt} = 0$ then $-\tau_{ijt} - \lambda_{it} + \lambda_{jt} \leq 0$

(6.23) if $s_{it} > 0$ then $-k_{it} - \lambda_{it} + \lambda_{i,t+1} = 0$ for $t = 1,\ldots,T-1$

(6.24) if $s_{it} = 0$ then $-k_{it} - \lambda_{it} + \lambda_{i,t+1} \leq 0$ for $t = 1,\ldots,T-1$

Analogous to the argumentation in footnote 8, and using (6.19) and (6.20), it follows immediately that $s_{iT} = 0$. From these conditions we derive the following properties – compare *Equilibrium properties* 5.1 to 5.3 in Section 5.2.

<u>*Equilibrium property* 6.1:</u> For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:

a) In the optimal solution of (6.14) - (6.16) $\pi_{it}(y_{it}) \leq p_{it}(x_{it})$.

b) If in the optimal solution of (6.14) - (6.16) $\pi_{it}(y_{it}) < p_{it}(x_{it})$, then $x_{it} = 0$ or $y_{it} = 0$.

c) If in the optimal solution of (6.14) - (6.16), supply and demand are both positive, $x_{it} > 0$ and $y_{it} > 0$, then the prices necessarily satisfy $p_{it}(x_{it}) = \pi_{it}(y_{it})$.

Proof: See Appendix 1.

Equilibrium property 6.2: Let in the optimal solution of (6.14) - (6.16) transport in period t take place from a market i to a market j, so $x_{ijt} > 0$, with $i,j \in \{1,2, ...,n\}$, $j \neq i$, $t \in \{1,\ldots,T\}$, then:

a) in period t, no cereals are transferred from other regions into market i, i.e. $x_{sit} = 0$, for all $s \neq i$

b) in period t, no cereals are transported from market j to other regions, i.e. $x_{jst} = 0$, for all $s \neq j$

c) in period t, the producer supply in region i in period t satisfies, $x_{it} > 0$, or the stock remaining from the previous period is positive, $s_{i,t-1} > 0$.

d) in period t, the consumer demand in region j in period t satisfies $y_{jt} > 0$, or the quantity put in stock in region j is positive, $s_{jt} > 0$.

Proof: See Appendix 1.

Equilibrium property 6.3: For region i and j, $i,j \in \{1,\ldots,n\}$, $i \neq j$, and period $t \in \{1,\ldots,T\}$:

a) In the solution of (6.14) - (6.16) $\pi_{jt}(y_{jt}) \leq p_{it}(x_{it}) + \tau_{ijt}$.

b) If in the optimal solution of (6.14) - (6.16) $\pi_{jt}(y_{jt}) < p_{it}(x_{it}) + \tau_{ijt}$, then $x_{it} = 0$ or $x_{ijt} = 0$ or $y_{jt} = 0$.

c) If in the optimal solution of (6.14) - (6.16) supplies in region i, transport between region i and j, and demand in region j are positive, $x_{it} > 0$ and $x_{ijt} > 0$ and $y_{jt} > 0$, then the optimal prices necessarily satisfy $\pi_{jt}(y_{jt}) = p_{it}(x_{it}) + \tau_{ijt}$.

Proof: See Appendix 1.

Equilibrium property 6.4: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\text{-}1\}$:

a) In the optimal solution of (6.14) - (6.16) $\pi_{i,t+1}(y_{i,t+1}) \leq p_{it}(x_{it}) + k_{it}$. Analogously, for $\tau \in \{t+1,\ldots,T\}$: $\pi_{i\tau}(y_{i\tau}) \leq p_{it}(x_{it}) + \kappa_{it\tau}$.

b) If in the optimal solution of (6.14) - (6.16) $\pi_{i,t+1}(y_{i,t+1}) < p_{it}(x_{it}) + k_{it}$, then $x_{it} = 0$ or $s_{it} = 0$ or $y_{i,t+1} = 0$. Analogously, for $\tau \in \{t+1,\ldots,T\}$ – see also (6.12): if $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$, then any optimal solution of (6.14) - (6.16) has to satisfy $x_{it} = 0$ or $s_{it} = 0$ or $s_{i,t+1} = 0$ … or $s_{i,\tau-1} = 0$ or $y_{i\tau} = 0$.
c) If in the optimal solution of (6.14) - (6.16) supplies in period t, stock levels at the end of period t, and demand in period t+1 are positive, $x_{it} > 0$ and $s_{it} > 0$ and $y_{i,t+1} > 0$, then the optimal prices necessarily satisfy $\pi_{i,t+1}(y_{i,t+1}) = p_{it}(x_{it}) + k_{it}$.
d) If in the optimal solution of (6.14) - (6.16), supplies in period t, storage from period t to the end of period τ-1, and demand in period τ are positive, $x_{it} > 0$, $s_{it} > 0$, $s_{i,t+1} > 0,\ldots, s_{i,\tau-1} > 0$ and $y_{i\tau} > 0$, then the optimal prices satisfy $\pi_{i\tau}(y_{i\tau}) = p_{it}(x_{it}) + \kappa_{it\tau}$, for $\tau \in \{t+1,\ldots,T\}$.

<u>Proof:</u> See Appendix 1.

The results of equilibrium model (6.14) - (6.16) are more or less similar to those of model (5.16) - (5.19) in Section 5.2. Summarizing:

- For a situation in which the optimal solution results in supply and demand in region i in period t, so $x_{it} > 0$ and $y_{it} > 0$ for $i \in \{1,\ldots,n\}$ and $t \in \{1,\ldots,T\}$, the producer price and consumer price are the same, $\pi_{it}(y_{it}) = p_{it}(x_{it})$.
- If in period t, commodities are supplied in region i, transported between region i and j, and demanded in region j, so $x_{it} > 0$, $x_{ijt} > 0$ and $y_{jt} > 0$ for $i,j \in \{1,\ldots,n\}$, $i \neq j$ and $t \in \{1,\ldots,T\}$, then $\pi_{jt}(y_{jt}) = p_{it}(x_{it}) + \tau_{ijt}$.
- If in region i, commodities are supplied in period t, stored from period t to the end of period τ-1 and demanded in period τ, so $x_{it} > 0$, $s_{it} > 0,\ldots,s_{i,\tau-1} > 0$, and $y_{i\tau} > 0$ for $i \in \{1,\ldots,n\}$, $t \in \{1,\ldots,T-1\}$ and $\tau \in \{t+1,\ldots,T\}$, then $\pi_{i\tau}(y_{i\tau}) = p_{it}(x_{it}) + \kappa_{it\tau}$.

One may wonder whether traders are interested to buy x_{it} from the producers, transport x_{ijt}, store s_{it}, and sell y_{it} to the consumers $i,j \in \{1,\ldots,n\}$, $i \neq j$ and $t \in \{1,\ldots,T\}$. This follows from the following theorem.

Theorem 6.1:

Let x_{it}, y_{it}, x_{ijt}, s_{it}, $i,j \in \{1,\ldots,n\}$, $i \neq j$, $t \in \{1,\ldots,T\}$, be an optimal solution of the equilibrium model (6.14) - (6.16). Let $\pi_{it} = \pi_{it}(y_{it})$, $p_{it} = p_{it}(x_{it})$. The solution:

(6.25) $q_{it} = x_{it}$; $r_{it} = y_{it}$; $q_{ijt} = x_{ijt}$; $v_{it} = s_{it}$ for $i,j \in \{1,\ldots,n\}$, $i \neq j$, $t \in \{1,\ldots,T\}$

is an optimal solution of trader decision problem (6.10). The value of the objective function is equal to 0, meaning that the trader makes no profits or losses.

Proof: See Appendix 1.

From Theorem 6.1 it follows that it is optimal for the traders to buy, sell, transport, and store the equilibrium quantities. They will make no losses from these transactions. The equilibrium model can be used to analyse the optimal sales, purchase, transport and storage behaviour of producers, consumers and traders on a competitive market. For a monopolistic market situation, the results will be different. This will be discussed in the next section.

6.3 Monopolistic behaviour of traders

If the market is not competitive, but the trader is a monopolist, the trader strategies change. In Section 5.3 we described the market equilibrium solution for a monopolistic trader. In this section we will extend this approach for a situation in which the trader may store commodities. The multi-period model describing the monopolist's objectives to maximize profits may be written as - compare (5.28):

$$\underset{y_{it},x_{it},x_{ijt},s_{it}}{Max} \sum_{t=1}^{T}\sum_{i=1}^{n}\left\{\pi_{it}(y_{it})y_{it} - p_{it}(x_{it})x_{it} - \sum_{\substack{j=1\\j\neq i}}^{n}\tau_{ijt}x_{ijt} - k_{it}s_{it}\right.$$

(6.26) $$\left| \; x_{it} + \sum_{\substack{j=1\\j\neq i}}^{n} x_{jit} + s_{i,t-1} = y_{it} + \sum_{\substack{j=1\\j\neq i}}^{n} x_{ijt} + s_{it}; \right.$$

$$\left. x_{it}, y_{it}, x_{ijt}, s_{it} \geq 0,\ i,j = 1,\ldots,n,\ t = 1,..,T \quad \right\}$$

Formulating the Lagrangean function and taking the Kuhn-Tucker conditions, analogous to the analysis in Section 5.3, we can derive some properties to which the optimal solution of the monopolistic trader model will apply. Analogous to model (6.14) - (6.16), it follows immediately that $s_{iT} = 0$. Instead of *Equilibrium properties* 6.1 - 6.4, we can write the following properties:

Monopoly property 6.1: If $x_{it} > 0$ and $y_{it} > 0$, then: $p_{it} + x_{it} \cdot p'_{it}(x_{it}) = \pi_{it} + y_{it} \cdot \pi'_{it}(y_{it})$.

This condition says that if a monopolist purchases and sells in the same region, his marginal revenue equals his marginal cost. It follows due to (6.5) that in that case:

$$\pi_{it} - p_{it} = p'_{it}x_{it} - \pi'_{it}y_{it} \geq 0$$

Monopoly property 6.2: In the solution, let transport in period t take place from a market i to a market j, i.e. let $x_{ijt} > 0$, with $i,j \in \{1,2, ...,n\}$, $j \neq i$, $t \in \{1,\ldots,T\}$, then (compare *Equilibrium property* 6.2):

a) in period t, no cereals are transferred from other regions into market i, i.e. $x_{sit} = 0$, for all $s \neq i$

b) in period t, no cereals are transported from market j to other regions, i.e. $x_{jst} = 0$, for all $s \neq j$

c) in period t, the producer supply in region i in period t satisfies, $x_{it} > 0$, or the stock remaining from the previous period is positive, $s_{i,t-1} > 0$.
d) in period t, the consumer demand in region j in period t satisfies $y_{jt} > 0$, or the quantity put in stock in region j is positive, $s_{jt} > 0$.

<u>*Monopoly property* 6.3:</u> If $x_{it} > 0$, $y_{jt} > 0$ and $x_{ijt} > 0$, then necessarily:

$$\tau_{ijt} = y_{jt}\,\pi'_{jt}(y_{jt}) + \pi_{jt} - x_{it}\,p'_{it}(x_{it}) - p_{it}$$

It follows, due to (6.5), that:

$$\pi_{jt} - p_{it} = \tau_{ijt} - y_{jt}\,\pi'_{jt}(y_{jt}) + x_{it}\,p'_{it}(x_{it}) \geq \tau_{ijt}$$

<u>*Monopoly property* 6.4:</u> If $x_{it} > 0$, $y_{i,t+1} > 0$ and $s_{it} > 0$, then necessarily:

$$k_{it} = y_{i,t+1}\,\pi'_{i,t+1}(y_{i,t+1}) + \pi_{i,t+1} - x_{it}\,p'_{it}(x_{it}) - p_{it}$$

It follows, due to (6.5), that:

$$\pi_{i,t+1} - p_{it} = k_{it} - y_{i,t+1}\,\pi'_{i,t+1}(y_{i,t+1}) + x_{it}\,p'_{it}(x_{it}) \geq k_{it}$$

As can be seen from the *Equilibrium properties* 6.1 to 6.4, in the competitive case discussed in Section 6.2, it was not possible that in the optimal solution π_{it} - $p_{it} > 0$ or π_{jt} - $p_{it} > \tau_{ijt}$ or $\pi_{i,t+1}$ - $p_{it} > k_{it}$. Like in Section 5.3, traders will make positive profits on a monopolistic market, whereas traders play even on a competitive market.

7 Spatial equilibrium on *n* markets in *T* periods: stochastic future prices.

In the previous chapter we discussed the set-up of an equilibrium model for n regions and T periods, for a situation in which the level of future prices was not an uncertain factor for traders, producers and consumers. They were supposed to know future prices, on which they based their storage, supply and demand decisions. In reality, however, producers, consumers and traders do not have full knowledge on what will happen in the future. In fact, their decisions are based on their observations of the market, and on their *expectations* for the future. In this chapter we will analyse the market situation in which future prices are stochastic.

In Section 7.1 we will first deal with the optimal strategies of the individual market agents. In a period $t \in \{1,\dots,T\}$, the optimal strategies of the consumers are assumed to depend only on the observed consumer price in period t. In this period, the optimal strategy of the producers depends on the observed producer price in period t, and the uncertain producer prices in the periods t+1 to T. Finally, the optimal trader strategies in period t, depend on both observed producer and consumer prices in period t, and on uncertain producer and consumer prices in the periods t+1 to T. In Section 7.2 we discuss a stochastic, multi-region, multi-period, equilibrium model to analyse cereal price formation in a situation in which future prices are stochastic. For each period $t \in \{1,\dots,T\}$, a model will be set up in which producer prices, consumer prices, supplies by the producers, demands by the consumers, and quantities transported and stored by the traders are computed for all regions $i = 1,\dots,n$. Furthermore, also future supplies, demands and quantities transported and stored which are expected to be optimal for future periods are derived. The objective function of this model is set up in such a way, that the optimal equilibrium quantities for the period t correspond to the optimal strategies of the individual agents in this period. This means that for p_{it} and π_{it} the computed equilibrium producer and consumer price in the region i, the optimal supplies of all individual producers together, optimal demand of all individual consumers together, and optimal strategies of all individual traders together, are equal to the computed equilibrium quantities.

7.1 Strategies of producers, consumers and traders

Consumer strategies

Again, we deal with a situation in which a country is divided in n regions, and a year is divided in T periods, from one harvest to the other. Consumer demand strategies are reflected by demand functions. Anticipating on the empirical implementation of the model for cereal trade in Burkina Faso, we will assume that *demand* in period t only depends on current prices, and not on prices in previous or expected prices in future periods, so $y_{it} = y_{it}(\pi_{it})$. In analogy with (6.1) and (6.2) it is assumed that:

(7.1) $$u_{it}(y_{it}) = \int_0^{y_{it}} \pi_{it}(\xi)d\xi \qquad i = 1,2,\ldots,n,\ t = 1,\ldots,T.$$

Producer strategies

To describe producer supply strategies, we follow a different approach than in the Chapters 5 and 6. We make use of the variables and parameters introduced in (6.1), and of the parameters:

(7.2) w_{i0} available produce at the beginning of period 1 in region i

and of the variables

(7.3) w_{it} the quantity in stock at the end of period t by the producers of region i

for $i \in \{1,\ldots,n\}$ and $t \in \{1,\ldots,T\}$. We assume that the producers in region i can supply during the T periods, from one harvest to the other, at most a quantity w_{i0}, which is known at the beginning of the first period. w_{i0} may contain the harvest and the commodities still in store from the previous year. In the previous chapter, prices

p_{it} for all periods $t = 1,...,T$, were assumed to be known at the beginning of period 1. In this section, we assume that in a certain period $t \in \{1,...,T\}$, prices are known for the periods $1,...,t$, but future prices for the periods $t+1,...,T$ are random variables, of which the probability distributions are assumed to be known. Introduce, for $i = 1,...,n$, $t = 1,...,T$:

(7.4) P_{it} random future producer price for period t in region i.

We will first assume that $P_{i1},..., P_{iT}$ are independent random variables, and that P_{it}, for $t \in \{1,\ldots,T\}$ has a discrete distribution, with possible price realizations p_{it}^k, for $k = 1,\ldots,K$. Define:

(7.5) $\Pr\left(P_{it} = p_{it}^k\right) = f_{it}^k$, for $t \in \{1,\ldots,T\}$, $k \in \{1,\ldots,K\}$, $i \in \{1,...,n\}$

with probabilities f_{it}^k satisfying $0 \le f_{it}^k \le 1$ and $\sum_{k=1}^{K} f_{it}^k = 1$. $EP_{it} = \sum_{k=1}^{K} f_{it}^k p_{it}^k$ is the expected price in region i and period t, for $t \in \{1,\ldots,T\}$, $i \in \{1,...,n\}$.

Different from the approach followed in the previous chapters, we assume that at the beginning of the first period, producers do not make final decisions on the optimal supplies for all periods. Based on w_{i0}, the observed price p_{i1}, and random future prices, $P_{i2},...,P_{iT}$, at $t = 1$, producers decide on the optimal supplied quantity x_{i1} in the first period. In each period $t \in \{2,...,T\}$, producers decide on the optimal supplies x_{it} for the period t. These decisions depend on the quantity remaining from the previous period, $w_{i,t-1}$, the observed price p_{it}, and the distribution of the random future prices $P_{i,t+1},...,P_{iT}$. Supplies in period t are constrained by the produce which is in stock at the end of the previous period. If storage losses are not taken into account, w_{it} can be written as:

(7.6) $w_{it} = w_{i,t-1} - x_{it} = w_{i0} - x_{i1} - \ldots - x_{it}$, for $t=1,\ldots,T$, $i = 1,...,n$

The condition $w_{it} \geq 0$, for $t = 1,\ldots,T$, implies that:

(7.7) $x_{it} \leq w_{i,t\text{-}1}$, for $t = 1,\ldots,T$, $i = 1,\ldots,n$.

To choose between selling now or later, the producer balances net revenues from current sales and expected net revenues from selling later. He maximizes in each period t his revenues for that period, $p_{it} \cdot x_{it}$, minus the costs made to sell x_{it}, called $c_{it}(x_{it})$, plus the *expected* net revenues for future periods. We will assume here that the cost function can be written as :

(7.8) $c_{it}(x_{it}) = c_{it}x_{it}$, for $i = 1,\ldots,n$, $t = 1,\ldots,T$.

with $c_{it} > 0$ a constant. The parameter c_{it} may contain among other things, production costs per unit, transport costs to the market place, and costs to store the goods until period t. Consequently $c'_{it}(x_{it}) > 0$ and $c''_{it}(x_{it}) = 0$, for $i = 1,\ldots,n$, $t = 1,\ldots,T$, see also Section 4.1. It will be discussed below why we assume that $c_{it}(x_{it})$ is a linear function.

In each period $t \in \{1,\ldots,T\}$, a producer optimizes his revenues for the period t, plus his expected revenues for the future periods t+1 to T, knowing the current price p_{it} and available stock $w_{i,t\text{-}1}$. Define for $i \in \{1,\ldots,n\}$ and $t \in \{1,\ldots,T\}$:

(7.9) $z_{it}^{pr}\left(w_{i,t-1}, p_{it}\right)$ the optimal current plus expected future net revenues of the producer in region i for period t.

Optimal producer supply in period T

The sequential decision process can be modelled using a so called recourse model. The model structure is the same as in model (3.48) in Section 3.5. Consider first the producer's supply in the last period $t = T$. In the last period the producer knows the level of the stocks remaining from the previous period, $w_{i,T\text{-}1}$, and also p_{iT} is assumed to be known. The producer in region i maximizes his net revenues for that period.

Produce remaining at the end of period T, x_{iT} - $w_{i,T\text{-}1}$, is assumed not to yield any future revenues. The decision problem for period T may then be written as:

$$
(7.10) \quad \begin{aligned} z_{iT}^{pr}\left(w_{i,T-1}, p_{iT}\right) &= \underset{x_{iT}}{Max}\left\{p_{iT} x_{iT} - c_{iT}\left(x_{iT}\right) \mid 0 \leq x_{iT} \leq w_{i,T-1}\right\} \\ &= \underset{x_{iT}}{Max}\left\{\left(p_{iT} - c_{iT}\right) x_{iT} \mid 0 \leq x_{iT} \leq w_{i,T-1}\right\} \end{aligned}
$$

It is easily seen that the optimal supply level x_{iT}, for i = 1,...,n, is given by – see also (3.42) - (3.44):

$$
(7.11) \quad \begin{cases} \text{if } p_{iT} < c_{iT} & \text{then } x_{iT} = 0 \\ \text{if } p_{iT} \geq c_{iT} & \text{then } x_{iT} = w_{i,T-1} \\ \text{if } p_{iT} = c_{iT} & \text{then any solution } x_{iT} \text{ between 0 and } w_{i,T-1} \text{ is optimal} \end{cases}
$$

Optimal producer supply for the periods T-1 *to* 1

In the producers' decision problems for the periods t = T-1, T-2,…,1, producers in region i are assumed to know the level of the stock remaining from the previous period, $w_{i,t\text{-}1}$ – see (7.6), the producer price p_{it}, and the probability distribution of future prices – see (7.5). In a period $t \in \{1,\ldots,T-1\}$, a producer optimizes his profits for that period plus expected future profits. His decision problem for period t can be written as:

$$
(7.12) \quad \begin{aligned} & z_{it}^{pr}\left(w_{i,t-1}, p_{it}\right) = \\ & \qquad \underset{x_{it}, w_{it}}{Max}\left\{\left(p_{it} - c_{it}\right) x_{it} + Ez_{i,t+1}^{pr}\left(w_{it}, P_{i,t+1}\right) \mid 0 \leq x_{it} \leq w_{i,t-1}, w_{it} = w_{i,t-1} - x_{it}\right\} \end{aligned}
$$

$Ez_{i,t+1}^{pr}\left(w_{it}, P_{i,t+1}\right)$ refers to the expectation of $z_{i,t+1}^{pr}$ with respect to the random price $P_{i,t+1}$, i.e. the expectation of the optimal revenues for period t+1 plus expected future revenues for the periods t+2 to T. Define $x_{i,t+1}^{k}$ and $w_{i,t+1}^{k}$ for k = 1,…,K, the supply in period t+1 and the stock at the end of period t+1, if the producer price in period t+1 takes the value $p_{i,t+1}^{k}$. Analogous to (3.49), $Ez_{i,t+1}^{pr}(w_{it}, P_{i,t+1})$ in (7.12) is:

$$Ez_{i,t+1}^{pr}\left(w_{it}, P_{i,t+1}\right) = \underset{x_{i,t+1}^{k}, w_{i,t+1}^{k}}{Max} \left\{ \sum_{k=1}^{K} f_{i,t+1}^{k} \left[\left(p_{i,t+1}^{k} - c_{i,t+1} \right) x_{i,t+1}^{k} + Ez_{i,t+2}^{pr}\left(w_{i,t+1}^{k}, P_{i,t+2} \right) \right] \right. \tag{7.13}$$
$$\left. \mid 0 \le x_{i,t+1}^{k} \le w_{it},\ w_{i,t+1}^{k} = w_{it} - x_{i,t+1}^{k} \right\}$$

Since we assumed above that remaining stocks at the end of period T will not yield future revenues, $Ez_{i,T+1}^{pr}(\cdot)$ = 0. The supply problem (7.12) for period t, t = T-1,…,1, can be written as, with $w_{i,t+1}^{k}$ = $w_{it} - x_{i,t+1}^{k}$, see (3.50):

$$z_{it}^{pr}\left(w_{i,t-1}, p_{it}\right) = \underset{x_{it}, x_{i,t+1}^{k}, w_{it}}{Max} \left\{ \left(p_{it} - c_{it}\right) x_{it} + \sum_{k=1}^{K} f_{i,t+1}^{k} \left[\left(p_{i,t+1}^{k} - c_{i,t+1} \right) x_{i,t+1}^{k} + \right. \right. \tag{7.14}$$
$$\left. \left. Ez_{i,t+2}^{pr}\left(w_{it} - x_{i,t+1}^{k}, P_{i,t+2} \right) \right] \right| \ 0 \le x_{i,t+1}^{k} \le w_{it},\ 0 \le x_{it} \le w_{i,t-1}, w_{it} = w_{i,t-1} - x_{it} \left. \vphantom{\sum} \right\}$$

Optimal producer supply for period T-1

Consider the optimal supply for period T-1, $x_{i,T\text{-}1}$. For each region i = 1,…,n, the set of possible realizations {1,…,K} in period T can be divided in two subsets – see (7.11):

$$\begin{aligned} K_{iT}^{1} &= \left\{ k \in \{1,...,K\} \,\middle|\, p_{iT}^{k} < c_{iT} \right\} \\ K_{iT}^{2} &= \left\{ k \in \{1,...,K\} \,\middle|\, p_{iT}^{k} \ge c_{iT} \right\} \end{aligned} \tag{7.15}$$

Due to (7.11), it follows that the optimal solution x_{iT}^{k} in (7.14) may be written as x_{iT}^{k} = 0 for $k \in K_{iT}^{1}$, and x_{iT}^{k} = $w_{i,T\text{-}1} = w_{i,T\text{-}2} - x_{i,T\text{-}1}$ for $k \in K_{iT}^{2}$. For the k for which p_{iT}^{k} = x_{iT} the solution is not unique. Let $(a)^{+}$ = max(a;0), the positive point of a. Define for each region i = 1,…,n:

$$(7.16)\quad \Psi_{iT} = \sum_{k \in K_{i2}} f_{iT}^{k}\left(p_{iT}^{k} - c_{iT}\right) = \sum_{k=1}^{K} f_{iT}^{k}\left(p_{iT}^{k} - c_{iT}\right)^{+} = E\left[P_{iT} - c_{iT}\right]^{+}$$

For $t = T$-1, (7.14) is:

$$\begin{aligned} z_{i,T-1}^{pr}\left(w_{i,T-2}, p_{i,T-1}\right) = \underset{x_{i,T-1}, w_{i.T-1}}{Max} \Big\{ &\left(p_{i,T-1} - c_{i,T-1}\right)x_{i,T-1} + \\ &+ \sum_{k \in K_{iT}^{1}} f_{iT}^{k}\left(p_{iT}^{k} - c_{iT}\right)0 + \sum_{k \in K_{iT}^{2}} f_{iT}^{k}\left(p_{iT}^{k} - c_{iT}\right)\left(w_{i,T-2} - x_{i,T-1}\right) \\ (7.17)\qquad &\Big|\ 0 \le x_{i,T-1} \le w_{i,T-2},\ w_{i,T-1} = w_{i,T-2} - x_{i,T-1} \Big\} = \\ = \underset{x_{i,T-1}, w_{i,T-1}}{Max} \Big\{ &\left(p_{i,T-1} - c_{i,T-1} - \Psi_{iT}\right)x_{i,T-1} + \Psi_{iT} w_{i,T-2} \\ &\Big|\ 0 \le x_{i,T-1} \le w_{i,T-2},\ w_{i,T-1} = w_{i,T-2} - x_{i,T-1} \Big\} \end{aligned}$$

Ψ_{iT} can be interpreted as the expected net revenues of selling one kg in period T or not selling it at all. Ψ_{iT} is a constant. The solution of model (7.17) depends on the difference between the current net revenues, $p_{i,T\text{-}1} - c_{i,T\text{-}1}$, and the expected net revenues for the next period Ψ_{iT}:

$$(7.18)\quad \begin{cases} \text{if } p_{i,T-1} - c_{i,T-1} < \Psi_{iT} \text{ then } x_{i,T-1} = 0 \\ \text{if } p_{i,T-1} - c_{i,T-1} \ge \Psi_{iT} \text{ then } x_{i,T-1} = w_{i,T-2} \\ \text{if } p_{i,T-1} - c_{i,T-1} = \Psi_{iT} \text{ then any solution } x_{i,T-1} \text{ between 0 and } w_{i,T-2} \text{ is optimal} \end{cases}$$

Optimal producer supply for period T-2

To determine an optimal solution of model (7.14) for the period T-2, analogous to the derivation of optimal supplies for period T-1, the set $\{1,\ldots,K\}$ can be subdivided in two subsets, see also (7.15):

$$K_{i,T-1}^{1} = \left\{ l \in \{1,\dots,K\} \middle| \; p_{i,T-1}^{l} < c_{i,T-1} + \Psi_{iT} \right\}$$

$$K_{i,T-1}^{2} = \left\{ l \in \{1,\dots,K\} \middle| \; p_{i,T-1}^{l} \geq c_{i,T-1} + \Psi_{iT} \right\}$$

The optimal solution for period T-1 may be written as, $x_{i,T-1}^{l} = w_{i,T-2} = w_{i,T-3} - x_{i,T-2}$, for $l \in K_{i,T-1}^{2}$. In that case nothing will be supplied in period T. For the l for which $p_{i,T-1}^{l} - c_{i,T-1} = \Psi_{iT}$, the solution is not unique. Finally, nothing will be supplied in period T-1, $x_{i,T-1}^{l} = 0$, if $l \in K_{i,T-1}^{1}$. In that case, supplies in period T will be equal to $x_{iT}^{k} = 0$ for $k \in K_{iT}^{1}$, and $x_{iT}^{k} = w_{i,T-2} = w_{i,T-3} - x_{i,T-2}$ for $k \in K_{iT}^{2}$ – see (7.15). Filling in model (7.14) for period T-2, the optimal supplies for the periods T-1 and T – see (7.11) and (7.18) – it can be derived that the optimal supply in period T-2 satisfies:

$$(7.19)\quad \begin{cases} \text{if } p_{i,T-2} - c_{i,T-2} < \Psi_{i,T-1} \text{ then } x_{i,T-2} = 0 \\ \text{if } p_{i,T-2} - c_{i,T-2} \geq \Psi_{i,T-1} \text{ then } x_{i,T-2} = w_{i,T-3} \\ \text{if } p_{i,T-2} - c_{i,T-2} = \Psi_{i,T-1} \text{ then any solution } x_{i,T-2} \text{ between 0 and } w_{i,T-3} \text{ is optimal} \end{cases}$$

with – see (7.16):

$$\begin{aligned} \Psi_{i,T-1} &= \sum_{l \in K_{i,T-1}^{1}} \sum_{k \in K_{iT}^{2}} f_{i,T-1}^{l} f_{iT}^{k} \left(p_{iT}^{k} - c_{iT} \right) + \sum_{l \in K_{i,T-1}^{2}} f_{i,T-1}^{l} \left(p_{i,T-1}^{l} - c_{i,T-1} \right) \\ &= \sum_{l=1}^{L} \sum_{k \in K_{iT}^{2}} f_{i,T-1}^{l} f_{iT}^{k} \left(p_{iT}^{k} - c_{iT} \right) + \sum_{l \in K_{i,T-1}^{2}} f_{i,T-1}^{l} \left(p_{i,T-1}^{l} - c_{i,T-1} - \sum_{k \in K_{iT}^{2}} f_{iT}^{k} \left(p_{iT}^{k} - c_{iT} \right) \right) \\ &= \Psi_{iT} + \sum_{l=1}^{K} f_{i,T-1}^{l} \left(p_{i,T-1}^{l} - c_{i,T-1} - \Psi_{iT} \right)^{+} \end{aligned}$$

$\Psi_{i,T-1}$ are the expected net revenues of selling one kg not in period T-2 but in one of the later periods or not selling it at all.

Optimal producer supply for the periods 1 *to T*

The optimal supplies for the periods T-3 to 1 can be determined in a similar way. In Appendix 2 the optimal supplied quantities are derived step by step for a slightly different supply decision problem for cereal producers in Burkina Faso, in which a year is divided in four periods, so $T = 4$. Define Ψ_{it}, $t \in \{1,\ldots,T\}$, the expected net revenues of selling one kg not in period t-1, but in one of the later periods or not selling it at all. Analogous to (7.16), Ψ_{it} can be written for $t \in \{1,\ldots,T\}$, as – see Appendix 2:

$$(7.20)\quad \Psi_{it} = \Psi_{i,t+1} + \sum_{k=1}^{K} f_{it}^{k}\left(p_{it}^{k} - c_{it} - \Psi_{i,t+1}\right)^{+}$$

with $\Psi_{i,T+1} = 0$. Since both terms on the right hand side are positive, it follows that $\Psi_{it} \geq \Psi_{i,t+1}$, $t \in \{1,\ldots,T\}$. In other words, expected net revenues from selling in one of the periods t to T exceed expected net revenues from selling in one of the periods t+1 to T. Analogous to (7.17), the producer supply model (7.14) for period $t \in \{1,\ldots,T\}$ is:

$$(7.21)\quad \begin{aligned} & z_{it}^{pr}\left(w_{i,t-1}, p_{it}\right) = \\ & \qquad \underset{x_{it}, w_{it}}{Max}\left\{\left(p_{it} - c_{it} - \Psi_{i,t+1}\right)x_{it} + \Psi_{i,t+1}w_{i,t-1} \,\middle|\, 0 \leq x_{it} \leq w_{i,t-1};\, w_{it} = w_{i,t-1} - x_{it}\right\} \end{aligned}$$

The optimum supply levels x_{it}, for the periods $t = 1,\ldots,T$, satisfy – see Appendix 2:

$$(7.22)\quad \begin{cases} \text{if } p_{it} - c_{it} < \Psi_{i,t+1} \text{ then } x_{it} = 0 \\ \text{if } p_{it} - c_{it} \geq \Psi_{i,t+1} \text{ then } x_{it} = w_{i,t-1} \\ \text{if } p_{it} - c_{it} = \Psi_{i,t+1} \text{ then any solution } x_{it} \text{ between 0 and } w_{i,t-1} \text{ is optimal} \end{cases}$$

These results indicate that the producer will sell his entire stock $w_{i,t-1}$, if the net revenues from sales in period t exceed the expected net revenues from sales in a later period. Else he will sell nothing. This result will be used in the next section to develop multi-period, spatial equilibrium models in which future prices are stochastic variables.

If we did not suppose in (7.8) that $c''_{it}(x_{it})$ = 0, but that $c''_{it}(x_{it}) \geq 0$, then Ψ_{iT} should be written as:

$$\Psi_{iT}\left(x_{i,T-1}\right) = \sum\nolimits_{k \in K_{i2}} f_{iT}^{k}\left(p_{iT}^{k} - c'_{iT}\left(x_{i,T-1}\right)\right)$$

with $K_{i2}(x_{i,T\text{-}1})$ = $\left\{k \in K \middle| \, p_{iT}^{k} \geq c'_{iT}\left(x_{i,T-1}\right)\right\}$. In that case K_{i2} would be a dynamic set depending on $x_{i,T\text{-}1}$, complicating the analysis considerably. Although it is possible to derive the optimal producer supply in a similar way as above, this will not be done in this paper.

<u>*Some characteristics of trader strategies:*</u>

To show how the behaviour of the traders depends on uncertain future prices, we make use of the variables and parameters introduced in (6.6) and (6.7). The aggregate trader operates on a competitive market, and can not influence producer and consumer prices. We assume that in period t, he does know the prices p_{it} and π_{it}, but that future prices are random variables of which the probability distributions are known. As for the producers, the basic characteristic of the trader's strategy to cope with uncertain prices, and uncertain supply and demand, is the sequential nature of the decision process. The trader's decision problem has many similarities with the producer's supply problem discussed above. The structure of the models to analyse their decision problem is similar to the structure of the producer model discussed above and of model (3.48) in Section 3.5. In period $t \in \{1,...,T\}$ he decides on the optimal strategies for this period, taking into account the strategies which he *expects* to be optimal in future periods. His decisions in period t are based on the observed current market prices, p_{it} and π_{it}, and the probability distribution of possible prices for the future periods $\tau = t+1,\ldots,T$. Introduce, for $i \in \{1,\ldots,n\}$, $t \in \{1,\ldots,T\}$:

(7.23) P_{it} random future producer price for period t in region i
Π_{it} random future consumer price for period t in region i

We assume for the moment that random producer prices $P_{i1},\ldots,P_{iT}$ are mutually independent, and that also random consumer prices $\Pi_{i1},\ldots,\Pi_{iT}$ are mutually independent. Random producer prices in a period t are not necessarily mutually independent from the random consumer prices in period t. Assume that future prices have a discrete empirical probability distribution, which are written as, for $t \in \{1,\ldots,T\}$, $k \in \{1,\ldots,K\}$:

$$\text{(7.24)} \quad \Pr(\Pi_{1t} = \pi_{1t}^k \,;\, P_{1t} = p_{1t}^k ;\ldots; \Pi_{it} = \pi_{it}^k \,;\, P_{it} = p_{it}^k ;\ldots; \Pi_{nt} = \pi_{nt}^k \,;\, P_{nt} = p_{nt}^k) = g_t^k$$

with possible price realizations p_{it}^k and π_{it}^k, and corresponding probabilities g_t^k.[9] Expected prices are:

$$\text{(7.25)} \quad E\Pi_{it} = \sum_{k=1}^{K} g_t^k \pi_{it}^k \text{ and } EP_{it} = \sum_{k=1}^{K} g_t^k p_{it}^k \text{ , } t \in \{1,\ldots,T\},\ i \in \{1,\ldots,n\}.$$

Given current producer and consumer prices, p_{it} and π_{it}, the probability distributions of random future prices, $P_{i\tau}$ and $\Pi_{i\tau}$, $\tau = t+1,\ldots,T$, and the stock level at the end of period t-1, the trader decides in period t on the quantity of goods q_{it} he purchases from the producers, the quantity q_{ijt} he transfers to other regions, the quantity r_{it} he sells to the consumers, and the level of the stock v_{it} at the end of period t to be sold later – see (6.6).

[9] (7.24) may also be written as: $\Pr(\Pi_{1t} = \pi_{1t}^{k_\pi^1} \;;\; P_{1t} = p_{1t}^{k_p^1};\ldots;\Pi_{nt} = \pi_{nt}^{k_\pi^n} \;;\; P_{nt} = p_{nt}^{k_p^n}) = \hat{g}_{it}^{k_\pi^1, k_p^1,\ldots,k_\pi^n, k_p^n}$, for $k_\pi^i \in \{1,\ldots,K_\pi^i\}$ and $k_p^i \in \{1,\ldots,K_p^i\}$. We prefer to use (7.24), with $k \in \{1,\ldots,K\} = \left\{1,\ldots,\prod_{i=1}^{n} K_\pi^i \cdot K_p^i\right\}$.

In each period $t \in \{1,\ldots,T\}$ the trader optimizes his current revenues for period t, plus the expected future revenues for the periods t+1 to T, knowing the current producer and consumer prices, p_{it} and π_{it}, and the trader's stock $v_{i,t-1}$ in the region $i = 1,\ldots,n$. Define for $t \in \{1,\ldots,T\}$:

(7.26) $z_t^{tr}\left(\pi_{it}, p_{it}, v_{i,t-1} \middle| i \in \{1,...,n\}\right)$ the optimal current plus expected future revenues of the trader in period t.[10]

Like the producer supply problem, the trader's sequential decision process can be modelled using a recourse model. For period t the problem can be written as – see also (6.10) and (3.48):

(7.27)
$$
\begin{aligned}
& z_t^{tr}\left(\pi_{it}, p_{it}, v_{i,t-1} \middle| i \in \{1,\ldots,n\}\right) = \\
& \underset{\substack{r_{it}, q_{it}, \\ q_{ijt}, v_{it}}}{Max} \left\{ \sum_{i=1}^{n} \left[\pi_{it} r_{it} - p_{it} q_{it} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ijt} q_{ijt} - k_{it} v_{it} \right] + E z_{t+1}^{tr}\left(\Pi_{i,t+1}, P_{i,t+1}, v_{it} \middle| i \in \{1,\ldots,n\}\right) \right. \\
& \qquad \left| \; q_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{jit} + v_{i,t-1} = r_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ijt} + v_{it}; \;\; 0 \le q_{it} \le x_{it}; \right. \\
& \qquad \left. 0 \le r_{it} \le y_{it}; \;\; q_{ijt}, v_{it} \ge 0, \;\; i, j = 1,\ldots,n, \;\; j \neq i \right\}
\end{aligned}
$$

The quantities $v_{i,t-1}$ are the known stocks remaining from the previous period. In analogy with (6.10), in (7.27) the producer supply x_{it} and consumer demand y_{it}, i = 1,...,n, are given upperbounds on the traders' purchases from the producers and sales to the consumers, with x_{it} the optimal solution of model (7.10) for period T and of

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[10] $z_t^{tr}\left(\pi_{it}, p_{it}, v_{i,t-1} \middle| i \in \{1,\ldots,n\}\right)$ is a short notation for $z_t^{tr}\left(\pi_{1t},\ldots,\pi_{nt}, p_{1t},\ldots,p_{nt}, v_{1,t-1},\ldots,v_{n,t-1}\right)$.

model (7.12) for period t = 1,…,T-1, and in which $y_{it} = y_{it}(\pi_{it})$. $Ez^{tr}_{t+1}(\cdot)$ refers to the expectation of $z^{tr}_{t+1}(\cdot)$, with $\pi_{i,t+1}$ and $p_{i,t+1}$ replaced by the random prices $\Pi_{i,t+1}$ and $P_{i,t+1}$, see (3.49). We assume that $Ez^{tr}_{T+1}(\cdot)$ = 0, i.e. stocks remaining at the end of period T are assumed not to yield any future revenues. Define $q^k_{i,t+1}, r^k_{i,t+1}, q^k_{ij,t+1}$ and $v^k_{i,t+1}$ as the purchased, sold, transported and stored quantities, and $x^k_{i,t+1}$ and $y^k_{i,t+1}$ the upperbounds on the traders' purchases and sales, if producer and consumer prices in period t+1 and region i would be $p^k_{i,t+1}$ and $\pi^k_{i,t+1}$. Analogous to (3.49), $Ez^{tr}_{t+1}(\cdot)$ can be rewritten as – see also (7.13):

(7.28)
$$
\begin{aligned}
&Ez^{tr}_{t+1}\left(\Pi_{it}, P_{it}, v_{it} \,\middle|\, i \in \{1,\ldots,n\}\right) = \\
&\underset{\substack{q^k_{i,t+1}, r^k_{i,t+1}, \\ q^k_{ij,t+1}, v^k_{i,t+1}}}{Max} \left\{ \sum_{k=1}^{K} g^k_{t+1} \left[\sum_{i=1}^{n} \left[\pi^k_{i,t+1} r^k_{i,t+1} - p^k_{i,t+1} q^k_{i,t+1} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \tau_{ij,t+1} q^k_{ij,t+1} - k_{i,t+1} v^k_{i,t+1} \right] + \right.\right. \\
&\qquad\qquad \left. Ez^{tr}_{t+2}\left(\Pi_{i,t+2}, P_{i,t+2}, v^k_{i,t+1} \,\middle|\, i \in \{1,\ldots,n\}\right) \right] \\
&\qquad \left| \; q^k_{i,t+1} + \sum_{\substack{j=1 \\ j\neq i}}^{n} q^k_{ji,t+1} + v_{it} = r^k_{i,t+1} + \sum_{\substack{j=1 \\ j\neq i}}^{n} q^k_{ij,t+1} + v^k_{i,t+1};\; 0 \le q^k_{i,t+1} \le x^k_{i,t+1}; \right. \\
&\qquad\qquad \left. 0 \le r^k_{i,t+1} \le y^k_{i,t+1};\; q^k_{ij,t+1}, v^k_{i,t+1} \ge 0;\; i,j = 1,\ldots,n,\; j \neq i,\; k = 1,\ldots,K \right\}
\end{aligned}
$$

with $Ez^{tr}_{t+2}\left(\Pi_{i,t+2}, P_{i,t+2}, v^k_{i.t+1} \middle| i \in \{1,...,n\}\right) = \sum_{l=1}^{K} g^l_{t+2}\, z^{tr}_{t+2}\left(\pi^l_{i,t+2}, p^l_{i,t+2}, v^k_{i,t+1} \middle| i \in \{1,...,n\}\right)$, in which $\pi^l_{i,t+2}$ and $p^l_{i,t+2}$ are the possible price realizations in period t+2, for $l \in \{1,\ldots,K\}$. Purchased, sold, transported and stored quantities in period t+2, if producer and consumer prices in period t+1 and t+2 are equal to, respectively, $p^k_{i,t+1}$, $\pi^k_{i,t+1}$,

$p^l_{i,t+2}$ and $\pi^l_{i,t+2}$, can be defined as $q^{k,l}_{i,t+2}, r^{k,l}_{i,t+2}, q^{k,l}_{ij,t+2}$ and $v^{k,l}_{i,t+2}$, for k = 1,…,K, l = 1,…,K.

Introduce λ_{it} the Lagrange multipliers of the equilibrium constraints of model (7.27), $\lambda^k_{i,t+1}$ the Lagrange multipliers of the equilibrium constraints in $Ez^{tr}_{t+1}(\cdot)$, and $\lambda^{k,l}_{i,t+2}$ the Lagrange multipliers of the equilibrium constraints in $Ez^{tr}_{t+2}(\cdot)$, for t= 1,…,T-1, k,l = 1,…,K. A part of the Kuhn-Tucker conditions of model (7.27) read – see (3.38) - (3.40) and (3.42) - (3.44):

$$(7.29)\quad \begin{cases} \text{If } q_{it}=0 \text{ then } \lambda_{it}\le p_{it} \\ \text{If } 0<q_{it}<x_{it} \text{ then } \lambda_{it}=p_{it} \\ \text{If } q_{it}=x_{it} \text{ then } \lambda_{it}\ge p_{it} \end{cases} \qquad \begin{cases} \text{If } q^k_{i,t+1}=0 \text{ then } \lambda^k_{i,t+1}\le g^k_{t+1}p^k_{i,t+1} \\ \text{If } 0<q^k_{i,t+1}<x^k_{i,t+1} \text{ then } \lambda^k_{i,t+1}=g^k_{t+1}p^k_{i,t+1} \\ \text{If } q^k_{i,t+1}=x^k_{i,t+1} \text{ then } \lambda^k_{i,t+1}\ge g^k_{t+1}p^k_{i,t+1} \end{cases}$$

$$(7.30)\quad \begin{cases} \text{If } r_{it}=0 \text{ then } \lambda_{it}\ge \pi_{it} \\ \text{If } 0<r_{it}<y_{it} \text{ then } \lambda_{it}=\pi_{it} \\ \text{If } r_{it}=y_{it} \text{ then } \lambda_{it}\le \pi_{it} \end{cases} \qquad \begin{cases} \text{If } r^k_{i,t+1}=0 \text{ then } \lambda^k_{i,t+1}\ge g^k_{t+1}\pi^k_{i,t+1} \\ \text{If } 0<r^k_{i,t+1}<y^k_{i,t+1} \text{ then } \lambda^k_{i,t+1}=g^k_{t+1}\pi^k_{i,t+1} \\ \text{If } r^k_{i,t+1}=y^k_{i,t+1} \text{ then } \lambda^k_{i,t+1}\le g^k_{t+1}\pi^k_{i,t+1} \end{cases}$$

$$(7.31)\quad \begin{cases} \text{If } q_{ijt}=0 \text{ then } \lambda_{jt}\le \lambda_{it}+\tau_{ijt} \\ \text{If } q_{ijt}>0 \text{ then } \lambda_{jt}=\lambda_{it}+\tau_{ijt} \end{cases}$$

$$(7.32)\quad \begin{cases} \text{If } v_{it}=0 \text{ then } \sum_k \lambda^k_{i,t+1}\le \lambda_{it}+k_{it} \\ \text{If } v_{it}>0 \text{ then } \sum_k \lambda^k_{i,t+1}= \lambda_{it}+k_{it} \end{cases}$$

$$\begin{cases} \text{If } v^k_{i,t+1}=0 \text{ then } \sum_l \lambda^{k,l}_{i,t+2}\le \lambda^k_{i,t+1}+g^k_{t+1}k_{i,t+1} \\ \text{If } v^k_{i,t+1}>0 \text{ then } \sum_l \lambda^{k,l}_{i,t+2}= \lambda^k_{i,t+1}+g^k_{t+1}k_{i,t+1} \end{cases}$$

To derive condition (7.32) for $v^k_{i,t+1}$, write out $Ez^{tr}_{t+2}(\cdot)$, in which $v^k_{i,t+1}$ occurs only in the equilibrium constraint, with Lagrange multiplier $\lambda^{k,l}_{i,t+2}$, for $l \in \{1,…,K\}$. The Kuhn-Tucker conditions show that the results of model (7.27) satisfy a number of properties. Like in Section 6.1, these properties show the influence of the producer and consumer price levels, on the traders' optimal purchased, sold, transported and stored quantities. The interpretation of the properties is equal to the interpretation of the *Trader properties* 6.1 to 6.4 in Section 6.1.

Trader property 7.1: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:
a) If $\pi_{it} < p_{it}$, then any optimal solution of (7.27) satisfies $q_{it} = 0$ or $r_{it} = 0$.
b) If $\pi_{it} \geq p_{it}$, then an optimal solution of (7.27) exists, satisfying the condition $q_{it} = x_{it}$ or $r_{it} = y_{it}$. For $\pi_{it} = p_{it}$, other optimal solutions of (7.27) may exist, not satisfying this condition.

Proof: See Appendix 1.

Trader property 7.2: Let q_{it}, r_{jt}, q_{ijt}, v_{it}, $j \neq i$, $i,j = 1,\ldots,n$, $t = 1,\ldots,T$, be an optimal solution of (7.27). Let a trader transport in a period t from region i to j, so $q_{ijt} > 0$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, $t \in \{1,\ldots,T\}$, then:
a) no goods are transported from a region $s = 1,\ldots,n, s \neq i$, to region i, $q_{sit} = 0$
b) no goods are transported from region j to a region $s = 1,\ldots,n$, $,s \neq j$, $q_{jst} = 0$.
c) purchases in region i are positive, $q_{it} > 0$, or the stock remaining from the previous period is positive, $v_{i,t-1} > 0$.
d) sales in region j are positive, $r_{jt} > 0$, or the quantity put in stock in region j is positive, $v_{jt} > 0$.

Proof: See Appendix 1.

Trader property 7.3: For region $i,j \in \{1,\ldots,n\}$, $i \neq j$, and period $t \in \{1,\ldots,T\}$:
a) If $\pi_{jt} < p_{it} + \tau_{ijt}$, then any optimal solution of (7.27) has to satisfy $q_{it} = 0$ or $q_{ijt} = 0$ or $r_{jt} = 0$.
b) If $\pi_{jt} \geq p_{it} + \tau_{ijt}$, and $q_{it} > 0$, $q_{ijt} > 0$ and $r_{jt} > 0$, then an optimal solution of (7.27) exists satisfying $q_{it} = x_{it}$ or $r_{jt} = y_{it}$. For $\pi_{jt} = p_{it} + \tau_{ijt}$, an optimal solution of (7.27) is not unique.

Proof: See Appendix 1.

We recall that storing from period t to the end of period τ-1 costs $\kappa_{it\tau}$ per unit – see (6.12). Analogous to the results of (6.10), it follows that $v_{iT} = 0$.

Trader property 7.4: For region $i \in \{1,\dots,n\}$, and period $t \in \{1,\dots,T\text{-}1\}$:

a) If $E\pi_{i,t+1} < p_{it} + k_{it}$, then any optimal solution of (7.27) has to satisfy $q_{it} = 0$ or $v_{it} = 0$ or $r^{k}_{i,t+1} = 0$ for at least one $k \in \{1,...,K\}$.

b) Analogously, if $E\pi_{i,t+2} < p_{it} + \kappa_{it,t+2}$, see also (6.12), then any optimal solution of (7.27) has to satisfy $q_{it} = 0$ or $v_{it} = 0$ or $v^{k}_{i,t+1} = 0$ or $r^{k,l}_{i,t+2} = 0$ for at least one k, $l \in \{1,...,K\}$. Analogous properties can be derived for storage until the periods $\tau = t+3,...,T$ if $E\pi_{i\tau} < p_{it} + \kappa_{it\tau}$.

c) If $E\pi_{i,t+1} \geq p_{it} + k_{it}$ and $q_{it} > 0$ and $s_{it} > 0$ and $r^{k}_{i,t+1} > 0$ for all $k = 1,\dots,K$, then an optimal solution of (7.27) exists satisfying $q_{it} = x_{it}$ or $r^{k}_{i,t+1} = y^{k}_{i,t+1}$ for at least one $k \in \{1,...,K\}$. For $E\pi_{i,t+1} = p_{it} + k_{it}$, an optimal solution of (7.27) is not unique.

d) Analogously: if $E\pi_{i,t+2} \geq p_{it} + \kappa_{it,t+2}$ and $q_{it} > 0$ and $v_{it} > 0$, $v^{k}_{i,t+1} > 0$ and $r^{k,l}_{i,t+2} > 0$, for all k, $l \in \{1,\dots,K\}$, then a solution of (7.27) which satisfies $q_{it} = x_{it}$ or $r^{k,l}_{i,t+2} = y^{k,l}_{i,t+2}$, for at least one k, $l \in \{1,...,K\}$, is an optimal solution. For $E\pi_{i,t+2} = p_{it} + \kappa_{it,t+2}$, an optimal solution of (7.27) is not unique. Analogous properties can be derived for storage until the periods $\tau = t+3,...,T$ if $E\pi_{i\tau} \geq p_{it} + \kappa_{it\tau}$.

Proof: See Appendix 1.

In this problem, in which decisions are taken on the basis of expectations on future prices, it is possible that a trader makes a loss out of a transaction. If the price in period t is lower than the price for period t he expected in earlier periods ($p_{it} < EP_{it}$), then it is possible that he makes a loss out of the sales from his stock $v_{i,t-1}$. In the previous chapter, this was not possible. In that situation he knew with certainty future prices, giving him the possibility to know in advance the profits he could get from storage.

These conditions will be used in the next section to verify whether the results of the stochastic equilibrium model to be developed satisfy the optimal strategies of the market agents.

7.2 Maximizing welfare; stochastic future prices

In this section we extend equilibrium model (6.14) - (6.16), to take into account stochastic future prices. In the multi-period, spatial equilibrium model of Section 6.2, optimal quantities and prices are determined for all T periods at once. In the model set up in this section, the optimal strategies of producers and traders in a period $t \in \{1,...,T\}$ depend on known past strategies, on observed current prices in period t, and on stochastic future prices. Therefore, in a period $t \in \{1,...,T\}$, we can only determine strategies which are optimal for the current period t, and provisional strategies which are *expected* to be optimal for future periods. We set up an equilibrium model for each period $t = 1,...,T$, in which the optimal values of the following variables are determined for $i = 1,...,n$: producer prices p_{it} and consumer prices π_{it} for period t, producer supply x_{it} for period t, consumer demand y_{it} for period t, total transported quantities x_{ijt} to the various regions, and stock levels s_{it} at the end of period t – see (6.13). These quantities depend on known stock levels at the end of period t-1, $s_{i,t-1}$, on the available producer stock $w_{i,t-1}$, see (7.6), and on uncertain future prices. In the equilibrium model also future transacted quantities are determined, which are expected to be optimal at the stochastic future prices. The structure of this model is comparable to the structure of model (3.48) in Section 3.5

In analogy with the Sections 5.2 and 6.2, we will discuss the set-up and results of the stochastic, multi-period, spatial equilibrium model, and show that the equilibrium quantities are in line with the optimal strategies of the individual producers, consumers and traders at the equilibrium producer and consumer prices.

Stochastic, multi-period, spatial equilibrium model

In the stochastic, multi-period, spatial equilibrium model for period t, we optimize current semi-welfare for period t plus *expected* future semi-welfare for the periods t+1 to T. In principle, we are interested in current semi-welfare. After all, expected future strategies can be adapted in later periods. However, since the strategies of the producers and traders in a period t depend on what they expect to be optimal in the future periods, expected future semi-welfare must be considered as well. In Section 5.2 semi-welfare was defined as the sum of consumer, plus producer, plus trader 'net revenues', with consumer 'revenues' defined as utility from consuming y_{it} minus the costs to purchase y_{it}: $\pi_{it} y_{it}$ – see (5.18). Likewise, in this section, current semi-welfare for period t can be defined as consumer, plus producer, plus trader revenues in period t. Current net revenues in period t can be written as:

- Current consumer net revenues: utility from consuming y_{it} minus costs to purchase y_{it}:

(7.33) $$\sum_{i=1}^{n}\left(u_{it}\left(y_{it}\right)-\pi_{it}y_{it}\right)$$

- Current producer net revenues: revenues minus costs from supplying x_{it} – see (7.8):

(7.34) $$\sum_{i=1}^{n}\left(p_{it}x_{it}-c_{it}\left(x_{it}\right)\right)=\sum_{i=1}^{n}\left(p_{it}x_{it}-c_{it}x_{it}\right)$$

- Current trader net revenues: revenues from selling y_{it}, minus costs to purchase x_{it}, minus costs to transport x_{ijt}, minus costs to store a quantity s_{it}, see (6.7):

(7.35) $$\sum_{i=1}^{n}\left(\pi_{it}y_{it}-p_{it}x_{it}-\sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ijt}x_{ijt}-k_{it}s_{it}\right)$$

Due to the properties of integrability of the utility and cost function and the properties of producer and consumer theory discussed in Chapter 4, utility from consuming y_{it} was equal to the integral of the inverse demand function (see (7.1)), and the costs of supplying x_{it} to the integral of the inverse supply function (see (6.3)). The properties of the supply problem discussed in Section 7.1 are, however, different from those in the Chapters 4 to 6. For that reason, the cost function in the objective of the stochastic equilibrium model can not be replaced by the integral of the inverse supply function. Consequently, current semi-welfare (7.33) + (7.34) + (7.35) can be written as:

$$(7.36) \quad \sum_{i=1}^{n} \left(\int_{0}^{y_{it}} \pi_{it}(\xi) d\xi - c_{it} x_{it} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ijt} x_{ijt} - k_{it} s_{it} \right)$$

where the variables, y_{it}, x_{it}, x_{ijt}, and s_{it} have to satisfy the constraint and non-negativity conditions:

$$(7.37) \quad x_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{jit} + s_{i,t-1} = y_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ijt} + s_{it}$$

$$(7.38) \quad 0 \leq x_{it} \leq w_{i,t\text{-}1}, y_{it} \geq 0, s_{it} \geq 0, x_{ijt} \geq 0, \quad i,j = 1, 2, ..., n; j \neq i.$$

for given stocks $s_{i,t\text{-}1}$ and $w_{i,t\text{-}1}$.

Expected future semi-welfare for the periods t+1 to T are the sum of expected consumer, expected producer, and expected trader revenues, with regard to random future producer prices $P_{i\tau}$ and consumer prices $\Pi_{i\tau}$, for $\tau = t+1$ to T. The perception of probability distributions of future prices may differ between producers, consumers and traders, depending on the information they have. Assume that producers have a price probability distribution function which is defined as in (7.5), and that the price probability distribution of the traders is defined as in (7.24). For consumers in region

i, we assume that the random prices $\Pi_{i1},...,\Pi_{iT}$ are mutually independent stochastic variables. The random price Π_{it} for period t is assumed to have a discrete, empirical distribution with possible realisations π_{it}^{k}, for k = 1,...,K, with:

(7.39) $$\Pr\left(\Pi_{it} = \pi_{it}^{k}\right) = h_{it}^{k}$$

with probabilities h_{it}^{k}. Expected future revenues for the consumers, producers, and traders can be written as:

- Expected future consumer revenues: since consumer demand satisfies the demand function $y_{it} = y_{it}(\pi_{it})$ in all periods $t \in \{1,\ldots,T\}$, see also (7.1), it follows that a consumer will demand in a period τ, $\tau = t+1,...,T$, a quantity $y_{i\tau}^{k} = y_{i\tau}\left(\pi_{i\tau}^{k}\right)$ if the consumer price is $\pi_{i\tau}^{k}$, for k = 1,...,K. Demand in a period τ is assumed not to depend on demanded quantities or prices in other periods. Optimal expected future revenues for the periods t+1 to T, $Ez_{i,t+1}^{c}\left(\Pi_{i,t+1}\right)$, can be defined as:

(7.40) $$\begin{aligned} Ez_{i,t+1}^{c}\left(\Pi_{i,t+1}\right) &= \sum_{k=1}^{K} h_{i,t+1}^{k}\left(u_{i,t+1}\left(y_{i,t+1}^{k}\right) - \pi_{i,t+1}^{k} y_{i,.t+1}^{k} + Ez_{i,t+2}^{c}\left(\Pi_{i,t+2}\right)\right) \\ &= \sum_{\tau=t+1}^{T} \sum_{k=1}^{K} h_{i\tau}^{k}\left(u_{i\tau}\left(y_{i\tau}^{k}\right) - \pi_{i\tau}^{k} y_{i\tau}^{k}\right) \end{aligned}$$

  with $Ez_{i,T+1}^{c}(\cdot)$ = 0.These revenues are a constant, since all elements are constants.

- Expected future producer revenues have already been discussed in Section 7.1. If a quantity x_{it} is supplied in period t, a quantity $w_{i,t-1}$ - x_{it} can be supplied in the remaining periods. In (7.13) we defined revenues expected to be earned in the future by the producers as:

(7.41)
$$Ez^{pr}_{i,t+1}\left(w_{i,t-1}-x_{it},P_{i,t+1}\right)=\underset{x^k_{i,t+1}}{Max}\left\{\sum_{k=1}^{K} f^k_{i,t+1}\left[\left(p^k_{i,t+1}-c_{i,t+1}\right)x^k_{i,t+1}+ Ez^{pr}_{i,t+2}\left(w_{i,t-1}-x_{it}-x^k_{i,t+1},P_{i,t+2}\right)\right] \,\middle|\, 0\le x^k_{i,t+1}\le w_{i,t-1}-x_{it}\right\}$$

- Expected future trader revenues have already been discussed in Section 7.1 – see (7.28). If the stock remaining from the period t is s_{it}, then traders expect to earn in the future revenues equal to (see also footnote 10):

(7.42)
$$\begin{aligned}&Ez^{tr}_{t+1}\left(\Pi_{i,t+1},P_{i,t+1},s_{it} \,\middle|\, i\in\{1,\dots,n\}\right)=\\&=\underset{\substack{q^k_{i,t+1},r^k_{i,t+1},\\ q^k_{ij,t+1},v^k_{i,t+1}}}{Max}\left\{\sum_{k=1}^{K} g^k_{t+1}\sum_{i=1}^{n}\left[\pi^k_{i,t+1}r^k_{i,t+1}-p^k_{i,t+1}q^k_{i,t+1}-\sum_{\substack{j=1\\ j\ne i}}^{n}\tau_{ij,t+1}q^k_{ij,t+1}-k_{i,t+1}v^k_{i,t+1}\right.\right.\\&\qquad\left.+Ez^{tr}_{t+2}\left(\Pi_{i,t+2},P_{i,t+2},v^k_{i,t+1} \,\middle|\, i\in\{1,\dots,n\}\right)\right]\\&\qquad\left| \; q^k_{i,t+1}+\sum_{\substack{j=1\\ j\ne i}}^{n}q^k_{ji,t+1}+s_{it}=r^k_{i,t+1}+\sum_{\substack{j=1\\ j\ne i}}^{n}q^k_{ij,t+1}+v^k_{i,t+1};\right.\\&\qquad\left. 0\le q^k_{i,t+1}\le \bar{x}^k_{i,t+1};\ 0\le r^k_{i,t+1}\le \bar{y}^k_{i,t+1};\ q^k_{ij,t+1},v^k_{i,t+1}\ge 0,\ i,j=1,\dots,n,\ j\ne i\right\}\end{aligned}$$

Note that the upperbound on supply $\bar{x}^k_{i,t+1}$ is a fixed bound, which is not necessarily equal to optimal future producer supply $x^k_{i,t+1}$ at price $p^k_{i,t+1}$. Below it will be explained why for these upper bounds not the variables $x^k_{i,t+1}$ should be taken, see Footnote 12 on page 108.

As will be proved later (see *Theorem* 7.2) welfare optimizing prices p_{it} and π_{it} in period t are formed in such a way, that it is optimal for the traders to purchase exactly the quantity the producers supply at producer price p_{it}, $q_{it} = x_{it}$, and to sell exactly the quantity consumers demand at consumer price π_{it}, $r_{it} = y_{it}(\pi_{it})$. In period t, producers plan for each possible future producer price $p_{i\tau}^{k}$, $k = 1,\ldots,K$, $\tau \in \{t+1,...,T\}$, to supply a quantity $x_{i\tau}^{k}$ which is optimal for them individually. Similarly, in period t, consumers plan for each possible consumer price $\pi_{i\tau}^{k}$ to demand in period $\tau \in \{t+1,...,T\}$ a quantity $y_{i\tau}^{k} = y_{i\tau}\left(\pi_{i\tau}^{k}\right)$ which is optimal for them individually. It is, however, not evident that it is also optimal for the traders to purchase the producer supply and to sell the consumer demand. It may be optimal for them to purchase or sell another quantity. Future strategies of the individual market agents are expected to be optimal for them individually, but this does not mean that they are also optimal for the other market agents. For that reason it is not possible to impose for each period t+1 to T and for each possible price realization a market equilibrium as defined in (7.37) for period t.

Define for each period $t \in \{1,\ldots,T\}$ the optimal current plus expected future semi-welfare, knowing the producer and trader stocks available at the beginning of period t, $w_{i,t-1}$ and $s_{i,t-1}$:

(7.43) $z_t(s_{i,t-1}, w_{i,t-1} \mid i \in \{1,\ldots,n\})$ optimal current plus expected future semi-welfare in period t.

Optimizing the sum of current semi-welfare for period t plus expected future semi-welfare, subject to the market equilibrium condition (7.37) and non-negativity conditions (7.38) for period t, and the supply upperbound $x_{it} \leq w_{i,t-1}$, results in the following stochastic, multi-period, spatial equilibrium model for period $t \in \{1,...,T\text{-}1\}$ as – see (7.36) - (7.38) and (7.40) - (7.42), see also (6.15), (6.16), and (3.48):

$$
\begin{aligned}
&z_t\left(s_{i,t-1}, w_{i,t-1} \middle| i \in \{1,\ldots,n\}\right) = \\
&\underset{\substack{y_{it},x_{it},\\ x_{ijt},s_{it}}}{Max}\left\{\sum_{i=1}^{n}\left[\int_0^{y_{it}} \pi_{it}(\xi)d\xi - c_{it}x_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ijt}x_{ijt} - k_{it}s_{it}\right] + \sum_{i=1}^{n} Ez^{c}_{i,t+1}\left(\Pi_{i,t+1}\right) + \right. \\
&\quad + \sum_{i=1}^{n} Ez^{pr}_{i,t+1}\left(w_{i,t-1} - x_{it}, P_{i,t+1}\right) + Ez^{tr}_{t+1}\left(\Pi_{i,t+1}, P_{i,t+1}, s_{it} \middle| i \in \{1,\ldots,n\}\right) \\
&\quad \left| \; x_{it} + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{jit} + s_{i,t-1} = y_{it} + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ijt} + s_{it} \right. \\
&\quad \left. x_{it} \le w_{i,t-1};\; x_{it}, y_{it}, x_{ijt}, s_{it} \ge 0, \;\; i,j = 1,\ldots,n, j \neq i \right\}
\end{aligned}
\tag{7.44}
$$

In period t, the initial stocks $s_{i,t\text{-}1}$ and $w_{i,t\text{-}1}$ are known parameters. $s_{i,t\text{-}1}$ is the quantity in stock at the end of period t-1, $w_{i,t\text{-}1}$ is the stocks of the producers – see (7.6). The model for period T is similar to (7.44), but without the terms for the expected future revenues. Note that $s_{i0} = 0$.

We will prove that the optimal supplied, demanded, transported and stored quantities resulting from model (7.44), are equal to, respectively, the optimal producer supply, consumer demand, trader transport flows, and trader stock levels, at the market equilibrium prices, as discussed in Section 7.1.

Optimal equilibrium prices and quantities for period T

Consider first the model for period T. Let x_{iT}, y_{iT}, x_{ijT}, and s_{iT}, $i,j = 1,\ldots,n$, $i \neq j$, be the optimal solution of model (7.44) for period T. Let λ_{iT} be the value of the Lagrange multiplier for the corresponding equilibrium condition. The Kuhn-Tucker conditions result in the following expressions – see (3.34) and (3.38) - (3.40):

$$
(7.45) \quad \begin{cases} \text{if } x_{iT} = 0 & \text{then } -c_{iT} + \lambda_{iT} \le 0 \\ \text{if } 0 < x_{iT} < w_{i,T-1} & \text{then } -c_{iT} + \lambda_{iT} = 0 \\ \text{if } x_{iT} = w_{i,T-1} & \text{then } -c_{iT} + \lambda_{iT} \ge 0 \end{cases}
$$

$$
(7.46) \quad \begin{cases} \text{if } y_{iT} = 0 & \text{then } \pi_{iT}(0) - \lambda_{iT} \le 0 \\ \text{if } y_{iT} > 0 & \text{then } \pi_{iT}(y_{it}) - \lambda_{iT} = 0 \end{cases}
$$

$$
(7.47) \quad \begin{cases} \text{if } x_{ijT} = 0 & \text{then } -\tau_{ijT} - \lambda_{iT} + \lambda_{jT} \le 0 \\ \text{if } x_{ijT} > 0 & \text{then } -\tau_{ijT} - \lambda_{iT} + \lambda_{jT} = 0 \end{cases}
$$

Analogous to the argumentation in footnote 6, it follows that $s_{iT} = 0$. The equilibrium model for period T results in optimal values for supplies, demand, transport and storage, from which optimal consumer price levels, $\pi_{iT}(y_{iT})$, follow. Due to the peculiar form of producer supply, see (7.11), (7.18), and (7.19), the model can not not determine a unique optimal value of the producer price, $p_{iT}(x_{iT})$. One may wonder for which producer price, producers are interested in supplying the equilibrium supply x_{iT}. This follows from the following theorem.

Theorem 7.1a:

Let in the optimal solution of the equilibrium model (7.44) for period T, $\hat{x}_{iT}$ be the optimal supply level and λ_{iT} be the corresponding optimal value of the Lagrange multiplier, for $i = 1,\ldots,n$. If the producer price in period T in region i is equal to:

(7.48) $p_{iT} = \lambda_{iT}$

then $x_{iT} = \hat{x}_{iT}$ is an optimal solution of model (7.10), the producer supply model for period T. In other words, the optimal equilibrium supply level is a supply level which gives the producers optimal profits in period T. Since the value of λ_{iT}, depends on the value of the equilibrium supply level, we write $p_{iT}(x_{iT}) = \lambda_{iT}$.[11]

Proof: See Appendix 1.

Like in Theorem 6.1, we can also prove that in period T traders are interested in buying x_{iT} from the producers, transporting x_{ijT}, and selling y_{iT} to the consumers. This is proved in the following theorem.

Theorem 7.2a:
Let x_{iT}, y_{iT}, x_{ijT}, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of equilibrium model (7.44) for period T. Let $\pi_{iT} = \pi_{iT}(y_{iT})$, $p_{iT} = p_{iT}(x_{iT}) = \lambda_{iT}$. The solution:

(7.49) $\quad q_{iT} = x_{iT} \;\; ; \;\; r_{iT} = y_{iT} \;\; ; \;\; q_{ijT} = x_{ijT} \qquad$ for $i,j \in \{1,\ldots,n\}$, $i \neq j$

is an optimal solution of the trader decision problem (7.27) for period T.

Proof: See Appendix 1.

Theorem 7.1a and 7.2a prove that the welfare optimal quantities for period T are in line with the optimal strategies of the individual agents.

Optimal equilibrium prices and quantities for period T-1
Consider now the equilibrium model for period T-1. Rewriting the expected future semi-welfare in (7.44), results in the following equilibrium model, with the variables $y_{i,T\text{-}1}$, $x_{i,T\text{-}1}$, $x_{ij,T\text{-}1}$, $s_{i,T\text{-}1}$, x_{iT}^{k} , r_{iT}^{k} , q_{iT}^{k} , q_{ijT}^{k} , and v_{iT}^{k} :

[11] $\lambda_{iT} > 0$ due to the assumption that $\pi_{iT}(0) > 0$.

$$z_{T-1}\left(s_{i,T-2}, w_{i,T-2} \,\middle|\, i \in \{1,\dots,n\}\right) =$$

(7.50)

$$\max \sum_{i=1}^{n} \left[\int_0^{y_{i,T-1}} \pi_{i,T-1}(\xi) d\xi - c_{i,T-1} x_{i,T-1} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ij,T-1} x_{ij,T-1} - k_{i,T-1} s_{i,T-1} \right]$$

$$+ \sum_{i=1}^{n} \sum_{k=1}^{K} h_{iT}^{k} \left(\int_0^{y_{iT}^{k}} \pi_{iT}(\xi) d\xi - y_{iT}^{k} \pi_{iT}^{k} \right) + \sum_{i=1}^{n} \sum_{k=1}^{K} f_{iT}^{k} \left(p_{iT}^{k} - c_{iT} \right) x_{iT}^{k}$$

$$+ \sum_{k=1}^{K} g_T^{k} \sum_{i=1}^{n} \left(\pi_{iT}^{k} r_{iT}^{k} - p_{iT}^{k} q_{iT}^{k} - \sum_{\substack{j=1 \\ j \neq i}}^{n} \tau_{ijT} q_{ijT} - k_{iT} v_{iT}^{k} \right)$$

subject to:

$$x_{i,T-1} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ji,T-1} + s_{i,T-2} = y_{i,T-1} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ij,T-1} + s_{i,T-1} \quad \left[\lambda_{i,T-1}\right];$$

$$q_{iT}^{k} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{jiT}^{k} + s_{i,T-1} = r_{iT}^{k} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ijT}^{k} + v_{iT}^{k} \quad \left[\lambda_{iT}^{k}\right]; \quad x_{i,T-1} + x_{iT}^{k} \le w_{i,T-2} \quad \left[\gamma_i^{k}\right];$$

$$0 \le r_{iT}^{k} \le \bar{y}_{iT}^{k} \quad \left[\mu_{iT}^{k}\right]; \quad 0 \le q_{iT}^{k} \le \bar{x}_{iT}^{k} \quad \left[\vartheta_{iT}^{k}\right];$$

$$x_{i,T-1}, y_{i,T-1}, x_{ij,T-1}, s_{i,T-1}, x_{iT}^{k}, q_{ijT}^{k} \ge 0, \quad i, j = 1,\dots,n, j \neq i$$

The terms between square brackets are the Lagrange multipliers. We will again prove that the optimal supplied, demanded, transported and stored quantities resulting from model (7.50), are equal to, respectively, the optimal producer supply, consumer demand, trader transport flows, and trader stock levels, discussed in Section 7.1. Let $y_{i,T-1}$, $x_{i,T-1}$, $x_{ij,T-1}$, $s_{i,T-1}$, x_{iT}^{k}, r_{iT}^{k}, q_{iT}^{k}, q_{ijT}^{k}, and v_{iT}^{k}, $i,j = 1,\dots,n$, $i \neq j$, be the optimal solution of model (7.50). Analogous to (3.56) - (3.60) in Section 3.5, the Kuhn-Tucker conditions of model (7.50) can be derived. They are presented in Appendix 1. With $L(\cdot)$ the Lagrange function and χ one of the variables of model (7.50), the Kuhn-Tucker conditions signify that – see (3.33) and (3.59):

$$(7.51)\quad \begin{cases} \text{if } \chi > 0 & \text{then } \dfrac{\partial L}{\partial \chi}(\cdot) = 0 \\ \text{if } \chi = 0 & \text{then } \dfrac{\partial L}{\partial \chi}(\cdot) \le 0 \end{cases}$$

Using the Kuhn-Tucker conditions, it follows that $v_{iT}^k = 0$ for all $i \in \{1,\ldots,n\}$ and $k \in \{1,\ldots,K\}$. Furthermore, similar to Theorem 7.1a and 7.2a, the following two theorem can be proved:

<u>Theorem 7.1b:</u>
Let in the optimal solution of the equilibrium model (7.50), $\hat{x}_{i,T-1}$ and $\hat{x}_{iT}^k$ be the optimal supply levels for period T-1 and T, respectively, and let $\lambda_{i,T\text{-}1}$, λ_{iT}^k and γ_i^k be the corresponding optimal values of the Lagrange multipliers, for $i = 1,\ldots,n$ and $k = 1,\ldots,K$. If the producer price in period T-1 in region i is equal to:

$$(7.52)\quad p_{i,T\text{-}1} = \lambda_{i,T\text{-}1}$$

then $x_{i,T\text{-}1} = \hat{x}_{i,T-1}$, and $x_{iT}^k = \hat{x}_{iT}^k$ are optimal solutions of model (7.14), the producer supply model for period T-1.[12] In other words, the optimal equilibrium supply levels give the producers optimal profits in period T-1. Since the value of $\lambda_{i,T\text{-}1}$, depends on the value of the equilibrium supply level, we write $p_{i,T\text{-}1}(x_{i,T\text{-}1}) = \lambda_{i,T\text{-}1}$.

<u>Proof:</u> See Appendix 1.

---

[12] Note that the upperbound on trader purchases in period T, $\bar{x}_{iT}^k$, has to be an exogenous upperbound, and may not be the variable x_{iT}^k. If the upperbound would be the variable x_{iT}^k, the Kuhn-Tucker conditions would change, in that way not well reflecting producer strategies.

Like in Theorem 7.2a, we can also prove that in period T-1 traders are interested in buying $x_{i,T\text{-}1}$ from the producers, transporting $x_{ij,T\text{-}1}$, storing $s_{i,T\text{-}1}$, and selling $y_{i,T\text{-}1}$ to the consumers. This is proved in the following theorem.

Theorem 7.2b:

Let $y_{i,T\text{-}1}$, $x_{i,T\text{-}1}$, $x_{ij,T\text{-}1}$, $s_{i,T\text{-}1}$, $\hat{r}_{iT}^{k}$, $\hat{q}_{iT}^{k}$, and $\hat{q}_{ijT}^{k}$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of model (7.50). Let $\pi_{i,T\text{-}1} = \pi_{i,T\text{-}1}(y_{i,T\text{-}1})$, $p_{i,T\text{-}1} = p_{i,T\text{-}1}(x_{i,T\text{-}1}) = \lambda_{i,T\text{-}1}$. The solution:

(7.53) $q_{i,T\text{-}1} = x_{i,T\text{-}1}$; $r_{i,T\text{-}1} = y_{i,T\text{-}1}$; $q_{ij,T\text{-}1} = x_{ij,T\text{-}1}$; $v_{i,T\text{-}1} = s_{i,T\text{-}1}$;
$r_{iT}^{k} = \hat{r}_{iT}^{k}$; $q_{iT}^{k} = \hat{q}_{iT}^{k}$; $q_{ijT}^{k} = \hat{q}_{ijT}^{k}$

for $k \in \{1,\ldots,K\}$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, is an optimal solution of the trader decision problem (7.27) for period T-1.

Proof: See Appendix 1.

Theorem 7.1b and 7.2b prove that the welfare optimal quantities for period T-1 are in line with the optimal strategies of the individual agents.

Optimal equilibrium prices and quantities for period $t \in \{1,\ldots,T\text{-}2\}$

Consider now the equilibrium model for period $t \in \{1,\ldots,T\text{-}2\}$. In this period optimal values of x_{it}, y_{it}, x_{ijt}, and s_{it} are determined, as well as optimal values of future consumer demand, producer supply, and trader purchases, sales transports and storage, for all possible future price realisations. We define:

- $y_{i,t+1}^{k_1}$, $x_{i,t+1}^{k_1}$, $r_{i,t+1}^{k_1}$, $q_{i,t+1}^{k_1}$, $q_{ij,t+1}^{k_1}$, $v_{i,t+1}^{k_1}$: optimal consumer demand, producer supply, and trader sales, purchases, transports and storage, for the period t+1, if consumer and producer prices in this period are $\pi_{i,t+1}^{k_1}$ and $p_{i,t+1}^{k_1}$, for $k_1 = 1,\ldots,K$.

- $y_{i,t+2}^{k_1,k_2}$, $x_{i,t+2}^{k_1,k_2}$, $r_{i,t+2}^{k_1,k_2}$, $q_{i,t+2}^{k_1,k_2}$, $q_{ij,t+2}^{k_1,k_2}$, $v_{i,t+2}^{k_1,k_2}$: optimal consumer demand, producer supply, and trader sales, purcases, transports and storage, respectively, for the period t+2, if consumer and producer prices in period t+1 and period t+2 are $\pi_{i,t+1}^{k_1}, p_{i,t+1}^{k_1}, \pi_{i,t+2}^{k_2}$ and $p_{i,t+2}^{k_2}$, for k_1, k_2 = 1,…,K.
- $y_{i,t+\tau}^{k_1,k_2,\dots,k_\tau}$, $x_{i,t+\tau}^{k_1,k_2,\dots,k_\tau}$, $r_{i,t+\tau}^{k_1,k_2,\dots,k_\tau}$, $q_{i,t+\tau}^{k_1,k_2,\dots,k_\tau}$, $q_{ij,t+\tau}^{k_1,k_2,\dots,k_\tau}$, $v_{i,t+\tau}^{k_1,k_2,\dots,k_\tau}$: optimal consumer demand, producer supply, and trader sales, purcases, transports and storage, respectively, for the period $t+\tau$, if consumer and producer prices in the periods t+1 to the period $t+\tau$ are $\pi_{i,t+1}^{k_1}, p_{i,t+1}^{k_1}, \pi_{i,t+2}^{k_2}, p_{i,t+2}^{k_2}, \dots \pi_{i,t+\tau}^{k_\tau}$ and $p_{i,t+\tau}^{k_\tau}$ for k_1, k_2,…,k_τ = 1,…,K, τ = 2,…,T-t.

Rewriting the expected future semi-welfare in (7.44) results in the following equilibrium model, with the variables y_{it}, x_{it}, x_{ijt}, s_{it}, $x_{i,t+1}^{k_1}$, $r_{i,t+1}^{k_1}$, $q_{i,t+1}^{k_1}$, $q_{ij,t+1}^{k_1}$, $v_{i,t+1}^{k_1}$, $x_{i,t+\tau}^{k_1,\dots,k_\tau}$, $r_{i,t+\tau}^{k_1,\dots,k_\tau}$, $q_{i,t+\tau}^{k_1,\dots,k_\tau}$, $q_{ij,t+\tau}^{k_1,\dots,k_\tau}$, $v_{i,t+\tau}^{k_1,\dots,k_\tau}$, τ = 2,…,T-t:

$$
\begin{aligned}
z_t\left(s_{i,t-1}, w_{i,t-1} \mid i \in \{1,\dots,n\}\right) = \max &\sum_{i=1}^{n}\left[\int_0^{y_{it}} \pi_{it}(\xi)d\xi - c_{it}x_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n} \tau_{ijt}x_{ijt} - k_{it}s_{it}\right. \\
&+\sum_{i=1}^{n}\left[\sum_{k_1=1}^{K} h_{i,t+1}^{k_1}\left(\int_0^{y_{i,t+1}\left(\pi_{i,t+1}^{k_1}\right)} \pi_{i,t+1}(\xi)d\xi - y_{i,t+1}\left(\pi_{i,t+1}^{k_1}\right)\pi_{i,t+1}^{k_1}\right) + \sum_{k_1=1}^{K} f_{i,t+1}^{k_1}\left(p_{i,t+1}^{k_1} - c_{i,t+1}\right)x_{i,t+1}^{k_1}\right] \\
&\left.+\sum_{k_1=1}^{K} g_{t+1}^{k_1}\sum_{i=1}^{n}\left(\pi_{i,t+1}^{k_1}r_{i,t+1}^{k_1} - p_{i,t+1}^{k_1}q_{i,t+1}^{k_1} - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ij,t+1}q_{ij,t+1}^{k_1} - k_{i,t+1}v_{i,t+1}^{k_1}\right)\right] \\
&+\sum_{i=1}^{n}\left[\sum_{k_1=1}^{K} h_{i,t+1}^{k_1} Ez_{i,t+2}^{c}\left(\Pi_{i,t+2}\right) + \sum_{k_1=1}^{K} f_{i,t+1}^{k_1} Ez_{i,t+2}^{pr}\left(w_{i,t-1} - x_{it} - x_{i,t+1}^{k_1}, P_{i,t+2}\right)\right] \\
&+\sum_{k_1=1}^{K} g_{t+1}^{k_1} Ez_{t+2}^{tr}\left(\Pi_{i,t+2}, P_{i,t+2}, v_{i,t+1}^{k_1} \mid i \in \{1,\dots,n\}\right)
\end{aligned} \tag{7.54}
$$

subject to

$$x_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{jit} + s_{i,t-1} = y_{it} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ijt} + s_{it} \qquad [\lambda_{it}];$$

$$q_{i,t+1}^{k_1} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ji,t+1}^{k_1} + s_{it} = r_{i,t+1}^{k_1} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ij,t+1}^{k_1} + v_{i,t+1}^{k_1} \qquad [\lambda_{i,t+1}^{k_1}];$$

$$q_{i,t+\tau}^{k_1,\ldots,k_\tau} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ji,t+\tau}^{k_1,\ldots,k_\tau} + v_{i,t+\tau-1}^{k_1,\ldots,k_{\tau-1}} = r_{i,t+\tau}^{k_1,\ldots,k_\tau} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ij,t+\tau}^{k_1,\ldots,k_\tau} + v_{i,t+\tau}^{k_1,\ldots,k_\tau} \qquad \left[\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}\right] \tau \in \{2,\ldots,T-t\};$$

$$0 \leq r_{i,t+1}^{k_1} \leq \bar{y}_{i,t+1}^{k_1} \quad [\mu_{i,t+1}^{k_1}]; \qquad 0 \leq q_{i,t+1}^{k_1} \leq \bar{x}_{i,t+1}^{k_1} \quad [\vartheta_{i,t+1}^{k_1}];$$

$$0 \leq r_{i,t+\tau}^{k_1,\ldots,k_\tau} \leq \bar{y}_{i,t+\tau}^{k_1,\ldots,k_\tau} \quad [\mu_{i,t+\tau}^{k_1}]; \qquad 0 \leq q_{i,t+\tau}^{k_1,\ldots,k_\tau} \leq \bar{x}_{i,t+\tau}^{k_1,\ldots,k_\tau} \quad \left[\vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau}\right] \ \tau \in \{2,\ldots,T-t\};$$

$$0 \leq x_{it} + x_{i,t+1}^{k_1} + x_{i,t+2}^{k_1,k_2} + \ldots + x_{i,t+\tau}^{k_1,k_2,\ldots,k_\tau} + \ldots + x_{iT}^{k_1,\ldots,k_{T-t}} \leq w_{i,t-1} \quad \left[\gamma_i^{k_1,k_2,\ldots,k_{T-t}}\right]; \ \tau \in \{3,\ldots,T-t-1\}$$

$$x_{it}, y_{it}, x_{ijt}, s_{it}, x_{i,t+\tau}^{k_1,\ldots,k_\tau}, r_{i,t+\tau}^{k_1,\ldots,k_\tau}, q_{i,t+\tau}^{k_1,\ldots,k_\tau}, q_{ij,t+\tau}^{k_1,\ldots,k_\tau}, v_{i,t+\tau}^{k_1,\ldots,k_\tau} \geq 0 \qquad \tau \in \{1,\ldots,T-t\},\ i,j \in \{1,\ldots,n\}, j \neq i$$

The terms between square brackets are the Lagrange multipliers. Analogous to the models for the periods T and T-1, by writing out the Kuhn-Tucker conditions (see Appendix 1), we can prove that optimal supplied, demanded, transported and stored quantities resulting from model (7.54), are equal to, respectively, optimal producer supply, consumer demand, trader transport flows, and trader stock levels, discussed in Section 7.1. Similar to Theorem 7.1a and b, and Theorem 7.2a and b we can formulate the following theorem.

<u>Theorem 7.1c:</u>
Let in the optimal solution of the equilibrium model (7.54) for period t, t = 1,…,T-2, $\hat{x}_t, \hat{x}_{t+1}^{k_1}, \hat{x}_{t+2}^{k_1,k_2}$, …, $\hat{x}_T^{k_1,\ldots,k_{T-t}}$ be the optimal supply levels and λ_{it} be the corresponding optimal value of the Lagrange multiplier of the equilibrium condition for period t, for i = 1,…,n, k_1,…,k_{T-t} = 1,…,K. If the producer price in period t in region i is equal to:

(7.55) $p_{it} = \lambda_{it}$

then $x_{it} = \hat{x}_{it}$, $x_{t+1}^{k_1} = \hat{x}_{t+1}^{k_1}, \ldots, x_T^{k_1,\ldots,k_{T-t}} = \hat{x}_T^{k_1,\ldots,k_{T-t}}$, for $k_1,\ldots,k_{T-t} = 1,\ldots,K$, is an optimal solution of model (7.14). In other words, the optimal equilibrium supply levels give the producers optimal profits. Since the value of λ_{it}, depends on the value of the equilibrium supply level, we write $p_{it}(x_{it}) = \lambda_{it}$.

Proof: See Appendix 1.

Theorem 7.2c:

Let $\hat{x}_{it}, \hat{y}_{it}, \hat{x}_{ijt}, \hat{s}_{it}, \hat{q}_{i,t+1}^{k_1}, \hat{r}_{i,t+1}^{k_1}, \hat{q}_{ij,t+1}^{k_1}, \hat{v}_{i,t+1}^{k_1}, \hat{q}_{i,t+\tau}^{k_1,\ldots,k_\tau}, \hat{r}_{i,t+\tau}^{k_1,\ldots,k_\tau}, \hat{q}_{ij,t+\tau}^{k_1,\ldots,k_\tau}, \hat{v}_{i,t+\tau}^{k_1,\ldots,k_\tau}$, $\tau = 2,\ldots,T-t$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of equilibrium model (7.54). Let $\pi_{it} = \pi_{it}(\hat{y}_{it})$, $p_{it} = p_{it}(\hat{x}_{it}) = \lambda_{it}$. The solution:

$$
\begin{aligned}
&q_{it} = \hat{x}_{it};\ r_{it} = \hat{y}_{it};\ q_{ijt} = \hat{x}_{ijt};\ v_{it} = \hat{s}_{it};\\
&q_{i,t+1}^{k_1} = \hat{q}_{i,t+1}^{k_1};\ r_{i,t+1}^{k_1} = \hat{r}_{i,t+1}^{k_1};\ q_{ij,t+1}^{k_1} = \hat{q}_{ij,t+1}^{k_1};\ v_{i,t+1}^{k_1} = \hat{v}_{i,t+1}^{k_1};\\
&q_{i,t+\tau}^{k_1,\ldots,k_\tau} = \hat{q}_{i,t+\tau}^{k_1,\ldots,k_\tau};\ r_{i,t+\tau}^{k_1,\ldots,k_\tau} = \hat{r}_{i,t+\tau}^{k_1,\ldots,k_\tau};\ q_{ij,t+\tau}^{k_1,\ldots,k_\tau} = \hat{q}_{ij,t+\tau}^{k_1,\ldots,k_\tau};\ v_{i,t+\tau}^{k_1,\ldots,k_\tau} = \hat{v}_{i,t+\tau}^{k_1,\ldots,k_\tau};\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{for } \tau \in \{2,\ldots,T-t\}
\end{aligned}
\tag{7.56}
$$

for $i,j \in \{1,\ldots,n\}$, $i \neq j$, is an optimal solution of the trader decision problem (7.27).

Proof: See Appendix 1.

We conclude that it is optimal for the individual agents to transact the equilibrium quantities. The stochastic, multi-period, multi-region equilibrium model can be used to analyse the optimal strategies of the market agents and price formation on a competitive market under uncertainty of future prices.

Some Equilibrium properties

From the Kuhn-Tucker conditions of model (7.44) we can derive the following properties – compare the *Equilibrium properties* 6.1 – 6.4 and the *Trader properties* 7.1 – 7.4:

<u>*Equilibrium property* 7.1:</u> For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:

a) In the optimal solution of (7.44) $\pi_{it}(y_{it}) \leq p_{it}(x_{it})$.

b) If in the optimal solution of (7.44) $\pi_{it}(y_{it}) < p_{it}(x_{it})$, then $y_{it} = 0$.

c) If in the optimal solution of (7.44), supply and demand are both positive, $x_{it} > 0$ and $y_{it} > 0$, then the prices satisfy necessarily $\pi_{it}(y_{it}) = p_{it}(x_{it})$.

<u>Proof:</u> See Appendix 1.

<u>*Equilibrium property* 7.2:</u> In the optimal solution of (7.44), let transport take place from market i to market j in period t, i.e. $x_{ijt} > 0$, with $i,j \in \{1,\ldots,n\}$, $j \neq i$, $t \in \{1,\ldots,n\}$, then:

a) no goods are transported from a region $s = 1,\ldots,n$, to region i, $x_{sit} = 0$, for $s \neq i$.

b) no goods are transported from region j to a region $s = 1,\ldots,n$, $x_{jst} = 0$, for $s \neq j$.

c) the producer supply in region i satisfies, $x_{it} > 0$, or the stock remaining from the previous period is positive, $s_{i,t-1} > 0$.

d) the consumer demand in region j satisfies, $y_{jt} > 0$, or the quantity in stock at the end of period t in region j is positive, $s_{jt} > 0$ (this is equal to the statement that the quantity in stock at the end of period t, to be sold in period τ, is positive for at least one period τ, $\dot{s}_{jt\tau} > 0$, $\tau \in \{t+1,\ldots,T\}$).

<u>Proof:</u> See Appendix 1.

<u>*Equilibrium property* 7.3:</u> For region i and j, $i,j \in \{1,\ldots,n\}$, $j \neq i$, and period $t \in \{1,\ldots,n\}$:

a) In the solution of (7.44) $\pi_{jt}(y_{jt}) \leq p_{it}(x_{it}) + \tau_{ijt}$.

b) If in the optimal solution of (7.44) $\pi_{jt}(y_{jt}) < p_{it}(x_{it}) + \tau_{ijt}$, then $x_{ijt} = 0$ or $y_{jt} = 0$
c) If in the optimal solution of (7.44) supplies in region i, transport between region i and j, and demand in region j are positive, $x_{it} > 0$, $x_{ijt} > 0$ and $y_{jt} > 0$, then the optimal prices satisfy necessarily $\pi_{it}(y_{it}) = p_{it}(x_{it}) + \tau_{ijt}$.

Proof: See Appendix 1.

Equilibrium property 7.4: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,n\}$, we can derive that:
a) If in the optimal solution of (7.44) $E\Pi_{i,t+1} < p_{it}(x_{it}) + k_{it}$, then $s_{it} = 0$ or $r_{i,t+1}^{k} = 0$ for at least one $k \in \{1,\ldots,K\}$.
b) If in the optimal solution of (7.44) $E\pi_{i,t+1} \geq p_{it} + k_{it}$, storage in period t, and planned sales in period t+1 are positive, $s_{it} > 0$, and $r_{i,t+1}^{k_1} > 0$ for all $k_1 \in \{1,\ldots,K\}$, then an optimal solution exists satisfying $q_{it} = x_{it}$ or $r_{i,t+1}^{k} = y_{i,t+1}^{k}$ for at least one $k \in \{1,...,K\}$. For $E\pi_{i,t+1} = p_{it} + k_{it}$, an optimal solution is not unique.

Proof: See Appendix 1.

In Section 6.2, a trader would not make profits from storage, $\pi_{i,t+1} \leq p_{it} + k_{it}$. In this section it is possible that a trader expects to make profits from storage. From *Equilibrium property* 7.4 follows that a trader will store if $E\pi_{i,t+1} \geq p_{it} + k_{it}$. On a competitive market, it is expected that traders will continue purchasing commodities in period t (so that p_{it} will increase), until $E\pi_{i,t+1} = p_{it} + k_{it}$. However, due to the (exogenous) upperbound on the trader's future sales ($r_{i,t+1} \leq \bar{y}_{i,t+1}^{k}$ – see (7.54)) it is possible that a trader will not continue purchasing in period t until $E\pi_{i,t+1} = p_{it} + k_{it}$.

The results of (7.44) are more or less similar to the results of the Equilibrium model in Section 6.2. However, the difference between the consumer and producer price π_{it} and p_{it} does, in this case, not influence the supply level, x_{it}. Supply only depends on the difference between the producer price p_{it} and the term $c_{it} + \Psi_{i,t+1}$. Differences between the situations discussed in the Sections 6.2 and 7.2 can be illustrated by considering the case in which $\pi_{it} < p_{it}$:

- In Section 6.2, it was possible that $y_{it} > 0$ for $\pi_{it} < p_{it}$. In that case $x_{it} = 0$, and demand in region i originated from transports to region i, or from stocks remaining from previous periods.
- If in the present situation $\pi_{it} < p_{it}$, then $y_{it} = 0$. It follows that, it is not possible to take goods from the stock in region i or to transport goods to region i, not even if $\pi_{it} = p_{jt} + \tau_{jit}$. In the present situation, necessarily $\pi_{it} = p_{it}$ if $y_{it} > 0$, see *Equilibrium Property* 7.1.

So, in both sections, goods will not be purchased and sold in the same region for $\pi_{it} < p_{it}$. In Section 6.2, goods demanded may originate from stocks or transports to region i, whereas in Section 7.2 no goods will be demanded for these prices. If in Section 7.2 demand in region i is positive, then necessarily $\pi_{it} = p_{it}$.

In the next chapter we discuss the cereal market situation in Burkina Faso. Using this information, we will discuss in the Chapters 9 and 10 a stochastic, temporal, spatial equilibrium model for cereal trade in Burkina Faso. Using this model, we can analyse the influence of for example storage and transport costs on cereal supply, demand, transport and storage in Burkina Faso.

7.3 Monopolistic behaviour of traders

If the market is not competitive, but the trader is a monopolist, trader strategies change. In that case the trader can set prices in each period $t = 1,\ldots,T$. Consequently, if he knows consumer and producer strategies as a function of market prices, future prices are not stochastic for him. The trader decision problem is comparable to the method discussed in Section 6.3. The difference is the price dependence of cereal supply by the producers. It has been argued in Section 7.1, that producers supply nothing in period t if $p_{it} < c_{it} + \Psi_{i,t+1}$ – see (7.22). They supply the entire stock $w_{i,t-1}$, if $p_{it} > c_{it} + \Psi_{i,t+1}$. If the price is equal to $p_{it} = c_{it} + \Psi_{i,t+1}$, then any supply x_{it} between 0 and $w_{i,t\text{-}1}$ is optimal for the producer. Since the monopolistic trader will pay the least possible for his purchases to the producer, he will offer a price $p_{it} = c_{it} + \Psi_{i,t+1}$ to the producer. Consequently, cereal purchases cost him: $(c_{it} + \Psi_{i,t+1})\cdot x_{it}$. The model describing the monopolist's decision problem may be written as – compare (6.26), (7.27) and (7.44):

$$\underset{y_{it},x_{it},x_{ijt},s_{it}}{Max}\sum_{t=1}^{T}\sum_{i=1}^{n}\left\{\left[\pi_{it}(y_{it})y_{it} - \left(c_{it} + \Psi_{i,t+1}\right)x_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ijt}x_{ijt} - k_{it}s_{it}\right]\right.$$

(7.57)
$$\left| \; x_{it} + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{jit} + s_{i,t-1} = y_{it} + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ijt} + s_{it};\; x_{it} \le w_{i,t-1};\; w_{it} = w_{i,t-1} - x_{it};\right.$$

$$\left. x_{it}, y_{it}, x_{ijt}, s_{it}, w_{it} \ge 0,\; i, j = 1,\dots,n,\; t = 1,\dots,T \right\}$$

in which $c_{it}\cdot x_{it} = c_{it}(x_{it})$ as in (7.8), and $\Psi_{i,t+1}$ is defined in (7.20). For the period $t = T$, the parameter $\Psi_{i,T+1} = 0$. Let x_{it}, y_{it}, x_{ijt} and s_{it}, $i,j = 1,\dots,n$, $i \neq j$, $t = 1,\dots,T$, be the optimal solution of model (7.57). Formulating the Lagrangean function and taking the Kuhn-Tucker conditions, we can write:

(7.58) If $x_{it} = 0$ then $-c_{it} - \Psi_{i,t+1} + \lambda_{it} \le 0$

(7.59) If $0 < x_{it} < w_{i,t\text{-}1}$ then $-c_{it} - \Psi_{i,t+1} + \lambda_{it} = 0$

(7.60) If $x_{it} = w_{i,t\text{-}1}$ then $-c_{it} - \Psi_{i,t+1} + \lambda_{it} \ge 0$

(7.61) If $y_{it} > 0$ then $\pi_{it}(y_{it}) + y_{it}\pi'_{it}(x_{it}) - \lambda_{it} = 0$

(7.62) If $y_{it} = 0$ then $\pi_{it}(y_{it}) + y_{it}\pi'_{it}(x_{it}) - \lambda_{it} \le 0$

(7.63) If $x_{ijt} > 0$ then $-\tau_{ijt} + \lambda_{jt} - \lambda_{it} = 0$

(7.64) If $x_{ijt} = 0$ then $-\tau_{ijt} + \lambda_{jt} - \lambda_{it} \le 0$

(7.65) If $s_{it} > 0$ then $-k_{it} - \lambda_{it} + \lambda_{i,t+1} = 0$

(7.66) If $s_{it} = 0$ then $-k_{it} - \lambda_{it} + \lambda_{i,t+1} \le 0$

Analogous to model (6.26), $s_{iT} = 0$. Like in Theorem 7.1, the optimal supply level x_{it} of model (7.57) will also be an optimal level for the producers in region i, if:

(7.67) $p_{it} = \lambda_{it}$.

Instead of *Equilibrium properties* 6.1 - 6.4, we can write the following properties:

Monopoly property 7.1: If $y_{it} > 0$, then – see (7.61) and (7.67):

$$p_{it} = \pi_{it} + y_{it}\pi'_{it}(y_{it}) .$$

According to this condition, marginal revenues from selling in region i equal marginal costs of purchasing in region i. It follows due to (6.5) that in that case: $\pi_{it} - p_{it} = -\pi'_{it}(y_{it})y_{it} \geq 0$

Monopoly property 7.2: Let in the solution transport in period t take place from a market i to a market j, i.e. let $x_{ijt} > 0$, with $i,j \in \{1,2, ...,n\}$, $j \neq i$, $t \in \{1,\ldots,T\}$, then (compare *Equilibrium property* 7.2):

a) in period t, no cereals are transferred from other regions into market i, i.e. $x_{sit} = 0$, for all $s \neq i$

b) in period t, no cereals are transported from market j to other regions, i.e. $x_{jst} = 0$, for all $s \neq j$

c) in period t, the producer supply in region i in period t satisfies, $x_{it} > 0$, or the stock remaining from the previous period is positive, $s_{i,t-1} > 0$.

d) in period t, the consumer demand in region j in period t satisfies $y_{jt} > 0$, or the quantity put in stock in region j is positive, $s_{jt} > 0$.

Monopoly property 7.3: If $y_{jt} > 0$ and $x_{ijt} > 0$, then necessarily – see (7.61), (7.63) and (7.67):

$$\tau_{ijt} = y_{jt}\,\pi'_{jt}(y_{jt}) + \pi_{jt} - p_{it}.$$

It follows, due to (6.5), that: $\pi_{jt} - p_{it} = \tau_{ijt} - y_{jt}\,\pi'_{jt}\left(y_{jt}\right) \geq \tau_{ijt}$

<u>*Monopoly property* 7.4</u>: If $y_{i,t+1} > 0$ and $s_{it} > 0$, then necessarily – see (7.61), (7.65) and (7.67):

$$k_{it} = \pi_{i,t+1} + y_{i,t+1}\pi'_{i,t+1}(y_{i,t+1}) - p_{it}.$$

It follows, due to (6.5), that: $\pi_{i,t+1} - p_{it} = k_{it} - y_{i,t+1}\pi'_{i,t+1}(y_{i,t+1}) \geq k_{it}$.

As can be seen from the *Equilibrium properties* 7.1 to 7.4, in the competitive case discussed in Section 7.2, it was not possible that in the solution π_{it} - $p_{it} > 0$, π_{jt} - $p_{it} > \tau_{ijt}$, or $\pi_{i,t+1} - p_{it} \geq k_{it}$. Like in Section 5.3 and 6.3, traders will make positive profits on a monopolistic market, whereas traders play even on a competitive market.

8 Cereal markets in Burkina Faso

Before we can analyse the inter-regional cereal flows in Burkina Faso, first the parameters of the models discussed in the previous section must be estimated. We have to estimate cereal supply and demand functions, storage costs and losses per stored unit per unit of time, transport costs per transported unit of cereals between the various markets, and the trading costs per unit of cereals sold. Estimation of these elements demands a careful review of the existing literature on these issues. In Section 8.1 a survey is given of empirical evidence of cereal supply and demand, both in terms of quantities and timing. It focuses on the major factors determining supply and demand, as well as on regional differences. This survey is based on the review in Appendix 3 of many studies focussing on cereal trade, production and consumption, which have been performed in Burkina Faso in the past. In the next chapter the cereal supply and demand functions are estimated using the data presented in this chapter. In Section 8.2 and Appendix 4 we discuss some studies which analyse the costs involved in cereal trade. We estimate the values of transport, storage and trading parameters of the equilibrium model.

8.1 Empirical evidence of supply and demand

For our analysis a planning period of one year is considered. The planning year is divided in four periods of three months each, starting at harvest time in October. During the planning year producers sell, traders purchase and sell, and consumers purchase the cereals harvested, as a function of the cereal prices in all four seasons. Farmers are both producers and consumers. For each period producer supply functions and consumer demand functions have to be estimated. In Figure 8.1 the planning year is presented. Supply and demand functions, discussed in Chapter 7, refer to aggregate regional supply and demand. The regions distinguished in this study are the 12 Burkinabé "agricultural extension" regions (the CRPA: Centre Regional de Promotion Agricole). Figure 8.2 shows a map of Burkina Faso with the provinces and CRPA's of the country. As planning year the reference year October

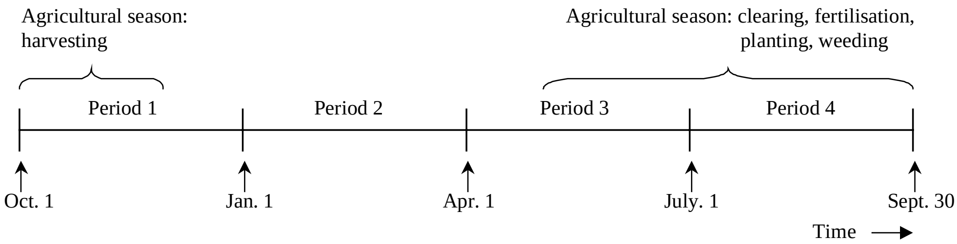


Figure 8.1 Schematic presentation of the planning year.

2000 to September 2001 will be chosen. Quarterly supply and demand functions will be estimated for this reference year.

A distinction is made between *production* and *marketable supply* functions. Production functions refer to the level of cereal production as a function of among other things rainfall, inputs and prices. Cereal supply functions, on the other hand, give the quantity of cereals which is supplied on the market as a function of e.g. production levels and market prices. For cash crops and industrial production, production and supply functions are often similar: the quantity produced is also sold. In Sahelian subsistence agriculture, however, both functions differ. In many households, certainly in the shortage regions, the largest part of the production is consumed on farm, while only a small part is sold. Production functions are not taken into account in this study. Cereal harvest levels are supposed to be known before farm households make their supply decisions. Supply decisions depend on cereal prices, harvest levels, and various other factors. For instance, distress sales often force households to sell a part of their production early in the season, even if prices are low (see e.g. Yonli, 1997). Furthermore, in some regions, merchants purchase from farmers only during some months of the year. During the rainy season villages may be inaccessible, consequently farmers will not be able to sell their supply. These examples show that supply functions must be set-up carefully, taking into account the particular characteristics and timing of Burkinabé agriculture and trade. The set-up of demand functions is also not straightforward. Urban households purchase all or the largest part of their consumption on the market. Rural households, on the other hand, only

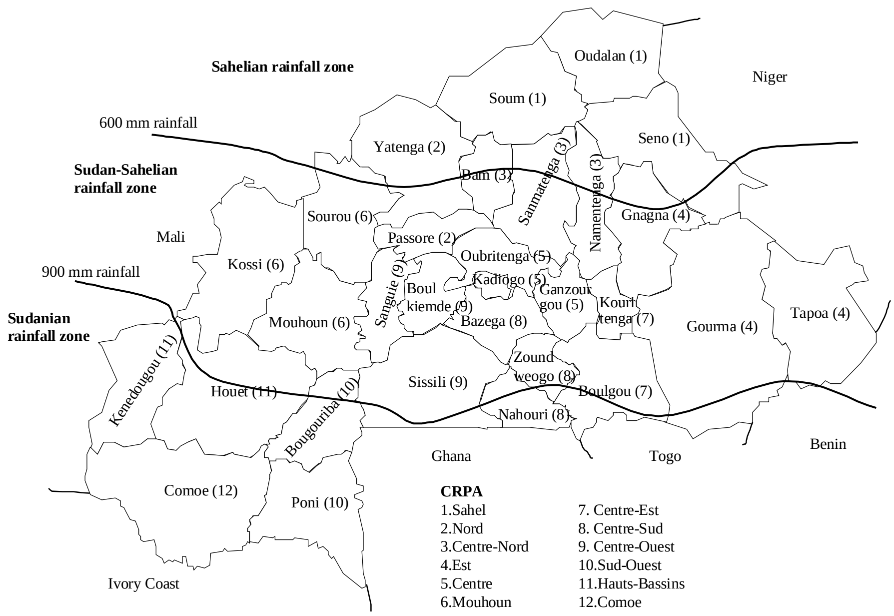


Figure 8.2 Map of Burkina Faso, showing the provinces and CRPA's

Notes: 1) In 1996, some of the 30 provinces were split in two or more new provinces, resulting in a total of 45 provinces. In this report, the 'old' provinces are still used, because most data refer to the old provinces; 2) The Sahelian rainfall zone is the climatic region with an average annual rainfall less then 600 mm, the Sudan-Sahelian rainfall zone is the region which receives on the average between 600 mm and 900 mm of rain per year, the Sudanian rainfall zone has an average annual rainfall exceeding 900 mm (see for example Laclavère, 1993).

purchase a small quantity of cereals on the market. Therefore, a distinction should be made between rural and urban demand.

It is recalled that in the equilibrium models of the previous chapters the supply and demand functions for each region *i* were written as a function of prices. In fact, as was seen above, these functions depend as well on other characteristics, which may differ from one region to the other. Population size, demographic growth and levels of production are obvious examples of such characteristics. In the Sections 8.1.1 and 8.1.2 first these characteristics will be discussed. In Section 8.1.1 and Appendix A3.1, for each region the size of the rural and urban population in the reference year 2000 will be estimated. These data will later be used in the process of estimating aggregate regional supply functions and rural and urban demand functions. In Section 8.1.2 and Appendix A3.2, for each region harvest levels are estimated. By making use of demographic data and regional cereal production data during a series of years (1984-1998), production per inhabitant and per rural inhabitant can be estimated. These data are compared with data on required cereal consumption per person in order to evaluate whether a region may be considered to be a shortage, a surplus or an ‘equilibrium’ region. Special attention will be put on the estimation of *expected* harvest levels in the reference year 2000, taking into account possible trends in production, yields and/or cultivated areas.

In Section 8.1.3 and Appendix A3.3 a review is given of empirical evidence at household level of cereal sales and their timing. It is discussed to which extent in a number of village level studies various characteristics have influenced cereal supply on the market. A distinction will be made between the annual supply on the market and the timing of the supply. Section 8.1.4 and Appendix A3.4 deal with cereal purchases by rural and by urban consumers. Special points of interest are the relation between cereal production levels and purchases. Both demand and supply of cereals depend on the household’s ability to earn an income from other sources. In Section 8.1.5 and Appendix A3.5 a review is given of data on household’s incomes and expenditures. For various regions average levels of household income are estimated. Finally, Section 8.1.6 and appendix A3.6 deal with cereal prices on markets in Burkina Faso. Especially seasonal price patterns are investigated. This information is used to estimate price probability distribution functions, and serves as a reference for validation of the calculation of (endogenous) prices in Chapter 10. A careful review of all possible sources, thus allows for the estimation of regional, quarterly cereal supply and demand functions in Chapter 9. Though not conform rigorous econometrical rules, data limitations do not enable another estimation procedure. It is recalled that in this paper *cereals* comprise millet, red sorghum, white sorghum and maize. Rice and fonio have not been taken into account.

8.1.1 Rural and urban population

The size of the urban and rural population in Burkina Faso in the reference year 2000 can be estimated using census data for 1996 and 1985 (INSD, 1995a,b, 1998). INSD data allow for the estimation of growth rates of the rural and urban population per CRPA. The total, rural and urban population can be estimated for each region for the year 2000, if it is supposed that the yearly urban and rural population growth between 1996 and 2000 is the same as between 1985 and 1996 (see Table 8.1 and Appendix A3.1). The population estimates are used in Section 8.1.2 to estimate the cereal production per person and the cereal production per head of the rural population, and in the Sections 9.1 and 9.2 to estimate the aggregate regional cereal demand and supply functions.

Table 8.1: Estimated urban and rural population in the reference year Oct 2000-Sept 2001.

CRPA	Total	urban	rural
Centre	1,787,175	843,454	943,721
Centre Nord	1,016,292	66,820	949,473
Centre Ouest	1,060,889	118,377	942,512
Centre Sud	518,920	18,343	50,0577
Sahel	792,889	31,499	761,390
Mouhoun	1,241,941	101,918	1,140,022
Est	1,049,317	55,799	993,518
Centre Est	836,249	103,372	732,877
Nord	1,039,819	104,111	935,708
Sud Ouest	543,289	19,221	524,068
Hauts Bassins	1,109,265	371,416	737,849
Comoe	359,652	76,996	282,655
Total	11,355,699	1,911,328	9,444,371

Estimates are based on 1985 and 1996 census data (INSD 1995a,b, 1998) data, see Table A4.1 in Appendix 4.

8.1.2 Cereal Production

Using the rural population estimates and production data which are published yearly by the Ministry of Agriculture, the expected cereal production for the year 2000 and the expected cereal production per head of the rural population can be estimated for each region. The estimated level of cereal production per rural inhabitant is used in Section 9.2 to estimate the annual cereal sales.

Table A3.2 in Appendix A3.2 shows production, cultivated area and yield data for the years 1984-‘98. For each CRPA it can be indicated whether in most years cereal production exceeds or is lower than the minimally required cereal consumption of 190 kg per person.[13] In this paper we adopt the definition that a region is in shortage if in most years average cereal production per person is lower than 170 kg (taken into account 15% grain losses). A region is a surplus region if average production exceeds 210 kg per head for most years. The other regions (with production levels between 170 and 210 kg per person or having alternately a surplus or shortage production) are called here equilibrium zones. In column (a) in Table 8.2 is indicated which CRPA are shortage, surplus or equilibrium regions. The entire country is on average just in equilibrium (between 1984 and 1998 the country had surplus in nine years, a shortage in three years, and was in equilibrium in three years). The northern CRPA Sahel and Nord, and the CRPA Centre (with the capital Ouagadougou) are in most years in a shortage situation. The cotton areas Mouhoun, Hauts Bassins (with Bobo-Dioulasso), Comoé and Sud-Ouest and the CRPA Centre Ouest, Centre Sud, and Est have in general a surplus. The other CRPA are in general in equilibrium.

Regression analyses executed on the data presented in Appendix A3.2 have demonstrated that although the production data feature a significant linear trend, it is risky to suppose that the cereal production for each CRPA in the reference year can be estimated by extrapolating the data. Regression results showed that yield levels, production and area cultivated are dependent upon rainfall (see Appendix A3.2 and Figure A3.3). Yield levels show a jump after 1991. Part of this jump can be explained by the good rainfall in 1991, 1994, 1995, 1997 and 1998. Yield levels were also high during the average rainfall years 1992 and 1993, but the period is too short to be able to draw conclusions upon the yield levels in the coming years. Furthermore,

[13] The norms for required minimal consumption per adult equivalent differ per source and depend on suppositions on the share of the different cereals in total consumption, and nutrient losses during food preparation. Estimated norms per person per year vary (see Bakker and Konaté, 1988) between 180 kg used by FAO, 220 kg calculated by Bakker and Konaté (taken into account the large losses due to meal preparation) and even 270 kg estimated by CILSS. We use here the average norm of 190 kg per person, applied by the ministry of agriculture to calculate the yearly consumption balances. It is noted that this norm is not a strict norm, and that therefore the bounds between surplus and shortage households are not strict.

regression analysis for the cultivated area showed a significant, positive, linear time trend for the cultivated area at a national level. For most CRPA's, however, this trend was not significant. In order to forecast for each CRPA the expected production levels in the reference year 2000-2001, average 1984-'98 yield levels are multiplied with forecasted cultivated area for each CRPA. This area is estimated as follows. The national cultivated area for the reference year is estimated, and for each CRPA the average 1984-'98 share in total cultivated land is calculated. Cultivated area per CRPA for the year 2000 is forecasted by multiplying these two (see Table 7.2). Yearly production is corrected for grains lost or used as seeds for the next season, which are supposed to be 15%.

The last two columns of Table 8.2 show the forecasted mean production per head of the total and per head of the rural population. If the norm for minimally required yearly consumption of 190 kg cereals is applied it is shown that in only 3 regions (Center, Sahel and Nord) farmers are expected to produce much less than their own consumption requirements. In the other regions, the farmers are expected to succeed fairly well in producing enough cereals for their own consumption. In five regions farmers are expected to produce a quantity of at least 50 kg of cereals above the norm. Farmers in these regions, which are all in the south-western part of the country, have the opportunity to sell a large quantity of their cereal production. It has to be noted that these farmers, usually also produce cotton. So, despite their cotton production activities, they also succeed in producing more cereals than needed. The data show that the northern regions are not self-sufficient and need to be provisioned by the surplus areas. The CRPA Centre, in which Ouagadougou is situated, is also in deficit. The low production per head of the total population in Hauts Bassins is caused by the large number of urban households in the city of Bobo-Dioulasso. Column (g) in Table 8.2 shows that for the reference year the CRPA Sahel, Nord, Centre, Centre Nord and Centre Est are expected to have a **shortage** production. The CRPA Mouhoun, Centre

Table 8.2: Forecasts of mean cereal production per capita for the total and for the rural population for the reference year 2000-2001.

	Sur/ Def/ Eq (a)	Average yield '84-'98 (kg/ha) (b)	Share of CRPA in cultivated land (c)	Forecasted cultivated area 2000 (ha) (d)	Forecasted mean production 2000 (tonnes) (e)	Production (d) –15% loss (tonnes) (f)	Production (e) per person (kg) (g)	Production (e) per rural inhabitant (kg) (h)
Centre	D	650	7%	222878	144926	123187	69	131
Centre Nord	E	597	10%	306996	183210	155728	153	164
Centre Ouest	S	668	11%	337521	225630	191786	181	203
Centre Sud	S	759	7%	210916	160008	136006	262	272
Sahel	D/E	483	9%	278245	134257	114118	144	150
Mouhoun	S	812	15%	467667	379850	322873	260	283
Est	S	808	10%	311393	251481	213759	204	215
Centre Est	E/S	771	7%	203259	156614	133122	159	182
Nord	D	584	9%	264955	154775	131559	127	141
Sud Ouest	S	814	7%	206583	168071	142860	263	273
Hauts Bassins	S	1211	7%	203726	246802	209782	189	284
Comoe	S	1171	2%	74872	87671	74520	207	264
Burkina Faso	S	740	100%	3089011	2293294	1949300	172	206

Notes: (a) According to the definition in the text, S = Surplus region, D = Deficit region, E = Equilibrium region; (b) Based on Table A3.2; (c) Average share of cultivated land of each CRPA in total cultivated area for the years 1984-98, see Table A3.2; (d) Total forecasted cultivated area is estimated on the basis of extrapolation of the national cultivated area between 1984-1995; see Appendix 3; (d) = Forecasted area Burkina Faso * (c); (e) = (b)*(d); (f) = 0.85*(e); (g) = (f)/forecasted total population per CRPA in 2000, see Table 8.1; (h) = (f)/forecasted rural population 2000, see Table 8.1.

Sud, and Sud Ouest are expected to be **surplus zones**. The other CRPA are expected to be more or less self-sufficient and are **equilibrium zones**. This does not say anything about the food security level of individual households, but shows whether in principle much has to be transported to these areas or not. The cereal balances differ from the official balances calculated by the government, since the last ones include rice and fonio.

8.1.3 Cereal sales

Many farm households, certainly those with a shortage, prefer not to sell cereals, but to earn an income by selling other crops, like groundnuts, cowpeas or cotton. However, as discussed above they sometimes have to sell cereals because of urgent cash needs. McCorkle (1987) speaks about a ‘code of honour’, which influences cereal sales. Referring to her research in Dankuie, a village in the province of Mouhoun, she reports that cereal sales to alleviate cash needs are usually disapproved, except in special cases (extraordinary surplus, sales to village cooperatives, sales in the lean period, just before the new harvest). Although the people in Dankuie usually produce a large surplus, harvests in the survey year were low. McCorkle uses a ‘commercial preference scale’ in order to classify the order in which products would be sold if people had them, in case of cash needs. Millet and sorghum occupy respectively the 17th and 18th place. Households prefer to sell cash crops (cotton is the most prefered commercial good in case of cash needs) or livestock (poultry is on the second place) or to borrow from parents (10th), farmer cooperatives (11th) or traders (16th). Despite this code of honour, cereal sales can be significant, especially when other ways to get an income are not conceivable.

In the past several marketing studies have been performed in Burkina Faso by for example the University of Michigan and the University of Wisconsin (McCorkle, 1987; Szarleta, 1987; Sherman et al., 1987), by CILSS (Pieroni, 1990), by ICRISAT (Reardon et al., 1987), by Yonli (1997) and by Broekhuyse (1988, 1998); see Appendix A3.3. A comparison of sales patterns reported by these studies, reveals that differences between years and regions as well as differences within regions are very large. The quantity of cereals sold depends to a large extent on production levels. In a good rainfall year, with a good harvest, more cereals can be sold. In bad rainfall years, households will not be inclined to sell many cereals, but they sometimes have to. For example, Szarleta reports a sale of 600 kg of cereals per household in the province of Houet in the bad rainfall season 1983-84, while Pieroni (1990) reports a household sale of 1806 kg in the same province in the abundant rainfall season 1986-87. Furthermore, households in surplus zones usually sell more than households in shortage areas (see for example Table A3.6). They sell both a larger quantity and a larger part of their production. Not only total cereal supplies, but also the type of crops offered differs per region. Szarleta (1987) shows that in the survey villages in the CRPA's Mouhoun and Hauts Bassins, large amounts of red sorghum but hardly any maize are offered.

The influence of prices on annual sales is weak. Szarleta (1987) concludes from a regression analysis among 5 villages scattered over Burkina Faso that, indeed, production is the most important determinant of annual cereal sales. Cereal prices, on the other hand, do not significantly influence cereal sales. In the long run production, and consequently also sales, may be influenced by prices. In the short run (for a period of only a few years), however, farmers will probably not immediately alter their production plans if cereal prices turned out to be different than expected. So, in the short run the dependence of supplies on prices can not be demonstrated. McCorkle (1987) argues that for some households prices do not influence sales decisions. For other households, however, prices are of influence, but not decisive. Using data collected between April 1983 and March 1984, Lang (1985) finds for the 4 surplus villages surveyed by SAFGRAD (see also Appendix A3.3) a relationship between prices and annual sales, which is negative for some and positive for other cereals, though it is not significant. Regression analysis per village shows that production is the most important determinant of annual sales. Prices do not significantly influence annual sales. Pardy also points at the importance of the household size. The larger the number of consumers, the lower the sales. Pieroni (1990) agrees with the strong link between production and sales (see Table A3.8 in

Appendix A3.3), but argues, based on his village studies in the surplus zones of Burkina Faso, that it is not a *law*. The production of a large surplus does, for some households, not correspond to large sales. Cereal sales depend not only on the surplus produced, but also on the need for capital, social relations, and market demand (Pieroni, 1990; p. 44, 45). Cash needs may also be satified by earnings from other activities (cash crop sales or non-cropping activities).[14] Market development and infrastructural conditions determine for a part the opportunity for such activities. Households closer to main roads or to busy markets, have more possibilities to sell handicrafts or processed food. Therefore, they also have the incentive to initiate activities, which can replace their cereal sales. Such activities attract new traders, and therefore enhance market competition and reduce marketing costs. Pieroni shows that households selling large quantities are those with more land, more modern techniques, less young children, and those closer to well functioning markets. This argument is in favour of improving market functioning in Burkina Faso. Developing other capital generating activities is, however, not only tied to the presence of roads and markets. Reardon et al. (1988a) show that households in the shortage Sahelian regions earn more income from non-cropping activities than households in the Sudanian equilibrium regions in the centre of the country (see Figure 8.2). Sahelian households provided most of their non-cropping income from livestock and temporary migration. Activities related to crop production (product processing) provided the largest part of the non-cropping income for most of the Sudanian households. Due to the dependence of these activities on crop production, earnings from these activities were low in years with bad harvest.

The ability to earn an income from other sources also influences the timing of sales. It is often said that West African farmers sell usually in the post-harvest, low-price season and buy in the pre-harvest, high-price season. Reasons for this are cash needs

[14] Non-cropping activities refer to all activities other than crop cultivation activities, like livestock raising, processing agricultural crops (*dolo* preparation, making millet porridge *bouillie*), handicrafts, trade, off-farm employment, temporary migration, etc..

for celebrations, urgent daily expenses, wage payments, debt repayments, etc. Various authors (see Appendix A3.3) observed that in general, most sales are effectuated during the post-harvest season. It appears that the number of households selling during the post-harvest season is higher than the number of households selling during the rainy season, when prices are higher. However, the quantity of cereals sold per selling household is smaller during the post-harvest season than during the rainy season. The results seem to indicate that households which do not have to sell cheap, prefer to wait until prices increase. Poorer households, who do not have other income generating sources have to sell (small quantities) during more periods. Sales of households posessing a large number of livestock seem to be dependent not only on the cereal market, but also on the livestock market (Pardy, 1987). These households may have the ability to postpone sales until prices increase. These patterns might support the hypothesis that sales during the harvest and post-harvest season are a function of cash needs (with a negative price elasticity of supply), whereas sales later in the year are a function of prices (with a positive price elasticity of supply).

To summarize, the different surveys indicate that cereal production levels are the most important determinant of annual cereal sales. Cereal prices do often not influence annual cereal sales significantly. Some juxtaposed effects reduce the total price effect which is overwhelmed by the effect of total production. Next to the surplus produced, also the need for capital, social relations, and the possibility to develop other income generating activities influences annual cereal sales. This also influences the timing of sales. A general recognized pattern is that most farmers sell during the post-harvest, low-price season. Some authors, however, observed that it are especially the poorer households who sell small quantities during all periods of the year. The wealthier households prefer to sell a larger quantity during the higher priced seasons.

The levels of cereal sales observed in the different surveys, are used to estimate in Section 9.2 the levels of annual cereal sales in each region. The information on sales patterns is used to estimate average levels of revenues from cereal sales in each period of the reference year which will be necessary to estimate the supply functions. Furthermore, the results from the equilibrium models can be validated using the information on cereal supply discussed here.

8.1.4 Cereal purchases

As only a part of the households sells cereals, most households purchase cereals on the market. A comparison of the same studies as mentioned above, learns us something about the purchase pattern of Burkinabé households (see also Appendix A3.4). Here a clear distinction must be made between urban and rural purchases. About 18% of the population lives in urban areas, who have to purchase almost all cereals consumed on the market. Urban households consume much more rice than rural households, and more often they purchase prepared food (millet porridge *bouillie*, bread, prepared meals).

Some studies only concentrated on urban consumption and demand. For example, Sherman et al. (1987) executed in 1983-84 a survey among 125 households in Ouagadougou and 108 households in Bobo-Dioulasso, and among 75 sellers of prepared food in Ouagadougou and 75 in Bobo-Dioulasso. They report that cereals, including rice, are the major staple of urban diets. White sorghum and millet are still consumed in largest quantities, but rice is increasingly consumed by urban households. Reardon et al. (1988b) report that rich households consume relatively more rice (32% of total cereal consumption) than poor households (19% of total cereal consumption). According to Sherman et al. (1987), red sorghum is not regularly eaten, but only as a grain of last resort. Households purchase the largest part from medium and small traders or from *vendeuses* (petty women traders) and to a lesser extent from large traders. Large traders usually do not sell in quantities less than 100 kg. The purchasing of 100 kg bags is typically reserved to civil servants and private sector employees who receive salaries periodically. Since purchases per bag are relatively cheaper than purchases in retail, the richer households can profit more from lower prices than poorer households. Only few households purchase directly from producers. Most prepared food sellers are women, who purchase and sell in small quantities. Apart from some of the *dolo* brewers, they purchase in the morning the cereals they need for preparing the food which is sold the same day. Purchases are often on credit. *Dolo* brewers usually purchase red sorghum in larger quantities, because it is not profitable to brew only small quantities.

The data presented in Table A3.18 in Appendix A3.4 show that almost all rural households purchase cereals on the market. Many rural households have to purchase large quantities in order to satisfy consumption requirements, certainly in the shortage areas.[15] As with sales patterns, purchases by rural households are dependent on production levels. The data clearly show that purchases are less in the higher production regions. So, on average, rural households in the surplus areas purchase less than those in the shortage areas. However, as Reardon et al. (1987) show, even in surplus zones there are many households who have to purchase large volumes of cereals to satisfy consumption needs. It regularly happens that the same type of cereals sold is rebought later in the season. This phenomenon is well known in Africa and sometimes called overcommercialization (see also Yonli, 1997). This is also the case for red sorghum, a part of which is consumed as *dolo* (see Table A3.19 in Appendix A3.4). Seasonal data confirm that most purchases take place during the lean, high price season. The data suggest, however, that richer farmers purchase earlier, when prices are still lower (in the surplus village of Baré most purchases were made between January and March, Ellsworth and Shapiro (1989)).

The observation that high production levels in surplus regions lead to lower purchases, may imply that purchases will be lower during the favourable rainfall years. The only source which reports on differences in purchases between production seasons is Reardon et al. (1987), who gives purchase data for 6 villages in three provinces in Burkina Faso. We will try to use Reardon's data to analyse the influence

[15] Most households, both in urban and in rural areas, receive cereal gifts. These gifts are not taken into account here, though they may be substantial (Szarleta, 1987; Broekhuyse, 1998; Appendix A3.5). It is supposed here that the quantity received is more or less equal to the quantity given to others. Therefore, they are not taken into consideration in this study.

of rainfall on purchases. The purchase data will be compared with rainfall data. For we do not have data on cereal production for these villages, and local cereal production levels may not be well represented by aggregate production data, we compare purchases with rainfall data from the rainfall stations closest to the survey villages. The data are obtained from the National Meteorological Institute of Burkina Faso. For the province of Soum, rainfall data from the rainfall station in Djibo (1981 458 mm, 1982 304 mm, 1983 322 mm, 1984 227 mm) show that only 1981 rainfall was above the 1970-93 average (of 333 mm). Table A3.23 in Appendix A3.4 shows that for the province of Soum purchases are higher if rainfall is low, although purchases in the 1983-84 season are rather low compared to the other low rainfall years. Purchases of the province of Passoré do not differ very much over the years, although the slightly increasing purchases correspond to the slightly decreasing rainfall in Kaya (1981 603 mm, 1982 583 mm, 1983 574 mm, 1984 533 mm; 1970-’93 average 615 mm). Finally, rainfall in the rainfall station in Dédougou in the province of Mouhoun are all below the 1970-‘96 average (1981 no observation, 1982 521 mm, 1983 621 mm, 1984 627 mm; 1970-’96 average 717 mm). The pattern of decreasing purchases from 1982 to 1984 corresponds to the pattern of increasing rainfall in this period. Although too few data are available to make conclusive statements, the data are not in contradiction with the hypothesis that purchases are inversely proportional to rainfall, and therefore also to production (in Appendix A3.2 it was shown that a positive, significant relation exists between rainfall and production).

To analyse the influence of income or prices on cereal demand, income elasticities of demand and price elasticities of demand are useful measures, see also Chapter 4. The price elasticity of demand measures the percentage change in demand if the price changes with 1%. Estimating elasticities is difficult since no time series data are available for a large number of respondents, and since many other factors influence demand behaviour as well. Because of the weakness of the available data in developing countries, many studies apply elasticities reported by different authors for similar situations in different countries. It is noted that elasticities also depend on the degree of commodity aggregation. The demand for cereals is expected to be less price elastic than the demand for white sorghum. If the price of white sorghum increases, other cereals can serve as substitute. If, however, the prices of all cereals increase, other types of food which can serve as a substitute, must be sought for. Some studies estimating price elasticities of demand and income elasticities of demand are discussed in Appendix A3.4.

The information on the timing of purchases is used in Section 9.1 to estimate the share of total purchases in each period. Furthermore, the purchase data discussed above are used to check the validity of the results from the equilibrium models. In Section 9.1 estimates are also made of the income elasticity of cereal demand in the different CRPA in Burkina Faso. These are necessary to estimate the share of the revenues spent on cereal purchases.

8.1.5 Revenues and expenditures

Cereal supply and demand decisions depend on the total household revenues and expenditures. If a household produces cash crops or has other sources of income, probably not many cereals will have to be sold. If household expenditures are high in a certain period, and if revenues (other than revenues from cereal sales) are not sufficient, cereals must probably be sold. If total revenues are low, not many cereals can be purchased for own consumption. In other words, household revenues and expenditures are decisive factors in view of the quantities of cereals that can or have to be sold or purchased.

Measurement of household revenues and expenses in developing countries is a notorious difficult task. For interviewers it may be difficult to get reliable answers to sensitive questions related to money. People are not always prepared to answer questions on for example their expenditure pattern during the last 12 or 6 months, people often forget many expenditures or (non-monetary) revenues, or give too low figures because they do not want other people to know their wealth. Reliable results can only be obtained if a relation of trust exists between the interviewer and the people interviewed, and if they are interviewed on a regular basis. The unreliability of the data clearly comes forward if revenue and expenditure data of different surveys are compared. Various surveys have been carried out in Burkina Faso. The most recent is a national poverty study by the national statistical and demographic institute (INSD, 1996a, 1996b). Other studies were performed by Broekhuyse (1988) in the province of Sanmatenga, Thiombano et al. (1988), Reardon et al. (1988a) and Lang et al. (1983) who performed village studies in different villages spread over the country. In Appendix A3.5 results of these studies are discussed. These studies will be used in Section 9.1 to estimate average expenditures on cereals and average income levels per capita per CRPA. The income per person turns out to be one of the major determinants of cereal purchases.

8.1.6 Agricultural prices

It has already been mentioned that many households sell cheap during the post-harvest season and purchase dear during the lean season. Cereal prices in Burkina Faso, as in many West African countries show a clear seasonal fluctuation. Prices are low immediately after the harvest, and increase considerably during the year, to start decreasing again just before the new harvest. Seasonal price increases are caused by demand and supply differences (supply is large during the post-harvest season, whereas demand is highest during the lean season) and storage costs which are charged in the prices. Prices in the lean season may be up to the double of post-harvest prices. In Burkina Faso price data are gathered by SIM/SONAGESS (Système d'Information sur les Marchés/Société Nationale de Gestion des Stocks de Sécurité). Since 1992 prices for all cereals are gathered on 37 markets scattered over the country. A distinction is made between producer and consumer prices. Producer prices ensue from transactions between producers and traders, consumer prices ensue from transactions between consumers and traders or between consumers and producers. For the analysis, we used prices for the crops white sorghum, millet and white maize. Data for red sorghum and yellow maize have been omitted because only very few data were available. In Appendix A3.6 the price data for the period 1992-1999 are analysed and some other studies discussing these price data are briefly discussed (Bassolet, 2000, Hoftijzer, 1998).

Four main conclusions can be drawn from the price analysis in Appendix A3.6:

1. Producer prices are lowest in the high production, surplus regions of the country, the CRPA Mouhoun and Hauts Bassins. Consumer prices are highest in the shortage regions Centre (with the capital Ouagadougou) and Sahel, See Table A3.36 and A3.37. Producer prices in the CRPA Nord are lower than expected. However, for this CRPA only data for Ouahigouya are available. This is a transit market, through which large amounts of cereals pass from Bobo Dioulasso towards the northern regions, and where supply from producers to traders is low. Producer prices in the regions Centre Est, Centre Ouest and Est and consumer prices in the regions Centre Sud and Centre Est are also high. This may be caused by demand from traders from the neighbouring countries Ivory Coast, Ghana and Togo. However, this can not be supported with data on cross-border trade. Consumer prices in these regions may also be high due to low cereal demand from the mainly rural population who usually can sell small surpluses – see Section 8.1.2.
2. Producer and consumer prices increased a lot after the devaluation of the Franc CFA in January 1994. This increase was not caused by lower cereal production in these years – see Section 8.1.2. On average producer and consumer prices in the period 1996 to 1999 were, respectively, 91% and 99% above the average prices between 1992 and 1994, see Table A3.39. Prices in the cotton producing areas have increased more than prices in the non-cotton areas, probably due to reforms in the cotton sector. It looks as if prices stabilized after October 1996.
3. Retail trade margins (the difference between the consumer and producer price in a region) increased significantly after the devaluation, see Table A3.39. Trade margins from transport from the surplus zones to Ouagadougou did not change a lot, whereas margins from transport to the other regions increased a lot. It looks as if competition on the wholesale markets in Ouagadougou has become more competitive, whereas traders make high profits from trade towards the retail markets in the shortage regions. A more detailed inquiry of trade costs and competition in Burkina Faso is needed to explain this.

4. In most years, consumer and producer prices reach their maximum in July and August. Minimum prices are reached in November or December. It can be concluded cautiously, that prices reach their minimum earlier if the harvest is bad, see Table A3.36 and Figure A3.5. Prices in the period July-September exceed prices between October and December on average with 17% and 18% for producer and consumer prices, respectively, see Table A3.36. Differences between CRPA are, however, large. It looks as if this did not change a lot after the devaluation.

To estimate the parameters for the supply and demand functions in Section 8.2 and 8.3, average producer and consumer prices for each quarter are used. Due to the huge price increase after the devaluation of the Franc CFA in 1994, we do not use '92-'99 averages, but average prices for the period October 1996 – September 1999. Although these averages are based on only a short time period, this is more realistic than using the prices for the entire period. Average cereal price levels for the period Oct '96 – Sept '99 are presented in Table 8.3.

8.2 Trading costs

Price differences between regions and between periods, are, as discussed in Chapter 5 to 7, caused by differences in supply and demand, and by the costs made by the trading agents. In Chapter 5 to 7, a distinction has been made between transport and storage costs. We distinguish also 'other trading costs', which include the costs which are made when the cereals are sold or purchased on the market. These costs include among other things costs to purchase bags, market taxes, and personnel costs. These costs are not included in the theory discussed in the Chapters 4 to 7, but are discussed here because they may amount to a significant part of the trading costs. In Appendix 4 the main conclusions and data of some studies on trading costs are discussed.

Table 8.3 Average seasonal cereal prices for the period Oct '96 – Sept '99 in FCFA per kg.

Oct '96 – Sept '99	Producer Cereal price				Year	Consumer Cereal price				Year
	Jan-Mar	Apr-Jun	Jul-Sept	Oct-Dec	Average	Jan-Mar	Apr-Jun	Jul-Sept	Oct-Dec	average
Centre						128	136	140	133	134
Centre Nord	104	104	116	100	103	120	129	137	119	126
Centre Ouest	115	125	125	102	115	121	131	134	114	125
Centre Sud	122	131	133	116	123	126	139	126	122	128
Sahel	105	103	93	96	100	133	144	149	131	139
Mouhoun	89	101	103	83	94	103	114	120	96	108
Est	106	113	116	102	109	112	125	133	108	120
Centre Est	111	118	126	117	117	125	132	133	119	127
Nord[1)]				99	111	114	124	128	110	119
Sud Ouest[1)]	83				83	134	143	152	129	139
Hauts Bassins	86	89	95	86	89	108	117	118	108	113
Comoe	105	116	114	122	113	119	134	140	123	129
Burkina Faso	103	109	110	99	104	119	129	133	116	125

Note: 1) Not enough data were available for these CRPA to estimate the average prices for all periods.

Source: Data from SIM/SONAGESS

8.2.1 Transport costs

In Burkina Faso the costs for transporting cereals between the place of production and the place of consumption, cause for a large part of the price differences between regions. Transport costs taken into account in this study only include the transport costs between markets, made by traders. Costs made by the producers to bring the produce from the field to the compound and from the compound to the market, as well as the costs made by the final consumers to transport their purchases from the market to their houses are not taken into account.

Transport costs are much influenced by the road conditions, which may be poor. In Burkina Faso, only a small portion of the road network is asphalted. Only the roads leading from Ouagadougou to Bobo-Dioulasso, to Ouahigouya, to Kaya and to Koudougou, and from Ouagadougou or Bobo-Dioulasso to the main border crossings with Ivory Coast, Ghana, Togo, Benin, Niger and Mali are asphalted. The other roads are unpaved roads. Some of the unpaved roads may be rather good, but others are only small trails (in our analysis we call them ‘dirt roads’) which are almost inaccesible for cars, not to mention trucks. Some roads, especially the dirt roads, which are passable during the dry season turn into mud trails during the rainy season inaccesible for cars. Some of the ‘good’ unpaved roads may also be closed for a few hours or days during the rainy season if the lower parts of the roads (which are sometimes constructed on purpose to prevent parts of the road to be washed away during showers by the swirling water running to lower places) are flooded.

Because of these bad road conditions, travelling time may be long and maintenance costs for trucks high. It regularly happens that trucks get stuck along the road because of breakdowns, which may delay the journey considerably. These problems cause transport costs to be high. If two villages are located along an asphalted road, transport costs between these villages may be cheap. However, sending a truck to a remote village in the Sahel during the rainy season may be a costly and risky undertaking.

Most traders (certainly the small and medium) do not own their own transport means, but have to rent a truck (if the traded quantities are large enough) or pay a certain transport price for each bag transported by a truck owner (which is often a merchant). Déjou (1987) reports that cereal transporters sometimes have monopolistic power. Merchants owning a truck may force other traders who are dependent upon this merchant for their transport, to respect mutual price agreements. However, according to Déjou (1987), transport is in general not the limiting factor for most regions. Competition is in most cases satisfactory. Transport costs between a number of markets are given in Appendix A4.1. It can be expected that transport became considerably more expensive after the devaluation of the Franc CFA in January 1994. However, a comparison of Table A4.3 and Table A4.4 in Appendix A4.1 does not give any evidence for such an increase. For some routes, the transport prices paid by the cereal traders to the carriers even decreased. This corresponds with the observation in Section 8.1.6 that the difference between the consumer price in Ouagadougou and the producer price in the surplus zones hardly increased. It is, however, in contradiction with observations that the difference between consumer prices in the northern shortage regions and producer prices in the south-western surplus zones increased considerably after the devaluation – see Section 8.1.6 and Appendix A3.1. It also corresponds to observations in other recent reports (Egg et al., 1997; Danida, 1999; UE, 1999), who also concluded on the basis of the SIM/SONAGESS price data, that trade margins from trade between the surplus zones and Ouagadougou, remained relatively stable after the devaluation, despite a substantial increase of cereal prices and prices of fuel and spare parts. The stability of margins for transport to Ouagadougou may be due to an increased competition on the cereal market. However, it causes difficulties for transporters who can not face the competition, and who may have difficulties of purchasing new vehicles in the near future (Danida, 1999, p. 13).

Transport prices also depend on the means of transport used. Sirpé (2000) makes a distinction between small pick-up trucks, 10-tonne trucks, and large 32-tonne trucks. Pick-ups are most often used to transport goods over short distances; for example, between villages, or from villages to the nearest city. The large, 32-tonne trucks are mainly used for international transport. Transport between the main commercial centres, and between the different provinces, mainly involves 10-tonne trucks. Sirpé makes a persistent distinction between the transport *costs* carriers make (i.e. costs for fuel, maintenance, personnel, depreciation), and transport *prices* to be paid to carriers by traders who rent transport services from them. Sirpé evaluates average transport *costs* per kilometer for each type of truck – see Appendix A4.1. The load rate (the part of the loading capacity of the truck which is filled) plays an important role in the costs. In this paper we only consider the distribution network between the main centres of each CRPA. It is assumed that only 10-tonne trucks are involved. Furthermore, the location of the carriers, an issue brought forward by Vogelzang (1996), may play a role in transport prices. Transporting cereals between two remote villages alongside tarmac roads in which no carriers are located will probably be relatively more expensive than between Ouagadougou and Bobo-Dioulasso. For example, if a carrier located in a city *C* has to transport goods from a village *A* to a village *B*, not only the costs to travel between *A* and *B*, but also the costs to go from *C* to *A* and to return from *B* to *C* have to be paid for. Furthermore, carriers transporting towards remote cities have less possibilities to find a freight for the return journey than carriers transporting between, for example, Ouagadougou and Bobo-Dioulasso. For that reason, transport between the more frequented markets will probably be cheaper than between more remote markets.

In this report we do not consider the transport costs made by the carriers. We only look at the transport costs cereal traders have to make (which are the transport prices charged by the carriers in the notation of Sirpé, see above). Using the studies discussed in Appendix 4, transport costs have been estimated. A difference has been made between 1) transport along the busy trade routes over asphalted roads (from Ouagadougou to Bobo Dioulasso, to Pouytenga, and to Koudougou and from Bobo-Dioulasso to Koudougou); 2) transport over less frequented trade routes over asphalted roads; 3) transport over (all-weather) unpaved roads, and 4) transport over (bad) dirt roads. Although most transport is done over asphalted and unpaved roads, some of the cities are connected by dirt roads (for example, the route Bobo-Dioulasso - Diebougou and a part of the route between Ouahigouya and Kaya). Transport along busy trade routes is cheaper than transport along less frequented routes, since the chance to have a return freight is larger for these routes. Transport over unpaved roads is more expensive than over asphalted roads. Transport over dirt roads is even more expensive. Furthermore, during the rainy season (July to September) transport over unpaved roads and dirt roads is more expensive than during the dry season (October to June). In this paper only transport between the main commercial centres is considered. For each CRPA one or two centres have been chosen, for which transport costs to other regions are estimated. The distance between each pair of cities has been estimated using the road map of Burkina Faso (see Table A4.5 in Appendix A4.1). For each road connection it has been estimated what part of the route is over busy asphalted roads, less frequented asphalted roads, unpaved roads or dirt roads. Next, transport costs per km per road type are estimated using the data in Appendix A4.1. By multiplying the costs per km with the distance, the costs to transport goods between two cities or two CRPA is estimated.

It is difficult to make balanced estimates of the transport costs. Most transport costs presented in the transport surveys do not make a distinction in costs per road type, although it is admitted that they differ a lot. Furthermore, the data of the different studies do not always correspond. For example, transporting 100 kg of cereals between Ouagadougou and Gorom-Gorom costs 2000 FCFA according to Déjou (1987), and 950 FCFA according to Bassolet (2000). Sirpé (2000) argues that transport costs depend on a lot of factors, of which the loading rate plays a major role. By comparing the different studies, we can make the following observations:

- Transporting along the busy trade routes, between the most animated markets (i.e. from Ouagadougou to Bobo-Dioulasso, to Pouytenga and to Koudougou and from Bobo-Dioulasso to Koudougou), costs the traders, according to Déjou (1987) and Bassolet (1997), less than 20 FCFA per kilometer per tonne (observations range between 11 and 21 FCFA). These costs are considerably lower than between the other markets. Reasons for these lower costs are that most carriers are located on these markets, and that they have more possibilities to find return freights.

- Transport costs between the other market depend on road type. Divide these trade routes, presented in Table A4.3 and Table A4.4 in Appendix A4.1, in routes over asphalted roads, routes which are both over asphalted and unpaved roads, and routes which are only over unpaved roads and dirt roads.[16] It can be observed that transport costs between markets connected by asphalted roads are on average 40 FCFA per tonne per km (observations range between 20 and 52 FCFA; standard deviation 15 FCFA). If it is also partly over unpaved roads, only a minor increase is observed (on average 41 FCFA, observations range between 20 and 76 FCFA, standard deviation 15 FCFA). If it is only over an unpaved road or if also dirt roads must be passed, costs per tonne per km increase on average to 52 FCFA (observations range between 24 and 82 FCFA, standard deviation 14 FCFA).
- It is striking that transport costs over unpaved roads and dirt roads paid by the cereal traders, increase less than the rise of transport costs made by the carrier reported in Table A4.2 in Appendix A4.1. Either the increases reported in the table are too high, or the transport price charged by the carriers is too low to cover their costs.
- The transport *price* charged by carriers of 112 FCFA, reported by Sirpé (2000) seems to be very high. It is not clear why his estimates are more than twice the averages observed by Déjou (1987) and Bassolet (1999).
- During the rainy season, the costs for transport over unpaved roads increases on average with 17%.

These considerations bring us to make the following estimations for the transport costs:

[16] For these calculations the more lively transport routes (Bobo-Dioulasso, Koudougou, Ouagadougou, Pouytenga) with lower transport costs and the short routes (less than 80 km) with higher transport costs are not taken into consideration.

1. Transporting cereals over the less frequented asphalted roads is estimated to cost during the dry and rainy season 40 FCFA per tonne per km.
2. For the busy routes between the CRPA Hauts-Bassins and Centre (Bobo-Dioulasso to Ouagadougou), Centre–Est and Centre (from Pouytenga to Ouagadougou) and between Koudougou and the CRPA Centre or Hauts Bassins costs per tonne are estimated at 25 FCFA per kilometer. For these routes it is supposed that transporters have return freights more often, so that transport costs can be lower. Although the market of Ouahigouya is one of the most important distribution centres of the country, transporting towards this market is not reported to be cheaper.
3. Average transport costs from Hauts Bassins and Centre towards the CRPA Centre Ouest, in which Koudougou is situated, exceeds 25 FCFA. To estimate the average transport costs to the CRPA Centre Ouest, the average is taken of transporting to Koudougou and to Leo. Since Leo is a less busy market than the market of Koudougou, transport costs from Ouagadougou and Bobo-Dioulasso towards Leo will exceed those for Koudougou.
4. Transport over unpaved roads is 20% more expensive than transport over the less frequented asphalted roads during the dry season, and 40% more expensive during the rainy season.
5. Transport over dirt roads is 40% more expensive than transport over the less frequented asphalted roads during the dry season, and 140% more expensive during the rainy season. Transporting over dirt roads during the rainy season is a risky undertaking. If the truck gets stuck, it may take a few days before the destination is reached.

Estimated transport costs between each CRPA are presented in Table 8.4.

Table 8.4 Estimation of the transport costs during the dry (October – June) and rainy season (July – September) in FCFA per 100 kg bag.

Dry Season	Centre	Centre Nord	Centre Ouest	Centre Sud	Sahel	Mouhoun	Est	Centre Est	Nord	Sud Ouest	Hauts Bassins	Comoe
Centre	0	392	517	408	1084	840	695	343	724	1366	890	1230
Centre Nord	392	0	909	800	773	1294	1070	718	918	1758	1282	1622
Centre Ouest	517	909	0	875	1601	924	1212	860	1118	934	1120	1460
Centre Sud	408	800	875	0	1492	1248	940	588	1132	1769	1298	1638
Sahel	1084	773	1601	995	0	1652	1473	1473	1062	2450	1974	2314
Mouhoun	840	1294	924	1248	1652	0	1535	1183	686	1838	1094	1434
Est	695	1070	1212	940	1473	1535	0	352	1419	2061	1585	1925
Centre Est	343	718	860	588	1473	1183	352	0	1067	1709	1233	1573
Nord	724	918	1118	1132	1062	686	1419	1067	0	2090	1614	1954
Sud Ouest	1366	1758	934	1769	2450	1838	2061	1709	2090	0	744	1084
Hauts Bassins	890	1282	1120	1298	1974	1094	1585	1233	1614	744	0	340
Comoe	1230	1622	1460	1638	2314	1434	1925	1573	1954	1084	340	0

Table 8.4 (continuation)

Rainy season	Centre	Centre Nord	Centre Ouest	Centre Sud	Sahel	Mouhoun	Est	Centre Est	Nord	Sud Ouest	Hauts Bassins	Comoe
Centre	0	392	583	428	1232	940	695	343	724	1702	890	1230
Centre Nord	392	0	975	820	986	1652	1454	1102	1574	2094	1282	1622
Centre Ouest	583	975	0	1099	1815	1078	1278	926	1213	1100	1493	1833
Centre Sud	428	820	1099	0	1660	1368	1008	656	1152	2462	1318	1658
Sahel	1232	986	1815	1107	0	2152	1660	1660	1573	2934	2122	2462
Mouhoun	940	1652	1078	1368	2152	0	1634	1282	801	2501	1277	1617
Est	695	1454	1278	1008	1660	1634	0	352	1419	2397	1585	1925
Centre Est	343	1102	926	656	1660	1282	352	0	1067	2045	1233	1573
Nord	724	1574	1213	1152	1573	801	1419	1067	0	2426	1614	1954
Sud Ouest	1702	2094	1100	2462	2934	2501	2397	2045	2426	0	1224	1564
Hauts Bassins	890	1282	1493	1318	2122	1277	1585	1233	1614	1224	0	340
Comoe	1230	1622	1833	1658	2462	1617	1925	1573	1954	1564	340	0

8.2.2 Storage and other trading costs

Price differences between periods are, for an important part, caused by storage costs. A trader will only store cereals if he expects to recover at least the storage costs. Market equilibrium theory shows that the price difference between two periods on a competitive market is expected to be equal to the storage costs if the traders store cereals (see Chapter 6). An evaluation of some studies on strategies of cereal traders (see Appendix A4.2, A4.3 and A4.4) showed that storage costs may include costs of the storehouses (rent or maintenance), costs for pesticides and insecticides, surveillance costs, and capital costs. Also storage losses must be considered. Many of these costs are difficult to estimate. Many cereals are not stored by the traders for a long time, but shipped quickly. Many of the storage costs are difficult to evaluate per bag, but have to be paid independent of the number of bags stored. Moreover, differences between traders are considerable.

The influence of capital costs on traders' storage decisions demands some extra explanation. In stead of capital costs, many authors take into account 'opportunity costs'. These are no 'real' costs to the traders. They reflect the foregone revenues if the trader would have invested the money value of his cereal stock in other activities, for instance put the money on the bank raising interest. In stead of calculating 'opportunity costs', which are rather difficult to determine, we prefer to estimate 'capital costs'. [17] Capital costs correspond to the interest payments a cereal trader should pay if he borrowed money from a bank to finance his cereal purchases. In each period he should pay interest costs, which are a certain percentage of the money

[17] In order to calculate opportunity costs, the traders' capital balance and activity portfolio should be considered. In that case, a comparison could be made between the possible investment opportunities and credit needs. For the moment we do not introduce this capital balance. Although we acknowledge that the development of credit facilities for cereal traders and farmers may be an important policy measure to improve the functioning of the market, it goes too far for this paper to evaluate the importance of credit costs in the strategies of cereal traders.

invested in the stock (i.e. of the quantity stored multiplied with producer price plus transport costs plus storage costs).

Other costs involved in trading may constitute an important part of the trading costs. These costs include personnel costs, costs to buy cereal bags, and taxes. Personnel costs are the salaries paid to intermediaries of merchants, who may be resident buyers and sellers or regional coordinators. They may receive a monthly salary or a commission. Personnel costs also include truck loading and unloading costs. Taxes include both trade and market taxes. The first category are the taxes which have to be paid to be allowed to operate as a trader. The second category are the taxes which have to be paid daily, weekly or monthly to be allowed to use the market infrastructure of a certain market place. The level of these taxes depends on the business size of the traders.

To calculate the equilibrium model discussed above, we also need estimates of storage and other trading costs per bag. No precise estimates can be made because of data limitations. For some of the services the costs per 100 kg bag are easy to estimate (personnel who are paid on a commission basis, costs for bags, loading and unloading costs), for others this turns out to be difficult. For example, personnel costs can be estimated per month, but costs per bag will differ considerably between months and traders. To estimate these costs per bag, not only the monthly costs, but also the number of bags traded must be known. These data are missing in some of the surveys available. Also costs for storehouses are difficult to estimate per kg. Monthly costs of a storehouse can be estimated, but to estimate the costs per bag, it should be kept up how long each bag is stored. Costs per bag are best described in Sherman et al. (1987) – see Appendix A4.4. Based on their estimates, and using the studies of Bassolet (2000) and Déjou (1987) as reference literature, we made estimates which are presented in Table 8.5. Because of the weakness of the data we do not make seperate estimates for the different CRPA. It is noted that the sensitivity of the model to the estimates must be analysed carefully. The costs estimated are:

1. Storage costs, including renting costs, surveillance and insecticides: Sherman does not present storage costs seperately, but places it under ‘sundry costs’, see Table A4.9 column (c) in Appendix A4.4, which are renting costs for warehouses, taxes, bribes and other costs. Sundry costs are on average 200 FCFA per bag. We suppose that half of these costs, so 100 FCFA per bag, are renting costs for warehouses and insecticides. Salaries paid to personnel engaged in storage is part of the personnel costs mentioned in column (b) in Table A4.9 in Appendix A4.4. The salary of a warehouseman per bag sold by the trader is estimated by Sherman between 40 FCFA and 200 FCFA per bag, with an average of 100 FCFA per bag. Suppose that also one of the apprentices of the trader (Sherman supposed that two apprentices are working for the trader) is half of his time occupied with controllling storage. Therefore, personnel costs for storage are 150 FCFA per bag. Total storage costs are 250 FCFA per bag
2. Storage losses: Bassolet observes storage losses of 8% per year, and Déjou of 15 to 20% per year. We take an average of 12% per year, so 3% per quarter, see Appendix A4.2.
3. Capital costs: the ongoing bank interest rate is 14% per year (3.5% per period) - see Appendix A4.3. Capital costs per quarter are estimated at 3.5% of the producer price of a bag of cereals. For the producer price we take the average producer prices, which are given in Table 8.3.[18] Producer prices for the CRPA Centre and missing producer prices for the regions Nord and Sud Ouest are supposed to be the average producer prices for Burkina Faso. The discount rate is assumed to be equal to 1/(1+*r*), with *r* the interest rate of 3.5% per period. Rounded off, the discount rate is 0.97.
4. Costs for bags: We adopt the estimate given by Sherman in Table A4.9 in Appendix A4.4, who gives an average cost of 200 FCFA per bag. This is a little

[18] We multiply the percentage capital costs with a predetermined, average value of the producer price, and not with the variable p_{it}. This last option would complicate the model considerably because it would result in an extra non-linear term ($p_{it} \cdot s_{it}$) in the objective function.

lower than the observations of Bassolet and Déjou in Appendix A4.3. Considering multiple uses of these bags 200 FCFA might still be high.

5. Annual and market taxes plus other trade costs: Bassolet and Déjou give trade taxes as percentage of profits and market taxes per day. The average amount of taxes paid per bag estimated by Bassolet in Table A4.10 and Table A4.11 in Appendix A4.4 is very low. We suppose that taxes plus other trade costs are half the ‘sundry costs’ given in column (c) in Table A4.9, so 100 FCFA per bag – see also under 1).
6. Personnel costs: Again, the personnel costs estimated by Bassolet are very low. Personnel costs and the payments to personnel paid on a commission basis are estimated by Sherman in the columns (a) and (b) in Table A4.9. His estimates are much higher than the estimates made by Bassolet, because Sherman includes the ‘salary’ of the trader. We also have to include this salary because the trade costs considered in our model have to account for the total difference between cereal consumer and producer prices. This difference includes the margin earned by the trader (i.e. his salary). Total personnel costs reported by Sherman vary between 400 and 1250 FCFA per bag. Part of these personnel costs are storage costs – see under 1). Other personnel costs (which are part of the trade costs) are estimated at 700 FCFA per bag.
7. Loading and unloading costs: Observations range between 50 FCFA per bag by Déjou, 250 FCFA per bag by Bassolet and 100 FCFA per bag by Sherman. We suppose it costs100 FCFA per bag to load or unload a truck. So, total loading and unloading costs are 200 FCFA per bag. We do not consider them to be trade costs, but treat them as transport costs.

Table 8.5 Trading costs in FCFA per 100 kg bag or in %.

1)	Storage costs per quarter, including renting costs, surveillance and instecticides: 250 FCFA per bag
2)	Storage losses: 3% per quarter
3)	Capital costs: $3.5\% * \tilde{p}_{it}$, in FCFA per 100 kg bag, with $\tilde{p}_{it}$ the average producer price of a 100 kg bag in region *i* in period *t* - see Table 8.3 for the producer prices per kg. Discount rate: 0.97.

CRPA	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sep	CRPA	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sep
Centre	3.47	3.59	3.81	3.86	**Est**	3.58	3.73	3.97	4.05
Centre Nord	3.50	3.64	3.64	4.05	**Centre Est**	4.10	3.88	4.14	4.39
Centre Ouest	3.59	4.03	4.37	4.39	**Nord**	3.47	3.59	3.81	3.86
Centre Sud	4.07	4.28	4.58	4.66	**Sud Ouest**	3.47	3.59	3.81	3.86
Sahel	3.36	3.69	3.60	3.25	**Hauts Bassins**	3.00	3.02	3.13	3.32
Mouhoun	2.89	3.10	3.52	3.60	**Comoe**	4.28	3.68	4.04	4.00

4)	Costs for bags: 200 FCFA per bag
5)	Taxes plus other costs: 100 FCFA per bag
6)	Personnel costs: 700 FCFA per bag
7)	Loading and unloading costs: 200 FCFA per bag

9 Estimation of cereal demand and supply functions for the case of Burkina Faso

For the analysis of the inter-regional cereal flows in Burkina Faso, use is made of the multi-period model (7.64) for a situation of a competitive market. The exogenous elements of the models are the storage costs and losses per stored unit per unit of time, the transport costs per transported unit of cereals between the various markets, the trading costs per unit of cereals sold, and the cereal supply and demand *functions*.[19] Storage, transport and trade costs have already been estimated in Section 8.2. Supply and demand functions are not readily available. In Section 9.1 regional demand functions will be estimated for each period, by choosing a functional form and then estimating the parameters with the aid of data and information discussed in Section 8.1. For the estimation of quarterly, regional producer supply in Section 9.2, the method discussed in Section 7.1 is extended. Annual supply is estimated, based on data on sales and production levels for each region discussed in Section 8.1. Annual supply depend on production levels and other factors, rather than on prices. As discussed in Section 8.1.3 the dependence of *yearly* supply on prices is weak. The distribution of the annual cereal supply over the year does depend on prices. The supply in each period is for a part influenced by cash needs, and for another part by the expected price development within a year.

9.1 Cereal demand functions

In the preceding chapter we discussed the cereal purchase behaviour of househols. In this section we will estimate cereal demand functions as a function of cereal prices for an ‘average’ consumer for each CRPA. Regional demand functions per CRPA are determined by aggregating the individual demand functions. First, in Section 9.1.1 a functional form for the demand functions is chosen. In Section 9.1.2 differences between the annual cereal demand functions for rural and for urban households are

[19] The supply and demand *functions* are exogenous elements of the model, supply and demand itself are endogenous elements.

discussed and the parameters for the annual cereal demand functions are estimated. Finally, in Section 9.1.3 the cereal demand functions per period are defined and the aggregate, regional demand functions are estimated.

9.1.1 Linear Expenditure System

The demand function adopted in this paper is derived from the widely applied Linear Expenditure System (LES) (see e.g. Roth (1986) for a discussion and application of the LES, see also Theil (1980) and Section 4.2). The LES is derived from the Stone-Geary utility function. It is widely applied because it is simple and has convenient properties. Although more elaborate demand systems exist (see for example Deaton and Muellbauer, 1980), data limitations prevent us from using them. To illustrate the principles of the LES, consider the case where a consumer can consume different commodities. The quantity consumed is purchased entirely on the market at consumer prices (so that consumption equals market demand). Define K the set of goods the consumers can purchase. Each consumer demands at least a minimally required, fixed quantity of each commodity (it is supposed that they can afford to buy this minimum quantity). This minimally required quantity may either be a minimum subsistence level of consumption or a minimum preferred quantity. The income remaining after purchasing all minimally required quantities, is divided in fixed shares over the commodities from the set K. This remaining income is also called 'supernumerary income', i.e. income after initial purchases. Introduce, for each commodity $k \in K$ the elements:

U	The utility level a consumer obtains from consuming the K commodities,
C_k	Consumption level of commodity k,
γ_k	The minimally required purchase level of commodity k,[20]

[20] This minimally required purchase level should not be compared with the minimally required quantity of nutrients which is necessary to remain healthy.

b_k Weighing coefficients corresponding to the preferences of consumption of commodity *k*.

then the Stone-Geary utility function can be written as:

$$(9.1) \qquad U = \sum_{k \in K} b_k \ln\left(C_k - \gamma_k\right), \quad \text{with} \quad \begin{cases} 0 < b_k < 1 \\ \sum_k b_k = 1 \\ C_k - \gamma_k > 0 \end{cases}$$

Manipulation of the first order conditions from utility maximization subject to an income constraint, gives the Linear Expenditure System. First introduce:

I Income level of a consumer which can be spent on buying the commodities from K

π_k Consumer price level of commodity *k*.

The Linear Expenditure System can now be written as:

$$(9.2) \qquad C_k = \gamma_k + \frac{b_k}{\pi_k}\left(I - \sum_{i \in K} \pi_i \gamma_i\right)$$

with $I \geq \sum_{i \in K} \pi_i \gamma_i$ and $0 < b_k < 1$, and $\sum_{k \in K} b_k = 1$.

The expression $I - \sum_{i \in K} \pi_i \gamma_i$ is the discretionary or supernumerary income which remains after due allowance for the minimum requirements. This income is allocated among the different goods in shares b_k/π_k. It follows that the parameter b_k is the share of supernumerary income spent on purchases of commodity *k*. It can also be interpreted as the marginal buget share $\frac{\partial(\pi_k C_k)}{\partial I}$, "which tells how expenditures on each commodity change as income changes" (Sadoulet et al., 1995). Expression (9.2) shows that expenditures on each good ($C_k\pi_k$) are linear in prices and income. Tastes, preferences, and subsistence requirements are implicitly included in the values of the parameters b_k and γ_k. The own-price, cross-price and income elasticity of demand of this demand function can be written as, respectively:

$$(9.3)\qquad \varepsilon_{kk} = \frac{\partial C_k}{\partial \pi_k}\frac{\pi_k}{C_k} = -1 + \frac{\gamma_k}{C_k}(1-b_k), \qquad \varepsilon_{kj} = \frac{\partial C_k}{\partial \pi_j}\frac{\pi_j}{C_k} = -\frac{\pi_j b_k \gamma_j}{\pi_k C_k},$$
$$\varepsilon_{kI} = \frac{\partial C_k}{\partial I}\frac{I}{C_k} = \frac{b_k I}{\pi_k C_k}.$$

From expression (9.3) it follows that -1 < ε_{kk} < 0 (since C_k > γ_k, see (9.1)). So, demand decreases if consumer prices increase, but less than proportionally. A consequence is that no inferior goods (for which ε_{kk} > 0) can be considered. This also follows from the expression of the income elasticity, which is always positive.

Now suppose that *cereals* (comprising red sorghum, white sorghum, millet and maize) is one of the commodities from the set K. The other commodities may contain among other things rice. Since we do not intend to analyse the role of prices of the other commodities on cereal distribution in Burkina Faso, it is not necessary to specify all commodities of set K. Only the budget share b_{cer}, the minimum cereal purchase level γ_{cer}, and the minimum expenses on the other commodities, $\sum_{\substack{k\in K\\ k\neq cer}} \pi_k\gamma_k$, need to be estimated.[21] The cereal demand function as a function of income I and cereal prices π_{cer}, can now be written as - see equation (9.2):

$$(9.4)\qquad C_{cer} = \gamma_{cer} + \frac{b_{cer}}{\pi_{cer}}(I - \pi_{cer}\gamma_{cer} - \xi) = \gamma_{cer}(1-b_{cer}) + \frac{b_{cer}}{\pi_{cer}}(I-\xi)$$

[21] If the influence of the price of rice on cereal distribution is to be analysed, then also the minimum rice demand, γ_{ri}, and the minimum expenses on all commodities aside from cereals and rice, $\sum_{\substack{k\in K\\ k\neq cer,ri}} \pi_k\gamma_k$, have to be estimated.

with $\xi = \sum_{\substack{k \in K \\ k \neq cer}} \pi_k \gamma_k$, and in which γ_{cer}, b_{cer} and ξ are exogenously given parameters.

9.1.2 Estimating cereal demand functions for Burkina Faso

To analyse cereal demand in Burkina Faso demand functions have to be estimated seperately for rural and urban consumers. Introduce the following set and variable:

$H = \{u, r\}$	set of urban (u) and rural (r) consumers
y^h	the level of cereal demand by a consumer of type h, for $h \in H$

For the sake of readibility, we do not present here cereal demand in region i as, y_i^h like we did in Chapter 5 and 6. The region index i has been skipped, and the variable y^h refers now to cereal demand of an individual consumer. Redefine also the parameters and variables introduced above, to indicate the type of consumer. For $h \in H$:

C^h	Cereal consumption level by a consumer of type h
γ^h	The minimally required cereal purchase level by a consumer of type h
b^h	Share of supernumerary income spent on cereal purchases by a consumer of type h
I^h	Income level of a consumer of type h
ξ^h	Minimally required expenses on all commodities except cereals by a consumer of type h
π	Cereal consumer price level.

For *urban* consumers it is supposed that they demand their entire consumption on the market. Their annual market demand is represented by (9.4), and can also be written as:

(9.5) $$y^u = C^u = \gamma^u\left(1-b^u\right)+\frac{b^u}{\pi}\left(I^u-\xi^u\right)$$

For the urban consumers budget shares b^u, minimum cereal purchase levels γ^u, and supernumerary income levels I^u - ξ^u have to be estimated. For *rural* households, account has to be taken of the on-farm consumption of self produced cereals. For them, consumption differs from demand on the market. Cereal consumption is the sum of purchased cereals and on-farm consumption of self produced cereals. We assume that their cereal production level is more than the minimally required cereal purchase level. Then it is not necessary to purchase this required quantity on the market, but it is taken from own stocks. So, the quantity purchased on the market depends not on the minimally required level, γ^r, but only on consumer price and income levels. So, for rural households the parameter γ^r is zero. Annual cereal demand of rural households, y^r, is- see (9.4):

(9.6) $$y^r = \frac{b^r}{\pi}\left(I^r-\xi^r\right)$$

Annual cereal consumption of rural households is, if OC^r is the on-farm consumption of self-produced cereals:

(9.7) $$C^r = OC^r + y^r$$

A consequence of this definition is that for rural households the income elasticity of cereal demand is equal to $I^r/(I^r\text{-}\xi^r) > 1$ (see equation (9.3)).

We estimate average values of γ^r and b^r. However, they depend for rural households in principle on rainfall. After a good rainfall season with a higher cereal production, on-farm consumption of self produced cereals will be higher and b^r lower, than after a bad rainfall season. The influence of rainfall on demand, and consequently market price levels, can be analysed with a sensitivity analysis.

To make the different estimates of the parameters, the data presented in Section 8.1 and in Appendix A3.5 are used. Differences between the various studies are enormous. For example, Reardon et al. (1988a) estimated household income in the Sudanian rainfall zone (see Figure 8.2) at 38,820 FCFA per year; Broekhuyse (1988) observed an average household income of 65,831 FCFA per year in the province of Sanmatenga; and INSD (1996a,b) came to an average monetary income in the Centre-Nord region (including the province of Sanmatenga) of 128,598 FCFA. Often samples are small, only one or two villages in a region are chosen, or income is estimated for only one year, so it is not strange that observed differences are large. Despite these problems, we will estimate the different parameters of the demand functions.

a) Average income per consumer, I^h.

To estimate the average income per consumer, I^h, the results of the 1994 INSD poverty surveys (INSD, 1996a,b, see Appendix A3.5) are used. The other studies discussed in Appendix A3.5 give income levels which are probably too low. If it is evaluated how many cereals can approximately be purchased with the income reported by Broekhuyse (1988) and Reardon et al. (1988a), it is seen that these possible purchases do not correspond with the purchases presented in Appendix A3.4. In the INSD studies, revenues and expenditures include both monetary and non-monetary revenues and expenses. The non-monetary terms include on-farm consumption of own production and gifts and payments in kind. It has been supposed here that the level of household revenues equals the level of expenditures. Average monetary revenues per person, I^h, are estimated as (see Table A3.31 for the values):

$$I^h = \frac{\text{(total household expenses)} \cdot \text{(monetary revenues as a percentage of total revenues)}}{\text{(average household size)}}$$

These estimates do not fully correspond with what could reasonably be expected. A closer look to these estimates, shows that the income for the CRPA Sud Ouest (in the INSD Area ‘Sud and Sud-Est’) is lower than what could be expected from the production per capita given in Table 8.1 and the situation of this CRPA (see Appendix A3). For that reason, the same estimate is chosen as for the CRPA Mouhoun and Hauts Bassins, which are in a more or less similar situation. For the CRPA Est a somewhat lower estimate is chosen, since there is no reason to believe that their income is much higher than for the other CRPA in the same rainfall zone.

The resulting estimate, gives income per person for the year 1994. Income in the reference year 2000 will be considerably higher, partly due to the devaluation of the Franc CFA in 1994, which caused cereal prices to double (see Section 8.1.6). Household income also increased, but no information is available on the percentage increase. It is known that salaries of civil servants did increase, but they did not double. Furthermore, also incomes of people working in private enterprises or in the informal sector, or from farmers earning an income from off-farm labour, increased but is is not known how much. For the moment we suppose that income in the year 2000, increased with 75% compared to income in 1994. The influence of income on market prices and demand and supply must be evaluated using a sensitivity analysis. The estimates of I^h are presented in Table 9.1, with estimates for rural inhabitants in column (a) and for urban inhabitants in column (b).

Table 9.1 Estimates of annual monetary income per person for each CRPA in FCFA.

CRPA	**Rural Annual income I^r (a)**	**Urban Annual income I^u (b)**	**CRPA**	**Rural Annual income I^r (a)**	**Urban Annual income I^u (b)**
Centre	24500	262500	**Est**	24500	152250
Centre Nord	25375	152250	**Centre Est**	24500	152250
Centre Ouest	24500	152250	**Nord**	25375	152250
Centre Sud	24500	152250	**Sud Ouest**	52500	152250
Sahel	38500	152250	**Hauts Bassins**	52500	262500
Mouhoun	52500	152250	**Comoe**	52500	152250

Source: Estimates are based on INSD (1996a,b) data and some additional assumptions, see above.

b) The share of supernumerary income spent on cereal purchases, b^h.

The parameter b^h can be estimated using the income elasticity of demand for a consumer of type *h*, $h \in H$, ε^h_{cerI} (see equation (9.3), (9.5) and (9.6)):[22]

$$(9.8) \qquad b^h = \varepsilon^h_{cerI} \frac{\pi y^h}{I^h}$$

To estimate this, introduce the share of *total* income spent on cereals by a consumer of type *h*, s^h. Note that the share parameter b^h differs from the share of *total* income spent on cereals, s^h. Cereal demand y^h can also be written as the income spent on cereal purchases ($I^h \cdot s^h$) divided by the cereal consumer price π:

$$(9.9) \qquad y^h = \frac{I^h s^h}{\pi}$$

Filling in (9.9) in expression (9.8) results in:

$$(9.10) \quad b^h = \varepsilon^h_{cerI} \; s^h$$

To estimate the income elasticities of cereal demand ε^h_{cerI} we use the estimates made by Roth (1986), which are presented in Table A3.28, see also Section 8.1.4. Roth presents income elasticities for all cereal types seperately. For our purpose we need estimates of income elasticities of demand for rural and urban households for the commodity *cereals* (comprising red sorghum, white sorghum, millet and maize). Elasticities for the rural households are supposed to be the same in all CRPA. Roth gives elasticities for maize demand which are lower than for the other cereal types. Since maize consumption is only a small part of cereal consumption, we suppose that

[22] The share parameter b^h can not be estimated with equation (9.6), because we can not estimate average values of y^h and $I^h - \xi^h$

the income elasticity of cereal demand is equal to the income elasticities Roth gives for sorghum and millet demand. For urban households not living in Ouagadougou or Bobo-Dioulasso we suppose that the income elasticity is a little higher than the elasticity for Ouagadougou. Estimates are given in Table 9.2.

Table 9.2 Estimates of income elasticities of cereal demand.

CRPA	Rural households ε^{r}_{cerI}	Urban households ε^{u}_{cerI}	
		Ouagadougou/ Bobo-Dioulasso	Other cities
Income elasticity of demand	0.95	0.7	0.75

Source: Estimates are based on Roth (1986) and some additional suppositions, see above.

The parameter S^h can be estimated using the INSD data of Table A3.31. We estimate the parameter s^h as:

$$s^h = \text{(monetary cereal expenses) / (monetary income)},$$

with:

Monetary cereal expenses = (total cereal expenses) * (monetary cereal expenses as a percentage of total cereal expenses); (see Table A3.31 for the values).

Monetary income = (total expenses) * (monetary revenues as a percentage of total revenues); (see Table A3.31 for the values).

Estimates of s^h and b^h are presented in Table 9.3.

The parameters b^h are lower than the cereal budget shares mentioned in Roth (1986; see Table A3.35, the cereal budget share is the sum of the shares for white sorghum, red sorghum, millet and maize), who also deals with both monetary and non-monetary expenses. If the budget shares from Roth are converted in monetary expenses using the figures given by the INSD survey (using the percentage cereal expenses and percentage monetary revenues given in Table A3.31), the difference between the Roth and INSD study is not that large.

Table 9.3 Estimates of the share of annual supernumerary income on cereal purchases for rural and for urban households per CRPA for an average rainfall year.

	Rural households			**Urban households**		
CRPA	s^r **(a)**	ε^r_{cerI} **(b)**	b^r **(a)*(b)**	s^u **(c)**	ε^u_{cerI} **(d)**	b^u **(c)*(d)**
Centre	0.15	0.95	0.14	0.08	0.7	0.06
Centre Nord	0.14	0.95	0.13	0.09	0.75	0.07
Centre Ouest	0.15	0.95	0.14	0.09	0.75	0.07
Centre Sud	0.15	0.95	0.14	0.09	0.75	0.07
Sahel	0.16	0.95	0.16	0.09	0.75	0.07
Mouhoun	0.09	0.95	0.09	0.09	0.75	0.07
Est	0.14	0.95	0.13	0.09	0.75	0.07
Centre Est	0.15	0.95	0.14	0.09	0.75	0.07
Nord	0.14	0.95	0.13	0.09	0.75	0.07
Sud Ouest	0.14	0.95	0.13	0.09	0.75	0.07
Hauts Bassins	0.09	0.95	0.09	0.08	0.7	0.06
Comoe	0.09	0.95	0.09	0.09	0.75	0.07

Notes: (a) and (c) based on table Table A3.31, see above. (b) and (d) given in Table 9.2.

c) Minimally required cereal purchases, γ^h .

Above it has been supposed that each *urban* consumer has to purchase at least a minimum amount of cereals. Since there are no data on which to found this estimate, it will be a rough estimate. If we consider a necessary cereal consumption to remain healthy of approximately 190 kg of cereals per person per year, of which an increasing part consists of rice, the minimum level of cereal purchases (of red sorghum, white sorghum, millet and maize) will not be very high. Certainly for consumers in Ouagadougou and Bobo-Dioulasso, who consume relatively more rice than consumers in other cities, the minimum requirements will be moderate. In Section 8.1.4 it has been discussed that rice consumption of urban households ranged between 19% and 32% of total cereal consumption in Ouagadougou in the early ‘80s.

The data in Table A3.17 show that rice consumption per person (divide the sum of rice production and imports by the urban population which is reported in Table 8.1) maybe even exceeds this percentage. Suppose now that urban consumers in Ouagadougou and Bobo Dioulasso consume each year about 90 kg of rice, and other urban consumers 70 kg. The remainder of the required consumption of 190 kg will consist of cereals. Suppose now that half of this remainder has to be purchased as the minimally required purchases. The exact estimate of γ^h is not very important, since its influence on the total purchased quantity is small[23]. So, the parameter γ^r is:

(9.11) For urban consumers in Ouagadougou or Bobo-Dioulasso: $\gamma^u = 50$ kg.
For urban consumers in other cities: $\gamma^u = 60$ kg.

For rural consumers it has been supposed that they produce the minimum requirement themselves. They do not have to purchase a minimum amount of cereals on the market. So,

(9.12) For rural consumers: $\gamma^r = 0$.

d) Supernumerary income, $I^h - \xi^h$.

To estimate the *supernumerary income* per person, $I^h - \xi^h$, we suppose that all other goods of the commodity set K are aggregated in one commodity, k. It follows that, if γ_k^h are the minimum requirements of commodity k for household h and π_k the

[23] For example, if the minimally required purchases increase from $\gamma^h = 50$ to 60, then the change of consumption is:

$$\frac{\left(60\left(1-b^h\right)+\frac{b^h\left(I^h-\xi^h\right)}{\pi^{\gamma=60}}\right)-\left(50\left(1-b^h\right)+\frac{b^h\left(I^h-\xi^h\right)}{\pi^{\gamma=50}}\right)}{\left(50\left(1-b^h\right)+\frac{b^h\left(I^h-\xi^h\right)}{\pi^{\gamma=50}}\right)}=\frac{\left(10\left(1-b^h\right)+b^h\left(I^h-\xi^h\right)\frac{\left(\pi^{\gamma=50}-\pi^{\gamma=60}\right)}{\pi^{\gamma=50}\pi^{\gamma=60}}\right)}{\left(50\left(1-b^h\right)+\frac{b^h\left(I^h-\xi^h\right)}{\pi^{\gamma=50}}\right)}$$

. The price difference between $\pi(\gamma^h = 50)$ and $\pi(\gamma^h = 60)$ will probably be small, because of which the second term in the numerator will be negligable. Consequently, the change of consumption will approximately be 1/5 kg.

consumer price of commodity k, then: $\xi^h = \pi_k \cdot \gamma_k^h$. We have no information available on which to found estimates of ξ^h. We therefore make a rough estimate, of which the importance will later be analysed by doing a sensitivity analysis. For the commodity k we suppose that for each consumer (rural and urban) the minimum requirement γ_k^h is half the total purchases of commodity k ($\gamma_k^h = ½ \cdot C_k^h$). Considering consumption function (9.2) for commodity k it follows that:

$$(9.13) \quad C_k^h = 2\gamma_k^h = \gamma_k^h + \frac{b_k^h}{\pi_k}\left(I^h - \pi_k\gamma_k^h - \pi\gamma^h\right) \quad \Rightarrow \quad \xi^h = \pi_k\gamma_k^h = \frac{b_k^h}{1+b_k^h}\left(I^h - \pi\gamma^h\right)$$

Since it is supposed that the entire income I^h is spent on cereals and commodity k ($b_k^h + b^h = 1$), the value of b_k^h can be determined from Table 9.3. Income levels I^h are given in Table 9.1, average cereal consumer price levels π are given in Table 8.3, and the value of γ^h is given in (9.11) and (9.12). Estimates of the supernumerary income I^h - ξ^h are given in Table 9.4.

It is recognized that the estimates made under a) to f) in Table 9.4 are rather unreliable. To analyse the impact of parameter changes, results will be analysed carefully by making use of a sensitivity analysis.

9.1.3 Cereal demand functions per period

The estimates made in the previous section result in cereal demand functions for an entire year. In order to come to quarterly demand functions some suppositions have to be made concerning the timing of purchases for rural and urban consumers. This will be discussed below.

Table 9.4 Estimation of annual supernumerary income I^h - ξ^h in FCFA.

CRPA	Rural				Urban					
	I^r (a)	b^r (b)	ξ^r (c)	$I^r - \xi^r$ (a) - (c)	I^u (d)	b^u (e)	γ^u (f)	π (g)	ξ^u (h)	$I^u - \xi^u$ (d) - (h)
Centre	24500	0.14	11322	13178	262500	0.06	50	134	124001	138499
Centre Nord	25375	0.13	11794	13581	152250	0.07	60	126	69810	82440
Centre Ouest	24500	0.14	11322	13178	152250	0.07	60	125	69847	82403
Centre Sud	24500	0.14	11322	13178	152250	0.07	60	128	69753	82497
Sahel	38500	0.16	17627	20873	152250	0.07	60	139	69437	82813
Mouhoun	52500	0.09	25033	27467	152250	0.07	60	108	70335	81915
Est	24500	0.13	11388	13112	152250	0.07	60	120	69979	82271
Centre Est	24500	0.14	11322	13178	152250	0.07	60	127	69776	82474
Nord	25375	0.13	11794	13581	152250	0.07	60	119	70010	82240
Sud Ouest	52500	0.13	24382	28118	152250	0.07	60	125	69858	82392
Hauts Bassins	52500	0.09	25033	27467	262500	0.06	50	113	124518	137982
Comoe	52500	0.09	25033	27467	152250	0.07	60	129	69720	82530

Note: Income (a) and (d) in Table 9.1; share parameters (b) and (e) in Table 9.3; minimum consumption level (f) in (9.11) and (9.12); consumer cereal price (g) in Table 8.3; minimum expenditures on commodity *k*, (c) and (h), see (9.13).

Expressions (9.5) and (9.6) give the annual cereal demand functions for urban and rural consumers, respectively. To define quarterly demand functions we slightly adapt the variables and parameters introduced. Define the set of time periods (see also figure 8.1):

$T = \{1, 2, 3, 4\}$

and introduce for $t \in T$:

	y_t^h	cereal demand level by a consumer of type *h* in period *t*
	$I_t^h - \xi_t^h$	supernumerary income level of a consumer of type *h* in period *t*
(9.14)	γ_t^h	minimally required cereal purchase level of a consumer of type *h* in period *t*
	b_t^h	share of supernumerary income spent on cereals in period *t*
	π_t	cereal consumer price in period *t*

The quarterly demand function for *urban* and *rural* consumers can now be written as - see (9.5) and (9.6):

(9.15) $$y_t^h = \gamma_t^h + \frac{b_t^h}{\pi_t}\left(I_t^h - \xi_t^h - \pi_t \gamma_t^h\right)$$

in which $\gamma_t^r = 0$ for *rural* consumers. For the minimum cereal demand level γ_t^u for *urban* consumers, we suppose that they have to purchase in each period at least a quarter of the annual level γ^u. So, - see (9.11):

(9.16) For urban consumers in Ouagadougou and Bobo-Dioulasso: $\gamma_t^u = 12.5$

For urban consumers in other cities: $\gamma_t^u = 15$.

Average quarterly cereal price levels π_t, are presented in Table 8.3. No data are available on income per period, but we suppose that each consumer, urban and rural, is able to spread his income equally over the year. So, we define:

$$(9.17) \quad I_t^h - \xi_t^h = ¼(I^h - \xi^h) \; ,$$

with the value of $I^h - \xi^h$ given in Table 9.4 - see Table 9.5 and Table 9.6. Although income levels are supposed to be the same in each period, expenses on cereal purchases differ per period. Consequently, also the share of income spent on cereals, b_t^h, differs per period.

For *rural* consumers the distribution of expenses on cereal purchases over the year may be different for each CRPA. The data of Table A3.23, Table A3.24, and Table A3.25 suggest that purchase patterns differ between the South-Western and the other CRPA. They show that households in the CRPA Mouhoun, Hauts-Bassins, Comoé and Sud-Ouest purchase on average approximately 15% of their cereals in the first period from October to December, 20% in the second period from January to March, 30% in the third period from April to June, and 35% in the fourth period from July to September. The other CRPA purchase on the average approximately 17.5% of their cereals during the first period, 17.5% during the second period, 25% during the third period, and 40% during the fourth period. These approximations are used to determine values of b_t^h for rural consumers.

To estimate the level of b_t^r, we can not apply the same method as in Section 9.1.2 (see (9.10)), for we have no data on elasticities or share parameter s^h per period. We therefore use the average annual demand level of a rural consumer y^r, which can be calculated with equation (9.6) and the estimates of π and $I^r - \xi^r$ given in Table 8.3 and Table 9.4, respectively - see Table 9.5. If the average quarterly cereal purchases as a percentage of the total purchased quantity is δ_t, then the average cereal demand in period t is $y_t^r = y^r \cdot \delta_t$. Values of δ_t are given in the text above. It follows from (9.15) and (9.12) that for rural consumers:

$$(9.18) \quad b_t^r = \frac{\pi_t y_t^r}{I_t^r - \xi_t^r} = \frac{\pi_t \delta_t y^r}{\frac{1}{4}\left(I_t^r - \xi_t^r\right)}$$

Estimates of b_t^r for *rural* consumers are presented in Table 9.5.[24]

Estimating the budget share b_t^u for *urban* consumers is done in a similar way as for *rural* consumers. For urban consumers it has been supposed that they purchase their entire consumption on the market, and that their cereal consumption is the same in each period. So, they purchase in each period a quarter of the yearly consumption. For we supposed in (9.17) that income for urban consumers is constant in each quarter, the share of total budget spent on cereal purchases has to increase during the year when cereal prices increase. To estimate the budget share b_t^u, we first calculate an average annual level of cereal demand for an urban consumer, y^u. This can be calculated using equation (9.5) and the estimates of γ^u, b^u, π and $I^u - \xi^u$ given in (9.11), Table 9.3, Table 8.3 and Table 9.4, respectively - see Table 9.6. The average level of cereal demand per quarter is: $y_t^u = ¼ \cdot y^u$. It follows from (9.15) that:

$$(9.19) \quad b_t^u = \frac{\pi_t\left(y_t^u - \gamma_t^u\right)}{I_t^u - \xi_t^u - \pi_t \gamma_t^u} = \frac{\frac{1}{4}\pi_t\left(y^u - \gamma^u\right)}{\frac{1}{4}\left(I^u - \xi^u - \pi_t \gamma^u\right)}$$

Estimates of b_t^u for *urban* consumers are presented in Table 9.6.

[24] Note that to estimate b_t^r, we need estimates of the parameter b^r. b^r is used to estimate the value of ξ^h and of ξ_t^h.

Table 9.5 Estimates of budget share for cereal purchases per period, b_t^r, for *rural* consumers.

CRPA	$I_t^r - \xi_t^r$ **(a)**	y_t^r **(b)**	b_1^r **(c)**	b_2^r **(d)**	b_3^r **(e)**	b_4^r **(f)**
Centre	3294	14	0.10	0.09	0.14	0.24
Centre Nord	3395	14	0.09	0.09	0.13	0.23
Centre Ouest	3294	15	0.09	0.10	0.15	0.24
Centre Sud	3294	14	0.09	0.10	0.15	0.22
Sahel	5218	23	0.10	0.10	0.16	0.27
Mouhoun	6867	23	0.05	0.07	0.11	0.14
Est	3278	14	0.08	0.09	0.14	0.23
Centre Est	3294	15	0.09	0.10	0.15	0.24
Nord	3395	15	0.09	0.09	0.14	0.23
Sud Ouest	7030	30	0.07	0.10	0.17	0.20
Hauts Bassins	6867	22	0.05	0.07	0.11	0.13
Comoe	6867	19	0.05	0.07	0.11	0.13

Note: (a) supernumerary income per period is defined in (9.17); (b) annual cereal demand is defined using (9.6), Table 8.3, Table 9.3 and Table 9.4; (c) - (f) budget shares are defined in (9.18), quarterly cereal prices per CRPA, π_t, are given in Table 8.3.

Table 9.6 Estimates of budget share for cereal purchases per period, b_t^u, for *urban* consumers.

CRPA	$I_t^u - \xi_t^u$ (a)	y_t^u (b)	b_1^u (c)	b_2^u (d)	b_3^u (e)	b_4^u (f)
Centre	34625	108	0.06	0.06	0.06	0.06
Centre Nord	20610	100	0.06	0.06	0.07	0.07
Centre Ouest	20601	100	0.06	0.07	0.07	0.07
Centre Sud	20624	99	0.06	0.07	0.07	0.07
Sahel	20703	96	0.06	0.06	0.07	0.07
Mouhoun	20479	107	0.06	0.06	0.07	0.08
Est	20568	102	0.06	0.06	0.07	0.08
Centre Est	20619	100	0.06	0.07	0.07	0.07
Nord	20560	102	0.06	0.06	0.07	0.07
Sud Ouest	20598	101	0.06	0.06	0.07	0.07
Hauts Bassins	34496	119	0.06	0.06	0.06	0.06
Comoe	20633	99	0.06	0.06	0.07	0.07

Note: (a) supernumerary income per period is defined in (9.17); (b) annual cereal demand is defined using (9.5), Table 8.3, Table 9.3 and Table 9.4; (c) - (f) budget shares are defined in (9.19), quarterly cereal prices per CRPA, π_t, are given in Table 8.3.

Using the above discussion on cereal demand functions for rural and urban consumers, we can now estimate the regional demand functions per CRPA. Introduce a set with all the twelve CRPA, *I,* and call for $i \in I$:

y_{it}^u	the cereal demand function for period *t* of an urban consumer in CRPA *i*
y_{it}^r	the cereal demand function for period *t* of a rural consumer in CRPA *i*
y_{it}	the regional cereal demand function for CRPA *i* for period *t*

The demand functions for rural and urban consumers is given in (9.15). Redefine the parameters in (9.14) with an index *i*, to indicate the CRPA concerned: $\gamma_{it}^h, \pi_{it}, I_{it}^h - \xi_{it}^h$ and b_{it}^h. The parameter values $\gamma_{it}^h, \pi_{it}, I_{it}^h - \xi_{it}^h$ and b_{it}^h are given in (9.12), (9.16), Table 8.3, Table 9.5 and Table 9.6. Define also:

Pop_i^u	the size of the urban population in CRPA *i*
Pop_i^r	the size of the rural population in CRPA *i*

Population size for each CRPA is given in Table 8.1. Now, total cereal demand in period *t* in CRPA *i*, $i \in I$, is given by:

(9.20) $y_{it} = Pop_i^u y_{it}^u + Pop_i^r y_{it}^r$.

Note that the demand function is in fact a simple demand function of the form

(9.21) $y_{it} = \alpha_{it} + \beta_{it}/\pi_{it}$,

with

$$\alpha_{it} = Pop_i^u \gamma_{it}^u \left(1 - b_{it}^u\right) \quad \text{and}$$

$$\beta_{it} = Pop_i^u b_{it}^u \left(I_{it}^u - \xi_{it}^u\right) + Pop_i^r b_{it}^r \left(I_{it}^r - \xi_{it}^r\right)$$

see (9.20) and (9.15).

9.2 Cereal supply functions

Estimating supply functions is a more complicated task than estimating demand functions. Supply differences between households are larger, and less is known about the influence of prices on supply. What is needed for our analysis are functions which, given cereal production levels, determine the distribution of cereal sales over the periods as a function of cereal prices, taken into account on-farm consumption of self produced cereals. In Chapter 7 it was argued that cereal supply in a period t depends on the (given) stock level at the beginning of the period, on the cereal producer price in period t, p_t, and on the uncertain prices in the future periods. In Section 9.2.1 and Appendix A2.1 the approach discussed in Section 7.1 is extended. Now, producers have to supply in each period at least a certain quantity, to satisfy cash needs. In Section 9.2.2 the parameters are estimated and the form of the resulting supply functions are discussed.

9.2.1 Cereal supply model

Each producer knows after the harvest, at the beginning of period 1, the level of his cereal production and how much he can sell during that year, w_0. As in the previous section we skip the region index i from the variables and parameters. So, for the annual supply of a producer in region i we write w_0 instead of w_{i0} - see (7.2). As in Section 7.1, the producer takes a decision on his cereal sales x_t in period t, when he knows the available stock level $w_{t\text{-}1}$ remaining from the previous period, and the current price p_t. Future prices for the periods t+1, t+2,..,4 are random variables, of which the simultaneous probability distributions are assumed to be known by the producers – see (7.4). Call the random future producer prices $P_{t+1}, P_{t+2}, \ldots, P_4$ – see

(7.3) and (7.4). In each period t the producer optimizes his revenues for that period, $p_t \cdot x_t$, minus the costs made to sell x_t, called $c_t(x_t)$ – see (7.8), plus the discounted, *expected* net revenues for future periods – see (7.12) and (7.13). Define σ the discount rate, which indicates the importance the producer attaches to future revenues. If σ is low, the producer puts a low value to future revenues. If $\sigma > 1$, the producer puts a higher value on future revenues than on current revenues. The discounted, expected, future revenues can also be interpreted as the present value of future revenues. In that case $\sigma = 1/(1+r)$, with r the interest rate. The value of σ will be discussed in Section 9.2.2.

Important in this problem are the costs which have been made to sell x_t: $c_t(x_t)$. In Chapter 7 a linear cost function was adopted, see (7.8). This differs from standard producer theory, in which it is usually supposed that: $c_t'(x_t) > 0$ and $c_t''(x_t) > 0$, see Chapter 4. However, not enough evidence is available to justify a cost function for Burkinabè farmers, for which the first and second derivatives are positive. It is, however, plausible to adopt a cost function which is linear in the quantity supplied. In that case, $c_t'(x_t) > 0$ but $c_t''(x_t) = 0$. If in period t a quantity x_t is sold, then the following costs are made:

1. Costs for supplying x_t on the market, i.e. the transaction costs (transport costs, negotiaton costs), assumed to be an amount α FCFA per kg of cereals sold during period t.
2. Financial storage costs are ρ FCFA per kg per period. Physical storage losses of keeping the quantity x_t in stock until period t are supposed to be a fraction $(1-\delta)$ per period, due to insects, rats and diseases. It is recalled that the periods t = 1,2,3,4 have all the same length of three months. To sell x_t, a producer stores at the beginning of the year a quantity x_t/δ^{t-1}. Despite the fact that the stock decreases in each period due to storage losses, we suppose that the producer has to pay in all the t-1 periods in which he stores approximately $\rho x_t/\delta^{t-1}$ FCFA per kg stored. So, if a producer sells at the beginning of each period, the storage costs which have been made to sell x_t are approximately:

$$(9.22)\quad (t-1)\frac{\rho x_t}{\delta^{t-1}}$$

3. Production costs per unit of cereals produced, amount β FCFA per kg. If one kg is sold in period t, then a quantity $1/\delta^{t-1}$ must be reserved from the quantity produced – see 2) above.

The values of the parameters α, β, δ and ρ are discussed in Section 9.2.2. Write the cost function: $c_t(x_t) = c_t \cdot x_t$, with c_t the costs per kg supplied, defined as:[25]

$$(9.23)\quad c_t = \alpha + \frac{\beta + (t-1)\rho}{\delta^{t-1}}$$

The sales in each period have to satisfy sales restrictions. First, as in Section 7.1, the producer can not sell more then what remains from previous periods. In each period a fraction 1-δ of the stock is lost. So, if we define w_t as the level of the stock at the end of period t, then $w_t = (w_{t-1} - x_t)\cdot\delta$. The initial stock is w_0. It follows, that – see also (7.6):

$$(9.24)\quad \begin{array}{ll} w_1 = (w_0 - x_1)\delta & w_3 = (((w_0 - x_1)\delta - x_2)\delta - x_3)\delta \\ w_2 = ((w_0 - x_1)\delta - x_2)\delta & w_4 = ((((w_0 - x_1)\delta - x_2)\delta - x_3)\delta - x_4)\delta \end{array}$$

Secondly, different from Section 7.1, based on observed practice in Burkina Faso, it is supposed that each producer sells in each period t, at least a minimum quantity, x_t^-.

[25] Note that the cost function does not reflect the costs which have been made in period t. For example, you include the costs which have been made in the previous periods to store x_t, but you do not include the costs which have to be made in period t to store the remainder. This approach is correct, if it is assumed that sales in period t take place in the beginning of the period and approxiamelty correct if the sales take place somewhere in period t.

In principle, these minimum sales are based on urgent cash requirements. If a producer needs an amount of capital, he sells a certain part of his stock. This would mean that his minimum sales depend on prices; if the price is high, his sales will be lower, than if prices are low – see Section 8.1.3. We do not take into account this price dependence of minimum sales. It would complicate the analysis considerably, and data are lacking to justify such a detailed approach. The requirement that sales should exceed x_t^- implies:

(9.25) $x_t \geq x_t^-$

The values of x_t^- will be discussed in Section 9.2.2.

We come back now to the producer's choice of x_t. He chooses x_t in such a way that the expected net revenues will be maximal. We first deal with the last period, t = 4. In period 4 the producer knows w_3 and p_4. The producer maximizes his net revenues for that period subject to the minimum sales x_4^-, and the available stock level, w_3. This problem may be written as - see also (7.10):

(9.26) $z_4(w_3; p_4) = \underset{x_4}{Max}\left\{(p_4 - c_4)x_4 \;\middle|\; x_4^- \leq x_4 \leq w_3\right\}$

This model results in the optimal supply for a producer for period 4. In period 3, the values of w_2 and p_3 are known. The value of the price in period 4 is uncertain, i.e. a random variable P_4, of which the probability distribution is assumed to be known. Supplies in period 3 should not be less than the minimum sales x_3^-, and not exceed the available stock minus the quantity which has to be reserved for future sales $w_2 - x_4^-/\delta$. Maximization of the net revenues in period 3, plus the expected revenues for the rest of the reference year, corresponds to the maximization problem – see (3.48) in Section 3.5 and (7.12):

$$z_3(w_2;p_3) = \underset{x_3,w_3}{Max}\left\{(p_3 - c_3)x_3 + \sigma E z_4(w_3, P_4)\right.$$

(9.27)

$$\left. \middle| \; x_3^- \le x_3 \le w_2 - \frac{x_4^-}{\delta},\; w_3 = (w_2 - x_3)\delta \ge 0 \right\}$$

$Ez_4(w_3;P_4)$ refers to the expectation of z_4 with regard to the random price P_4. It will be discussed below how it can be calculated. Model (9.27) results in the optimal supply for the producer in period 3 and in the quantity which is expected to be optimal for period 4. The optimal supply in period 4, calculated by (9.26) may differ from this expected supply, if prices turn out to be different than expected. In analogy with (9.27), for the periods 1 and 2 the following maximization problems have to be solved:

$$z_2(w_1;p_2) = \underset{x_2,w_2}{Max}\left\{(p_2 - c_2)x_2 + \sigma E z_3(w_2;P_3)\right.$$

(9.28)

$$\left. \middle| \; x_2^- \le x_2 \le w_1 - \frac{x_3^-}{\delta} - \frac{x_4^-}{\delta^2},\; w_2 = (w_1 - x_2)\delta \right\}$$

$$z_1(w_0;p_1) = \underset{x_1,w_1}{Max}\left\{(p_1 - c_1)x_1 + \sigma E z_2(w_1;P_2)\right.$$

(9.29)

$$\left. \middle| \; x_1^- \le x_1 \le w_0 - \frac{x_2^-}{\delta} - \frac{x_3^-}{\delta^2} - \frac{x_4^-}{\delta^3},\; w_1 = (w_0 - x_1)\delta \right\}$$

Model (9.28) results in optimal supply levels for period 2, x_2, and in supplies for the periods, 3 and 4, which are expected to be optimal. Analogously, model (9.29) gives optimal supply levels for period 1, x_1, and supplies for the periods 2, 3 and 4, which are expected to be optimal. In models (9.27) - (9.29) $Ez_{t+1}(\cdot)$ refers to the expectation of $z_{t+1}(\cdot)$ with regard to all uncertain prices $P_{t+1},\ldots,P_4$ – see (3.49) in Section 3.5.

The models (9.27) - (9.29) are typical examples of *dynamic programming* problems. In order to estimate $Ez_t(\cdot)$, t = 4,3,2, in (9.27) - (9.29) we assume producers know the probability distributions of random prices P_t, for the periods t = 2,3,4. For the model for period t, we assume that stochastic prices for the future periods t+1,…,4, are independent of p_t. We have made this assumption to simplify the computations.[26] However, stochastic prices for the period τ, for $\tau \in \{t+2,\ldots,4\}$, are assumed to depend on the stochastic price for the period τ–1. In Section 8.1.6 and Appendix A3.6, it has been discussed that producer prices show a clear seasonal pattern every year: they are low after the harvest between October and December, then gradually increase, to reach their maximum between July and September. Define $\overline{p}_t$, the average '96-'99 producer price in quarter t, see Table 8.3. We suppose that cereal producers (and traders) expect prices to follow more or less the same pattern every year. They expect the price in quarter t to be $\overline{p}_t - \overline{p}_{t-1}$ above the price in period t-1.

Period 3:
In model (9.27) for period 3, expected prices for period 4 are independent of p_3. We assume they are equal to a parameter $\hat{p}_3$, plus the expected increase $\overline{p}_4 - \overline{p}_3$:[27]

(9.30) $$\mathrm{E}P_4 = \hat{p}_3 + \left(\overline{p}_4 - \overline{p}_3\right)$$

We suppose that the random producer price is equal to the expected producer price plus a random disturbance. Define the discrete, random disturbance in period t, Θ_t, with $\mathrm{E}\Theta_t = 0$, t = 2,3,4. Assume that the random disturbances for the periods t = 2,3,4 are mutually independent, and have a discrete empirical distribution with possible realisations θ_t^k, for k = 1,…,K, and with

[26] If the probability distribution of the stochastic prices would depend on p_t, then the equilibrium models presented in Section 7.2 would be much more complicated and very difficult to handle. Moreover, in that case, the results of the equilibrium models would change, since p_t is a variable in these models. In that case the results of the equilibrium models would not correspond to the optimal strategies of the individual agents.
[27] How the parameter $\hat{p}_3$ is estimated, is discussed in the next section.

(9.31) $\Pr\left(\Theta_t = \theta_t^k\right) = f_t^{\,k}$

with $f_t^{\,k}$ satisfying $0 \leq f_t^{\,k} \leq 1$ and $\sum_{k=1}^{K} f_t^{\,k} = 1$. We can write in the model for period 3, the stochastic price for period 4 as:

(9.32) $P_4 = \hat{p}_3 + \left(\overline{p}_4 - \overline{p}_3\right) + \Theta_4$

Write the possible price realisations in period 4, p_4^k, as:

(9.33) $p_4^k = \hat{p}_3 + \left(\overline{p}_4 - \overline{p}_3\right) + \theta_4^k$

and define – see (9.30) - (9.33):

(9.34) $\Pr\left(P_4 = p_4^k\right) = f_4^{\,k}$

Estimation of the probability distributions is discussed in Section 9.2.2. Define x_4^k as the supply in period 4, if the price in this period is p_4^k, k = 1,…,K. Model (9.27) may be written as – see (3.49):

$$z_3\left(w_2; p_3\right) = \underset{x_3, w_3}{Max}\left\{(p_3 - c_3)x_3 + \sigma\sum_{k=1}^{K} f_4^k z_4\left(w_3, p_4^k\right) \;\middle|\; x_3^- \leq x_3 \leq w_2 - \frac{x_4^-}{\delta},\ w_3 = \left(w_2 - x_3\right)\delta\right\}$$

(9.35)
$$= \underset{x_3, x_4^k}{Max}\left\{\left(p_3 - c_3\right)x_3 + \sigma\sum_{k=1}^{K} f_4^k\left(p_4^k - c_4\right)x_4^k \;\middle|\; x_3 + \frac{x_4^k}{\delta} \leq w_2,\ x_3 \geq x_3^-,\ x_4^k \geq x_4^-,\ k = 1,\ldots,K\right\}$$

Period 2:

In model (9.28) for period 2, expected prices for period 3 and 4 are independent of p_2. We assume that expected prices for period 3 are equal to a parameter $\hat{p}_2$, plus the expected increase $\overline{p}_3 - \overline{p}_2$. Furthermore, the expected price for period 4 depends on the price realised in the previous period. We define:

$$
\text{(9.36)} \quad \begin{aligned} &\mathrm{E}P_3 = \hat{p}_2 + (\overline{p}_3 - \overline{p}_2) \\ &\mathrm{E}(P_4 | P_3 = p_3) = p_3 + (\overline{p}_4 - \overline{p}_3) \end{aligned}
$$

Assume again that the random producer price is equal to the expected producer price plus a random disturbance Θ_t – see (9.31). Define for $k,l = 1,\ldots,k$:

p_3^k the possible price realisations for period 3,

p_4^{kl} the possible price realisations in period 4 if the price in period 3 is p_3^k.

They are written as:

$$
\text{(9.37)} \quad \begin{aligned} &p_3^k = \hat{p}_2 + (\overline{p}_3 - \overline{p}_2) + \theta_3^k \\ &p_4^{kl} = p_3^k + (\overline{p}_4 - \overline{p}_3) + \theta_4^l = \hat{p}_2 + (\overline{p}_4 - \overline{p}_2) + \theta_3^k + \theta_4^l \end{aligned} .
$$

We define:

$$
P_3 = \hat{p}_2 + (\overline{p}_3 - \overline{p}_2) + \Theta_3 \qquad \text{and} \qquad P_4 = P_3 + (\overline{p}_4 - \overline{p}_3) + \Theta_4
$$

and write, see also (9.37):

$$\text{(9.38)} \quad \begin{aligned} &\Pr\left(P_3 = p_3^k\right) = \Pr\left(\Theta_3 = \theta_3^k\right) = f_3^k \\ &\Pr\left(P_4 = p_4^{kl} \,\middle|\, P_3 = p_3^k\right) = \Pr\left(\Theta_4 = \theta_4^l \,\middle|\, \Theta_3 = \theta_3^k\right) = \Pr\left(\Theta_4 = \theta_4^l\right) = f_4^l \end{aligned}$$

We can rewrite the model for the period $t = 2$ – see (9.28). Define x_3^k and x_4^{kl} as the supply in the periods 3 and 4, if prices in these periods are p_3^k and p_4^{kl}, respectively, for $k,l = 1,\ldots,K$ – see (3.53).

$$\text{(9.39)} \quad \begin{aligned} z_2(w_1; p_2) = &\underset{x_2, w_2}{Max}\left\{(p_2 - c_2)x_2 + \sigma\sum_{k=1}^{K} f_3^k z_3(w_2; p_3^k) \right. \\ &\left. \middle|\; x_2^- \le x_2 \le w_1 - \frac{x_3^-}{\delta} - \frac{x_4^-}{\delta^2},\; w_2 = (w_1 - x_2)\delta \right\} \\ = &\underset{x_2, x_3^k, x_4^{kl}}{Max}\left\{(p_2 - c_2)x_2 + \sigma\sum_{k=1}^{K} f_3^k\left((p_3^k - c_3)x_3^k + \sigma\sum_{l=1}^{K} f_4^l (p_4^{kl} - c_4)x_4^{kl}\right) \right. \\ &\left. \middle|\; x_2 + \frac{x_3^k}{\delta} + \frac{x_4^{kl}}{\delta^2} \le w_1,\, x_2 \ge x_2^-,\; x_3^k \ge x_3^-,\; x_4^{kl} \ge x_4^-, k,l = 1,\ldots,K \right\} \end{aligned}$$

Period 1:

Similarly for model (9.29) for period 1, we define expected future prices as:

$$\text{(9.40)} \quad \begin{aligned} &\mathrm{E}P_2 = \hat{p}_1 + (\bar{p}_2 - \bar{p}_1) \\ &\mathrm{E}(P_3 | P_2 = p_2) = p_2 + (\bar{p}_3 - \bar{p}_2) \\ &\mathrm{E}(P_4 | P_3 = p_3) = p_3 + (\bar{p}_4 - \bar{p}_3) \end{aligned}$$

Assume again that the random producer price is equal to the expected producer price plus a random disturbance Θ_t – see (9.31). Define for *k,l,m* = 1,…,*k:*

p_2^k the possible price realisations for period 2

p_3^{kl} the possible price realisation in period 3 if the price in period 2 is p_2^k

p_4^{klm} the possible price realisation in period 4 if the price in period 2 is p_2^k and the price in period 3 is p_3^{kl}

Write them as:

$$
\begin{aligned}
& p_2^k = \hat{p}_1 + (\overline{p}_2 - \overline{p}_1) + \theta_2^k \\
(9.41) \quad & p_3^{kl} = p_2^k + (\overline{p}_3 - \overline{p}_2) + \theta_3^l = \hat{p}_1 + (\overline{p}_3 - \overline{p}_1) + \theta_2^k + \theta_3^l . \\
& p_4^{klm} = p_3^{kl} + (\overline{p}_4 - \overline{p}_3) + \theta_4^m = \hat{p}_1 + (\overline{p}_4 - \overline{p}_1) + \theta_2^k + \theta_3^l + \theta_4^m .
\end{aligned}
$$

We define:

$$
\begin{aligned}
P_2 &= \hat{p}_1 + (\overline{p}_2 - \overline{p}_1) + \Theta_2 \\
P_3 &= P_2 + (\overline{p}_3 - \overline{p}_2) + \Theta_3 \\
P_4 &= P_3 + (\overline{p}_4 - \overline{p}_3) + \Theta_4
\end{aligned}
$$

and write:

$$
\begin{aligned}
& \Pr\left(P_2 = p_2^k\right) = \Pr\left(\Theta_2 = \theta_2^k\right) = f_2^k \\
& \Pr\left(P_3 = p_3^{kl} \mid P_2 = p_2^k\right) = \Pr\left(\Theta_3 = \theta_3^l \mid \Theta_2 = \theta_2^k\right) = \Pr\left(\Theta_3 = \theta_3^l\right) = f_3^l \\
(9.42) \quad & \Pr\left(P_4 = p_4^{klm} \mid P_2 = p_2^k, P_3 = p_3^{kl}\right) = \Pr\left(\Theta_4 = \theta_4^m \mid \Theta_2 = \theta_2^k, \Theta_3 = \theta_3^{kl}\right) = \\
& \qquad = \Pr\left(\Theta_4 = \theta_4^m\right) = f_4^m
\end{aligned}
$$

Define x_2^k, x_3^{kl}, and x_4^{klm} as the supply in the periods 2, 3 and 4, if prices in these periods are p_2^k, p_3^{kl}, and p_4^{klm}, respectively, for $k,l,m = 1,\ldots,K$. Model (9.29) can be written as:

$$z_1(w_0;p_1) = \underset{x_1,x_2^k,x_3^{kl},x_4^{klm}}{Max} \Big\{ (p_1 - c_1)x_1 + \sigma \sum_{k=1}^{K} f_2^k \Big[(p_2^k - c_2)x_2^k +$$

(9.43)
$$+\sigma \sum_{l=1}^{K} f_3^l \Big[(p_3^{kl} - c_3)x_3^{kl} + \sigma \sum_{m=1}^{K} f_4^m (p_4^{klm} - c_4)x_4^{klm} \Big]\Big]$$

$$\Big| \; x_1 + \frac{x_2^k}{\delta} + \frac{x_3^{kl}}{\delta^2} + \frac{x_4^{klm}}{\delta^3} \le w_0, \;\; x_1 \ge x_1^-,$$

$$x_2^k \ge x_2^-, \;\; x_3^{kl} \ge x_3^-, \; x_4^{klm} \ge x_4^-, \; k,l,m = 1,\ldots,K \;\; \Big\}$$

What can we learn from the above models:

- Solving model (9.43) results in optimal supply $x_1(w_0, \; p_1)$ for period 1 as a function of the producer price in period 1, p_1.
- Solving model (9.39) gives the optimal supply $x_2(w_1,p_2)$ in period 2, as a function of the available stock w_1 and the producer price for period 2, p_2.
- Solving model (9.35) gives the optimal supply $x_3(w_2,p_3)$ for period 3, as a function of the available stock w_2 and the producer price in the period 3, p_3.
- Solving model (9.26) gives the optimal supply $x_4(w_3,p_4)$ for period 4, as a function of the available stock w_3 and the producer price in the period 4, p_4.

In Appendix A2, the supply functions resulting from these models are derived. The supply functions are as follows:

Optimal supply in period 4:

(9.44)
$$\begin{cases} x_4(w_3;p_4) = x_4^- & \text{if } p_4 < c_4 \\ x_4^- \le x_4(w_3;p_4) \le x_4^+ & \text{if } p_4 = c_4 \\ x_4(w_3;p_4) = x_4^+ = (w_0 - x_1)\delta^3 - x_2\delta^2 - x_3\delta & \text{if } p_4 > c_4 \end{cases}$$

Optimal supply in period 3:

$$
(9.45)\quad \begin{cases} x_3(w_2;p_3)=x_3^- & \text{if } p_3<c_3+\Psi_4 \\ x_3^- \le x_3(w_2;p_3)\le x_3^+ & \text{if } p_3=c_3+\Psi_4 \\ x_3(w_2;p_3)=x_3^+ = w_2-\dfrac{x_4^-}{\delta}=(w_0-x_1)\delta^2-x_2\delta-\dfrac{x_4^-}{\delta} & \text{if } p_3>c_3+\Psi_4 \end{cases}
$$

with $\Psi_4=\sigma\delta\sum_{k=1}^{K} f_4^k\left(p_4^k-c_4\right)^+$ – see (A2.8) and (9.33), with $a^+=\max(a;0)$.

Optimal supply in period 2:

$$
(9.46)\quad \begin{cases} x_2(w_1;p_2)=x_2^- & \text{if } p_2<c_2+\Psi_3 \\ x_2^- \le x_2(w_1;p_2)\le x_2^+ & \text{if } p_2=c_2+\Psi_3 \\ x_2(w_1;p_2)=x_2^+ =(w_0-x_1)\delta-\dfrac{x_3^-}{\delta}-\dfrac{x_4^-}{\delta^2} & \text{if } p_2>c_2+\Psi_3 \end{cases}
$$

with $\quad\Psi_3=\sigma\delta\left[\sum_{k=1}^{K} f_3^k\Psi_4\left(p_3^k\right)+\sum_{k=1}^{K} f_3^k\left(p_3^k-c_3-\Psi_4\left(p_3^k\right)\right)^+\right]$ and

$\Psi_4\left(p_3^k\right)=\sigma\delta\sum_{l=1}^{K} f_4^l\left(p_4^{kl}-c_4\right)^+$ – see (A2.13) and (9.37).

Optimal supply in period 1:

$$
(9.47)\quad \begin{cases} x_1(w_0,p_1)=x_1^- & \text{if } p_1<c_1+\Psi_2 \\ x_1^- \le x_1(w_0,p_1)\le x_1^+ & \text{if } p_1=c_1+\Psi_2 \\ x_1(w_0,p_1)=x_1^+ = w_0-\dfrac{x_2^-}{\delta}-\dfrac{x_3^-}{\delta^2}-\dfrac{x_4^-}{\delta^3} & \text{if } p_1>c_1+\Psi_2 \end{cases}
$$

with: $$\Psi_2 = \sigma\delta\left[\sum_{k=1}^{K} f_2^{\,k}\Psi_3\left(p_2^k\right)+\sum_{k=1}^{K} f_2^{\,k}\left(p_2^k - c_2 - \Psi_3\left(p_2^k\right)\right)^+\right],$$

$$\Psi_3\left(p_2^k\right)=\sigma\delta\left[\sum_{l=1}^{K} f_3^{\,l}\Psi_4\left(p_3^{kl}\right)+\sum_{l=1}^{K} f_3^{\,l}\left(p_3^{kl} - c_3 - \Psi_4\left(p_3^{kl}\right)\right)^+\right] \text{ and}$$

$$\Psi_4\left(p_3^{kl}\right) = \sigma\delta\sum_{m=1}^{K} f_4^{\,m}\left(p_4^{klm} - c_4\right)^+ \text{ – see (A2.18) and (9.41).}$$

These functions show that optimal supply in each period has the following form:

- Supply in period t is the minimum quantity x_t^-, if the price is below the border price, $p_t < c_t + \Psi_{t+1}$.
- Supply in period t is the maximum possible quantity, taken into account minimum sales in the other periods, if the price is above the border price, $p_t > c_t + \Psi_{t+1}$.
- Supply in period t may take any value between the minimum and maximum supply levels, if the price is exactly equal to the border price, $p_t = c_t + \Psi_{t+1}$.

9.2.2 Estimating cereal supply functions for Burkina Faso

Before the supply functions (9.44) - (9.47) can be determined, first the values of the parameters w_0, x_t^-, α, δ, ρ and σ have to be estimated. The estimates will be discussed below one by one.

a) Annual supply, w_0

For the determination of the level of annual cereal supply w_0, use is made of cereal production levels. Cereal production per producer is supposed to be the forecasted mean production level for the year 2000 as presented in column (h) in Table 8.2. It is recalled that on the basis of evidence discussed in section 8.1.3 we assumed that annual supply w_0 does not depend on prices. Therefore, w_0 appears as a parameter in the models (9.43), (9.39), (9.35), and (9.26). Given production, it can be explored on the basis of the sales data presented in Appendix A3.3 which part of production can reasonably be sold (which means that not too much is sold that hardly anything remains to feed the own family, or that too little is sold so that stores are still full at the end of the year). The observations on sales and sales as a percentage of production give too little evidence to estimate per CRPA annual sales, w_0. Therefore, three different groups of CRPA are distinguished, with more or less the same sales and production characteristics. For the first group, the CRPA Sahel, Nord and Centre Nord, it is supposed that households sell on average 10% of their annual production. Households in the second group, the CRPA Centre, Centre Ouest, Centre Sud, Est, Centre Est, and Sud Ouest, sell on average 20%. Households in the third group, in the CRPA Mouhoun, Hauts Bassins, and Comoé, sell on average 35%. Using these estimates and the forecasted cereal production for the planning period 2000-2001, *annual* sales per person, w_0, can be calculated (see Table 9.8).

b) Minimally required supplies, x_t^-.

Minimally required supplies in each period are also estimated for the three different groups of CRPA. To estimate minimally required supplies, first, estimates are made of the average percentage of production sold in each quarter. Comparing the different surveys evaluated in Appendix A3.3 shows that the first group of CRPA sell on average approximately 26% of their cereals in period 1 from October to December, 30% in period 2 from January to March, 25% in period 3 from April to June, and 19% in period 4 from July to September – see Table 9.7. Producers from the second group of CRPA sell on average approximately 27%, 39%, 12% and 23% in period 1, 2, 3 and 4, respectively. Finally, producers from the third group of CRPA sell on average approximately 19%, 25%, 30% and 27% of their annual sales in period 1, 2, 3 and 4, respectively.

Table 9.7 Evaluation of sales per season as % of annual sales for some different studies.

	Author	Province	Year	Oct-Dec	Jan-Mar	Apr-June	July-Sept
CRPA group 1	Yonli	sanmatenga		38%	48%	11%	4%
	Reardon	Soum	81-82	35%	29%	18%	18%
			83-84	10%	45%	35%	10%
		Passoré	81-82	25%	25%	25%	25%
			82-83	21%	13%	46%	21%
			83-84	24%	24%	12%	40%
			Average	26%	30%	25%	19%
CRPA group 2	Pardy	Oubritenga	83-84	22%	40%	18%	21%
		Zoundweogo	83-84	48%	24%	10%	19%
		Gourma	83-84	11%	53%	9%	28%
			Average	27%	39%	12%	23%
CRPA group 3	Reardon	Mouhoun	81-82	19%	19%	44%	19%
			82-83	21%	21%	29%	29%
	Pardy	Kossi		16%	34%	17%	33%
			Average	19%	25%	30%	27%

Notes: See Appendix A3.3 for the details of the studies of Yonli (1997), Pardy (1987), and Reardon et al. (1987). Of these studies the years and provinces have been included of which most data were available.

Multiplying the average sales percentages given in Table 9.7 with the annual sales in Table 9.8 gives estimates of average supplies from cereal sales per period. Minimally required supplies from cereal sales to satisfy cash needs are now estimated as 60% of these average quarterly supplies for the first period and second period, and 40% for the third and fourth period – see Table 9.8. Minimally required supplies for the third and fourth period are assumed to be lower than for the other two periods, for it is supposed that non-cropping income is higher during these periods.

c) Discount rate: σ; Transaction costs: α; Production costs: β, Storage costs: ρ; Storage losses, δ.

Reliable estimates of α, β, δ, ρ and σ are difficult to obtain, due to lack of information on storage costs and transaction costs for the producers. The following values have been adopted for all periods and all CRPA. The storage costs, which include construction costs and time to build the stores, are ρ = 2.5 FCFA per kg per period. Transaction costs, which include transport towards the market, costs to obtain information on traders' demand and time invested in the sales activities, are estimated at α = 1 FCFA per kg per period. No data are available on production costs. We take a value equal to 75% of the average '96-'99 producer price, presented in Table 8.3. Storage losses are estimated to be 10% per year, so δ = 2.5% per period (see Section 8.4). For the discount rate, we suppose that it is equal to $\sigma = 1/(1+r)$, with r the interest rate. For r often the interest rate for lending money is chosen. These interest rates may differ a lot. Private money lenders charge much higher interest rates than official banks. Here an interest rate of 16% per year, so 4% per period is chosen (see Déjou (1987) Bassolet 2000), see also Appendix A4.2). Therefore: σ = 0.96 per period. The influence of these parameter values on the solution has to be analysed using a sensitivity analysis.

Table 9.8 Sales per person, minimum supplies per quarter and maximum sales in period 4.

	Annual sales[1] (kg/person)	**Minimally required supplies (as % of annual sales)[2]**			
	w_0	**Period 1 Oct-Dec** x_1^-	**Period 2 Jan-Mar** x_2^-	**Period 3 Apr-June** x_3^-	**Period 4 July-Sept** x_4^-
Centre	26	16%	23%	5%	9%
Centre Nord	16	15%	18%	10%	8%
Centre Ouest	41	16%	23%	5%	9%
Centre Sud	51	16%	23%	5%	9%
Sahel	15	15%	18%	10%	8%
Mouhoun	99	11%	15%	12%	11%
Est	43	16%	23%	5%	9%
Centre Est	36	16%	23%	5%	9%
Nord	14	15%	18%	10%	8%
Sud Ouest	55	16%	23%	5%	9%
Hauts Bassins	100	11%	15%	12%	11%
Comoe	92	11%	15%	12%	11%

Notes: 1) Annual sales are the average production per rural inhabitant (column (h) in Table 8.2) multiplied with the estimates of sales as percentage of production (see *a)* above); 2) Minimally required revenues per period are calculated as the annual sales multiplied with the estimated sales per period as %

of annual sales (see Table 9.7), and the minimum requirements per period (60% of average revenues in the first period and second period and 40% in the third and fourth period).

d) Random prices and probability distribution functions

In Chapter 7, it is argued that expected future revenues in the stochastic, multi-period equilibrium model can be estimated using the probability distribution of future producer prices. The discussion on cereal prices in Section 8.1.6 and Appendix A3.6, however, shows that estimating the probability that the price in one of the quarters reaches a certain level on the basis of the '92 – '99 price data, is not very accurate. Before the devaluation a cereal consumer price of 50 FCFA per kg was not very uncommon. After the devaluation in 1994 it is very rare. For that reason we do not estimate the probability distribution of prices directly, but via a detour.

Define again the time periods $t = 1,\ldots,4$, with $t = 1$ the post-harvest period from October to December, $t = 2$ the period from January to March, $t = 3$ the period from April to June, and $t = 4$ the hunger period from July to September, and define the years $Y = 1992,\ldots,1999$. Between 1992 and 1999 the average cereal prices per quarter changed a lot, see Table A3.36, but the distribution of the differences between observed and average prices remained more or less the same.[28] It is possible to estimate a discrete probability distribution function which gives the probability that the observed price deviates from the average price with a certain value. For that reason, we determine for each CRPA, each quarter and each year the average prices, and the deviations of the price observations from the averages.[29] Using this, we estimate the probability that the deviation from the average is within a certain interval. Define p_{it}^{Y}, the observed price in quarter $t = 1,2,3,4$, in CRPA $i \in I$, in year

[28] To analyse the dispersion of prices, for each year the standard deviation has been calculated of the relative difference between observed prices and the average price, $\left(P_{it}^{Y} - \overline{p}_{it}^{Y}\right)/\overline{p}_{it}^{Y}$, with P_{it}^{Y} a price observation and $\overline{p}_{it}^{Y}$ the average price for quarter t in CRPA i in year Y, $t = 1,\ldots,4$, Y = '92-'99. These standard deviations did not change significantly between 1992 and 1999. Furthermore, an analysis of the first, second, and third quartile distances and of the minimum and maximum relative difference between prices and average prices did not clearly show changes in time.

[29] Define the set of CRPA I = {Centre, Centre Nord, Centre Ouest, Centre Sud, Sahel, Mouhoun, Est, Centre Est, Nord, Sud Ouest, Hauts Bassins, Comoe}.

$Y \in$ {'92,…,'99}, $\overline{p}_{it}^{Y}$ the average price in CRPA i in quarter t in year Y, and introduce the nine intervals:

(9.48) $\Delta^1 = (-\infty;-21]$ $\Delta^2 = (-21;-15]$ $\Delta^3 = (-15;-9]$ $\Delta^4 = (-9;-3]$ $\Delta^5 = (-3;3]$
$\Delta^6 = (3;9]$ $\Delta^7 = (9;15]$ $\Delta^8 = (15;21]$ $\Delta^9 = (21;\infty]$.

For each of the quarters t = 2,3,4, we count the number of observations for which the difference between the observed and the averge price P_{it}^{Y} - $\overline{p}_{it}^{Y}$, was within the interval Δ^k, for k = 1,...,9. [30] The probability that the deviation from the average price in a certain quarter is within the interval Δ^k, is equal to the number of observations in the interval divided by the total number of observations in the quarter. For some of the CRPA, the number of observations was very small. For example, for the CRPA Nord, only 2 observations were available for the period July – September. Furthermore, the distributions did not differ a lot between the CRPA. For that reason we estimate for each quarter one probability distribution on the basis of price data for all CRPA, and suppose that it is the same for all regions. Define:

$$(9.49) \quad \Pr\left(P_{it}^{Y} - \overline{p}_{it}^{Y} \in \Delta^k\right) = f_t^{k\Delta}$$

for t = 2,3,4, k = 1,…,9, $i \in I$, $Y \in$ {'92,…,'99}, with $0 \leq f_t^{k\Delta} \leq 1$ and $\sum_k f_t^{k\Delta} = 1$. The probability distributions are given in Table 9.9.

In Section 9.2.1, random producer prices P_{it} have been introduced for each CRPA for the quarters t = 2, 3 and 4, to determine optimal revenues.[31] Expectations for future periods, change in each period. In period 1, a different price is expected for period 4, than in period 2. In Section 9.2.1, we assumed that in the decision model for period t, random prices for period t+1 do not depend on p_t, whereas random prices for the

[30] Note that we only need the probability distribution for the periods t = 2,3,4, see Section 9.2.1.
[31] In Section 9.2.1 the index i was skipped from the definition of the random price in quarter t.

period $\tau \in \{t+2,...,4\}$, depend on the price in the previous period, $P_{\tau-1}$. Possible realisations are supposed to depend on the expected price change between period t and t-1, $\overline{p}_{it} - \overline{p}_{i,t-1}$, and a discrete random disturbance, Θ_t, with $\mathrm{E}\Theta_t = 0$. Random disturbances Θ_t, whose probability distributions are defined in (9.31), are mutually independent. Random prices and probability distributions are defined for k,l,m = 1,...,9, as follows – see (9.32), (9.34), (9.36), (9.38), (9.40), and (9.42):

For the model for period 1, (9.29):

$$\begin{aligned}
&P_{i2} = \hat{p}_{i1} + (\overline{p}_{i2} - \overline{p}_{i1}) + \Theta_2 && \Pr\left(P_{i2} = p_{i2}^{k}\right) = f_2^k \\
(9.50)\quad &P_{i3} = P_{i2} + (\overline{p}_{i3} - \overline{p}_{i2}) + \Theta_3 && \Pr\left(P_{i3} = p_{i3}^{kl} \,\middle|\, P_{i2} = p_{i2}^{k}\right) = f_3^l \\
&P_{i4} = P_{i3} + (\overline{p}_{i4} - \overline{p}_{i3}) + \Theta_4 && \Pr\left(P_{i4} = p_{i4}^{klm} \,\middle|\, P_{i2} = p_{i2}^{k}, P_{i3} = p_{i3}^{kl}\right) = f_4^m
\end{aligned}$$

For the model for period 2, (9.28):

$$\begin{aligned}
(9.51)\quad &P_{i3} = \hat{p}_{i2} + (\overline{p}_{i3} - \overline{p}_{i2}) + \Theta_3 && \Pr\left(P_{i3} = p_{i3}^{k}\right) = f_3^k \\
&P_{i4} = P_{i3} + (\overline{p}_{i4} - \overline{p}_{i3}) + \Theta_4 && \Pr\left(P_{i4} = p_{i4}^{kl} \,\middle|\, P_{i3} = p_{i3}^{k}\right) = f_4^l
\end{aligned}$$

For the model for period 3, (9.27):

$$(9.52)\quad P_{i4} = \hat{p}_{i3} + (\overline{p}_{i4} - \overline{p}_{i3}) + \Theta_4 \qquad \Pr\left(P_{i4} = p_{i4}^{k}\right) = f_4^k$$

For $\hat{p}_{it}$ we made the following assumptions:

$$\begin{aligned}
&\hat{p}_{i1} = \overline{p}_{i1} \\
(9.53)\quad &\hat{p}_{i2} = p_{i1} + (\overline{p}_{i2} - \overline{p}_{i1}) \\
&\hat{p}_{i3} = p_{i2} + (\overline{p}_{i3} - \overline{p}_{i2})
\end{aligned}$$

with p_{i1} and p_{i2} the optimal equilibrium prices for the periods 1 and 2 – see Section 7.2. Assume that the random disturbance has for each period nine possible realisations, θ_t^k, for which we adopt the following values corresponding to the nine intervals introduced in (9.48):

(9.54) $\theta^1 = -24,\ \theta^2 = -18,\ \theta^3 = -12,\ \theta^4 = -6,\ \theta^5 = 0,\ \theta^6 = 6,\ \theta^7 = 12,\ \theta^8 = 18,\ \theta^9 = 24$

Suppose that the probability distribution of Θ_t is the same as the probability distribution of Δ_t, $f_t^{k\Delta}$, in (9.49). Define for $k = 1,\ldots,9$, $i \in I$, $t = 2,3,4$, see Table 9.9:

$$(9.55) \quad \Pr\left(\Theta_t = \theta^k\right) = f_t^k = f_t^{k\Delta}$$

with $0 \leq f_t^k \leq 1$ and $\sum_k f_t^k = 1$.

Table 9.9 Probability distribution functions of deviations from average producer prices – see (9.49) and of the disturbances – see (9.55), for the quarters t = 2, 3 and 4.[1]

$f_t^{k\Delta}$	f_t^k	**t = 2 Jan – Mar**	**t = 3 Apr – Jun**	**t = 4 Jul – Sept**
Δ^1 = (-∞ ; -21]	**θ^1 = -24**	0.018	0.027	0.026
Δ^2 = (-21 ; -15]	**θ^2 = -18**	0.028	0.034	0.047
Δ^3 = (-15 ; -9]	**θ^3 = -12**	0.079	0.071	0.096
Δ^4 = (-9 ; -3]	**θ^4 = -6**	0.228	0.230	0.192
Δ^5 = (-3 ; 3]	**θ^5 = 0**	0.300	0.307	0.259
Δ^6 = (3 ; 9]	**θ^6 = 6**	0.205	0.182	0.207
Δ^7 = (9 ; 15]	**θ^7 = 12**	0.099	0.086	0.113
Δ^8 = (15 ; 21]	**θ^8 = 18**	0.026	0.040	0.032
Δ^9 = (21 ; ∞)	**θ^9 = 24**	0.017	0.023	0.028

Note: 1 The numbers denote the probability the deviation from the average price is within one of the intervals Δ_k: $\Pr(p_{it} - \overline{p}_{it} \in \Delta^k)$ or the probability that the price in period t has the value $p_{i,t-1} + \left(\overline{p}_{it} - \overline{p}_{i,t-1}\right) + \theta^k$. The probability density function for quarter t = 1 is not used in the analysis, and therefore not shown in the table. Source: Price data from SIM/SONAGESS.

9.3 Review of the stochastic, multi-period, spatial equilibrium model

In Section 7.2 we argued, that we can analyse market functioning and market price formation by solving the stochastic, multi-period, spatial equilibrium model (7.46). In the previous sections we estimated consumer demand functions, discussed producer supply behaviour, and estimated the other parameters necessary to solve model (7.46). Before discussing the results of this model in the next chapter, in this section we shortly reconsider this model.

Some extra parameters for the trader's strategies

To simplify notation, in model (7.46) we only considered transport and storage costs. However, in Section 8.2.2 and 9.2 it has been argued that also storage losses and trading costs have to be taken into account, and that expected future profits have to be discounted to their present value. Trading costs are the costs a trader has to make for each unit of cereals transacted. They include costs for bags, personnel and taxes. We calculate total trading costs as the unit costs multiplied with the quantity sold to the consumers. In Section 9.2 storage losses and discount rates were already introduced for the producers. For traders we introduce the following parameters:

	$1-\hat{\delta}$	fraction of stock lost by the trader in each period due to insects, rats, diseases, etc, see Table 8.5 - 2.
(9.56)	$\hat{\sigma}$	discount factor for the traders, to calculate the present value of future profits, see Table 8.5 - 3.
	$\hat{\alpha}$	trading costs per unit of cereals transacted, see Table 8.5 - 4, 5, and 6.

These parameters may have different values than storage losses, δ, and discount rate, σ, for the producers, introduced in Section 9.2.2.

Depending on the number of periods, regions, and possible price realisations taken into account, the equilibrium model (7.64) can become very large. For example if 4 periods, 12 regions, and 9 possible combinations of price realisations in each period (i.e. K = 9) are considered, the number of variables in the equilibrium model for period 1 is almost 150,000, while we are only interested in the optimal values of 204 of them (the variables for the period 1). For that reason we are using a simplification of model (7.64) to analyse cereal price formation in Burkina Faso. In the model for period t, we assume the number of possible price realisations for period t+1 to be equal to K = 9. For the other periods t+2,…,T we assume that producers and traders consider only one possible price realisation, which is the expected price for that period, given the price realisation in the previous period. So, for the possible producer and consumer price realisations, taken into account by the traders and producers, we assume, like we did in Section 9.2, that traders and producers expect prices in each period t+1 to T, to increase with a fixed amount, and that the price in period t+1 may also increase with a random disturbance. We assume that traders expect the prices in the different regions to be linked, i.e. the disturbance on the expected price is the same for each region. This is a rough simplification of the model, but is necessary to avoid an unmanageable model. The possible prices taken into account by the traders and producers are – see (9.50) - (9.52):

For the model for period 1:

$$p_{i2}^k = \overline{p}_{i2} + \theta_2^k \qquad \pi_{i2}^k = \overline{\pi}_{i2} + \theta_2^k$$

$$(9.57) \quad p_{i3}^k = p_{i2}^k + \left(\overline{p}_{i3} - \overline{p}_{i2}\right) \qquad \pi_{i3}^k = \pi_{i2}^k + \left(\overline{\pi}_{i3} - \overline{\pi}_{i2}\right)$$

$$p_{i4}^k = p_{i3}^k + \left(\overline{p}_{i4} - \overline{p}_{i3}\right) \qquad \pi_{i4}^k = \pi_{i3}^k + \left(\overline{\pi}_{i4} - \overline{\pi}_{i3}\right)$$

For the model for period 2:

$$(9.58) \quad \begin{aligned} p_{i3}^k &= p_{i1} + \left(\overline{p}_{i3} - \overline{p}_{i1}\right) + \theta_3^k \qquad & \pi_{i3}^k &= \pi_{i1} + \left(\overline{\pi}_{i3} - \overline{\pi}_{i1}\right) + \theta_3^k \\ p_{i4}^k &= p_{i3}^k + \left(\overline{p}_{i4} - \overline{p}_{i3}\right) \qquad & \pi_{i4}^k &= \pi_{i3}^k + \left(\overline{\pi}_{i4} - \overline{\pi}_{i3}\right) \end{aligned}$$

For the model for period 3:

$$(9.59) \quad p_{i4}^k = p_{i2} + \left(\overline{p}_{i4} - \overline{p}_{i2}\right) + \theta_4^k \qquad \pi_{i4}^k = \pi_{i2} + \left(\overline{\pi}_{i4} - \overline{\pi}_{i2}\right) + \theta_4^k$$

The average producer and consumer prices $\overline{p}_{it}$ and $\overline{\pi}_{it}$ are given in Table 8.3. For the probability distributions of the random disturbances, we suppose that producers and traders have the same probability distribution – see (9.55):

$$\text{(9.60)}\quad \Pr\left(\Theta_t = \theta^k\right) = f_{it}^{\,k} = g_t^k$$

Consequently, in the model for period t = 1,2,3, for k = 1,…,9 – see (9.51) - (9.53):

$$\text{(9.61)}\quad \begin{array}{l} \Pr\left(P_{i,t+1} = p_{i,t+1}^k\right) = f_{i,t+1}^{\,k} \\ \Pr\left(P_{1,t+1} = p_{1,t+1}^k \wedge \Pi_{1,t+1} = \pi_{1,t+1}^k \wedge \ldots \wedge P_{n,t+1} = p_{n,t+1}^k \wedge \Pi_{n,t+1} = \pi_{n,t+1}^k\right) = g_{t+1}^k \end{array}$$

Parameters and variables:

Besides the parameters introduced in (9.56), the parameters used in the equilibrium models are, for regions i = 1,…,12, periods t = 1,…,4, household types h = *rural*, *urban*, price realisations k = 1,…,9:

Pop_i^h	size of the population of type h in region i – see Table 8.1
$I_{it}^h - \xi_{it}^h$	supernumerary income level of a consumer of type h, in region i, in period t, in FCFA – see (9.14), Table 9.5, and Table 9.6.
b_{it}^h	share of supernumerary income spent on cereals, for a consumer of type h, in region i, in period t – see (9.14), Table 9.5, and Table 9.6.
γ_{it}^h	minimally required cereal purchase level for a consumer of type h, in region i, in period t, in kg – see (9.12), (9.14), and (9.16).
δ	1 – storage losses for a producer – see page 187.
σ	discount factor for a producer – see page 187.
c_{it}	producer costs of cereals supplied, in FCFA/kg – see (9.23) and page 186.
w_{i0}	annual cereal supply for a producer in region i, in kg – see Table 9.8.

x_{it}^{-} minimally required cereal supplies in period t for a producer in region i, in kg – see Table 9.8.

τ_{ijt} costs to transport one kg in period t from region i to region j, in FCFA/kg – see (6.7) and Table 8.4.

k_{it} storage costs in region i in period t, in FCFA/kg – see (6.7) and Table 8.5–1,3.

s_{i0} initial trader stock; $s_{i0} = 0$ – see (6.7).

$\bar{y}_{it}^{k}, \bar{x}_{it}^{k}$ upper bounds on trader sales and purchases, in kg – see (9.62), (9.63) below.

f_{it}^{k} probability distribution of producer prices in region i in period t – see (9.61) and Table 9.9.

g_{t}^{k} trader probability distribution of producer and consumer prices in period t – see (9.61) and Table 9.9.

p_{it}^{k}, π_{it}^{k} possible producer and consumer price realisations in region i in period t, in FCFA/kg – see (9.57) - (9.59), (9.53), (9.54), and Table 8.3.

The variables in the equilibrium models are, for $i = 1,\ldots,12$, $t = 1,\ldots,4$, $\tau = t+1,\ldots,4$, $k = 1,\ldots,9$:

x_{it} Producer supply in region i in period t, in kg.

y_{it} Consumer demand in region i in period t, in kg.

x_{ijt} Trader's transported quantity from region i to j in period t, in kg.

s_{it} Trader's stored quantity in region i in period t, for $t = 1,2,3$, in kg.

p_{it} Producer price in region i in period t, in FCFA/kg.

π_{it} Consumer price in region i in period t, in FCFA/kg.

$x_{i\tau}^{k}$ Producer supply in region i in period τ, if future prices are $p_{i\tau}^{k}$, in kg.

$q_{i\tau}^{k}$ Trader purchases in region i in period τ, if future prices are $p_{i\tau}^{k}$ and π_{it}^{k}, in kg.

$r_{i\tau}^{k}$ Trader sales in region i in period τ, if the future prices are $p_{i\tau}^{k}$ and π_{it}^{k}, in kg.

$q_{ij\tau}^{k}$ Trader's transported quantity from region i to region j in period τ, if the future prices are $p_{i\tau}^{k}$ and π_{it}^{k}, in kg.

$v_{i\tau}^{k}$ Trader's stored quantity in region i in period τ if future prices are $p_{i\tau}^{k}$ and π_{it}^{k}, in kg.

In the equilibrium models, the inverse demand function can be written as – see (9.20):

$$\pi_{it}\left(y_{it}\right) = \frac{Pop_i^u b_{it}^u\left(I_{it}^u - \xi_{it}^u\right) + Pop_i^r b_{it}^r\left(I_{it}^r - \xi_{it}^r\right)}{y_{it} - Pop_i^u \gamma_{it}^u\left(1 - b_{it}^u\right)}$$

Note that future consumer strategies can be skipped from the model presentation, because it are constants.

Finally, we have to make estimates of the upperbounds on traders' sales and purchases, $\bar{y}_{it}^{k}, \bar{x}_{it}^{k}$. In the equilibrium model for period t, possible future trader sales and purchases, $r_{i\tau}^{k}$ and $q_{i\tau}^{k}$, are bounded from above by the consumer demand and producer supply. We can estimate the upperbound on traders' sales as follows, for τ = t+1,…,4 – see (9.20):

$$\text{(9.62)} \quad \bar{y}_{i\tau}^{k} = y_{i\tau}\left(p_{i\tau}^{k}\right) = Pop_i^u \gamma_{i\tau}^u\left(1 - b_{i\tau}^u\right) + \frac{Pop_i^u b_{i\tau}^u\left(I_{i\tau}^u - \xi_{i\tau}^u\right) + Pop_i^r b_{i\tau}^r\left(I_{i\tau}^r - \xi_{i\tau}^r\right)}{\pi_{i\tau}}$$

The upperbounds on trader purchases are estimated as the optimal producer supplies in the period τ, if they supply in period t the minimally required quantity x_{it}^{-} – see (9.44) - (9.47):

$$(9.63)\quad \bar{x}_{i\tau} = \begin{cases} Pop_i^r x_{i\tau}^- & \text{if } p_{i\tau}^k < c_{i\tau} + \Psi_{i,\tau+1} \\ Pop_i^r \left[\left(w_{t-1} - x_{it}^- \right) \delta - \sum\limits_{\nu=\tau+1}^{T} \dfrac{x_\nu^-}{\delta^{\nu-\tau}} \right] & \text{if } p_{i\tau}^k \geq c_{i\tau} + \Psi_{i,\tau+1} \end{cases}$$

Equilibrium models:

The equilibrium models for the periods 1, 2, 3 and 4 can be written as:

<u>For period 1</u>: Determine the optimal values of the following variables: y_{i1}, x_{i1}, x_{ij1}, $s_{i1}, x_{i\tau}^k, r_{i\tau}^k, q_{i\tau}^k, q_{ij\tau}^k$, $v_{i\tau}^k$ for $\tau = 2,3,4$, $k = 1,\ldots,K$, $i = 1,\ldots,n$:

$$(9.64)\quad \begin{aligned} z_1(s_{i0}, w_{i0}) = \max \sum_{i=1}^{n} & \left[\int_0^{y_{i1}} \pi_{i1}(\xi) d\xi - c_{i1} x_{i1} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \tau_{ij1} x_{ij1} - k_{i1} s_{i1} - \hat{\alpha} y_{i1} \right] \\ & + \sigma \sum_{i=1}^{n} \sum_{k=1}^{K} f_{i2}^k \left[\left(p_{i2}^k - c_{i2} \right) x_{i2}^k + \sigma \left[\left(p_{i3}^k - c_{i3} \right) x_{i3}^k + \sigma \left(p_{i4}^k - c_{i4} \right) x_{i4}^k \right] \right] \\ & + \hat{\sigma} \sum_{i=1}^{n} \sum_{k=1}^{K} g_2^k \left[\left(\left(\pi_{i2}^k - \hat{\alpha} \right) r_{i2}^k - p_{i2}^k q_{i2}^k - \sum_{\substack{j=1 \\ j\neq i}}^{n} \tau_{ij2} q_{ij2}^k - k_{i2} v_{i2}^k \right) + \hat{\sigma} \left[\left(\left(\pi_{i3}^k - \hat{\alpha} \right) r_{i3}^k - \right. \right. \right. \\ & \left. \left. \left. - p_{i3}^k q_{i3}^k - \sum_{\substack{j=1 \\ j\neq i}}^{n} \tau_{ij3} q_{ij3}^k - k_{i3} v_{i3}^k \right) + \hat{\sigma} \left(\left(\pi_{i4}^k - \hat{\alpha} \right) r_{i4}^k - p_{i4}^k q_{i4}^k - \sum_{\substack{j=1 \\ j\neq i}}^{n} \tau_{ij4} q_{ij4}^k - k_{i4} v_{i4}^k \right) \right] \right] \end{aligned}$$

subject to

$$x_{i1} + \sum_{\substack{j=1 \\ j\neq i}}^{n} x_{ji1} + \hat{\delta} s_{i0} = y_{i1} + \sum_{\substack{j=1 \\ j\neq i}}^{n} x_{ij1} + s_{i1}; \qquad q_{i2}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ji2}^k + \hat{\delta} s_{i1} = r_{i2}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ij2}^k + v_{i2}^k;$$

$$q_{i3}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ji3}^k + \hat{\delta} v_{i2}^k = r_{i3}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ij3}^k + v_{i3}^k; \qquad q_{i4}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ji4}^k + \hat{\delta} v_{i3}^k = r_{i4}^k + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ij4}^k + v_{i4}^k;$$

$$r_{i2}^k \le \bar{y}_{i2}^k; \quad q_{i2}^k \le \bar{x}_{i2}^k; \quad r_{i3}^k \le \bar{y}_{i3}^k; \quad q_{i3}^k \le \bar{x}_{i3}^k; \quad r_{i4}^k \le \bar{y}_{i4}^k; \quad q_{i4}^k \le \bar{x}_{i4}^k;$$

$$x_{i1} + \frac{x_{i2}^k}{\delta} + \frac{x_{i3}^k}{\delta^2} + \frac{x_{i4}^k}{\delta^3} \le w_{i0}; \quad x_{i1} \ge x_{i1}^-; \quad x_{i2}^k \ge x_{i2}^-; \quad x_{i3}^k \ge x_{i3}^-; \quad x_{i4}^k \ge x_{i4}^-;$$

$$x_{i1}, y_{i1}, x_{ij1}, s_{i1}, x_{i\tau}^k, r_{i\tau}^k, q_{i\tau}^k, q_{ij\tau}^k, v_{i\tau}^k \ge 0 \text{ for } \tau = 2,3,4,\ k = 1,\ldots,K$$

For period 2: Determine the optimal values of the following variables: y_{i2}, x_{i2}, x_{ij2}, s_{i2}, $x_{i\tau}^k, r_{i\tau}^k, q_{i\tau}^k, q_{ij\tau}^k$, v_{i3}^k for $\tau = 3,4$, $k = 1,\ldots,K$, $i = 1,\ldots,n$, with $w_{i1} = (w_{i0} - x_{i1})\cdot\delta$, and with s_{i1} and x_{i1} the optimal storage and producer supply for period 1:

$$\begin{aligned}
z_2(s_{i1}, w_{i1}) = \max \sum_{i=1}^{n} &\left[\int_0^{y_{i2}} \pi_{i2}(\xi)d\xi - c_{i2}x_{i2} - \sum_{\substack{j=1 \\ j\ne i}}^{n} \tau_{ij2}x_{ij2} - k_{i2}s_{i2} - \hat{\alpha}y_{i2} \right] \\
&+ \sigma \sum_{i=1}^{n}\sum_{k=1}^{K} f_{i3}^k \left[\left(p_{i3}^k - c_{i3}\right)x_{i3}^k + \sigma\left(p_{i4}^k - c_{i4}\right)x_{i4}^k \right] \\
&+ \hat{\sigma} \sum_{i=1}^{n}\sum_{k=1}^{K} g_3^k \left[\left(\left(\pi_{i3}^k - \hat{\alpha}\right)r_{i3}^k - p_{i3}^k q_{i3}^k - \sum_{\substack{j=1 \\ j\ne i}}^{n} \tau_{ij3}q_{ij3}^k - k_{i3}v_{i3}^k \right) + \right. \\
&\left. + \hat{\sigma} \left[\left(\pi_{i4}^k - \hat{\alpha}\right)r_{i4}^k - p_{i4}^k q_{i4}^k - \sum_{\substack{j=1 \\ j\ne i}}^{n} \tau_{ij4}q_{ij4}^k - k_{i4}v_{i4}^k \right] \right]
\end{aligned} \tag{9.65}$$

subject to

$$x_{i2} + \sum_{\substack{j=1 \\ j\ne i}}^{n} x_{ji2} + \hat{\delta}s_{i1} = y_{i2} + \sum_{\substack{j=1 \\ j\ne i}}^{n} x_{ij2} + s_{i2}; \quad q_{i3}^k + \sum_{\substack{j=1 \\ j\ne i}}^{n} q_{ji3}^k + \hat{\delta}s_{i2} = r_{i3}^k + \sum_{\substack{j=1 \\ j\ne i}}^{n} q_{ij3}^k + v_{i3}^k;$$

$$q_{i4}^k + \sum_{\substack{j=1 \\ j\ne i}}^{n} q_{ji4}^k + \hat{\delta}v_{i3}^k = r_{i4}^k + \sum_{\substack{j=1 \\ j\ne i}}^{n} q_{ij4}^k + v_{i4}^k; \quad r_{i3}^k \le \bar{y}_{i3}^k; \quad q_{i3}^k \le \bar{x}_{i3}^k; \quad r_{i4}^k \le \bar{y}_{i4}^k; \quad q_{i4}^k \le \bar{x}_{i4}^k;$$

$$x_{i2} + \frac{x_{i3}^k}{\delta} + \frac{x_{i4}^k}{\delta^2} \le w_{i1}; \qquad x_{i2} \ge x_{i2}^-; \qquad x_{i3}^k \ge x_{i3}^-; \qquad x_{i4}^k \ge x_{i4}^-;$$

$$x_{i2}, y_{i2}, x_{ij2}, s_{i2}, x_{i\tau}^k, r_{i\tau}^k, q_{i\tau}^k, q_{ij\tau}^k, v_{i\tau}^k \ge 0 \text{ for } \tau = 3,4,\ k = 1,\ldots,K$$

<u>For period 3</u>: Determine the optimal values of the following variables: y_{i3}, x_{i3}, x_{ij3}, s_{i3}, $x_{i4}^k, r_{i4}^k, q_{i4}^k, q_{ij4}^k$ and v_{i4}^k for $k = 1,\ldots,K$, $i = 1,\ldots,n$, with $w_{i2} = (w_{i1} - x_{i2})\cdot\delta$, and with s_{i2} and x_{i2} the optimal storage and producer supply for period 2:

(9.66)
$$z_3(s_{i2}, w_{i2}) = \max \sum_{i=1}^{n} \left[\int_0^{y_{i3}} \pi_{i3}(\xi) d\xi - c_{i3} x_{i3} - \sum_{\substack{j=1 \\ j \ne i}}^{n} \tau_{ij3} x_{ij3} - k_{i3} s_{i3} - \hat{\alpha} y_{i3} \right]$$
$$+ \sigma \cdot \sum_{i=1}^{n} \sum_{k=1}^{K} f_{i4}^k \left(p_{i4}^k - c_{i4} \right) x_{i4}^k + \hat{\sigma} \sum_{i=1}^{n} \sum_{k=1}^{K} g_4^k \left(\left(\pi_{i4}^k - \hat{\alpha} \right) r_{i4}^k - p_{i4}^k q_{i4}^k - \sum_{\substack{j=1 \\ j \ne i}}^{n} \tau_{ij4} q_{ij4}^k - k_{i4} v_{i4}^k \right)$$

subject to

$$x_{i3} + \sum_{\substack{j=1 \\ j \ne i}}^{n} x_{ji3} + \hat{\delta} s_{i2} = y_{i3} + \sum_{\substack{j=1 \\ j \ne i}}^{n} x_{ij3} + s_{i3}; \qquad q_{i4}^k + \sum_{\substack{j=1 \\ j \ne i}}^{n} q_{ji4}^k + \hat{\delta} s_{i3} = r_{i4}^k + \sum_{\substack{j=1 \\ j \ne i}}^{n} q_{ij4}^k + v_{i4}^k;$$

$$r_{i4}^k \le \bar{y}_{i4}^k; \qquad q_{i4}^k \le \bar{x}_{i4}^k; \qquad x_{i3} + \frac{x_{i4}^k}{\delta} \le w_{i2}; \qquad x_{i3} \ge x_{i3}^-; \qquad x_{i4}^k \ge x_{i4}^-;$$

$$x_{i3}, y_{i3}, x_{ij3}, s_{i3}, x_{i4}^k, r_{i4}^k, q_{i4}^k, q_{ij4}^k, v_{i4}^k \ge 0 \text{ for } k = 1,\ldots,K$$

<u>For period 4</u>: Determine the optimal values of the following variables: y_{i4}, x_{i4}, x_{ij4}, for $i = 1,\ldots,n$, with $w_{i3} = (w_{i2} - x_{i3})\cdot\delta$, and with s_{i3} and x_{i3} the optimal storage and producer supply for period 3:

(9.67)
$$z_4(s_{i3}, w_{i3}) = \max \sum_{i=1}^{n} \left[\int_0^{y_{i4}} \pi_{i4}(\xi) d\xi - c_{i4} x_{i4} - \sum_{\substack{j=1 \\ j \ne i}}^{n} \tau_{ij4} x_{ij4} - k_{i4} s_{i4} - \hat{\alpha} y_{i4} \right]$$

subject to

$$x_{i4} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ji4} + \hat{\delta} s_{i3} = y_{i4} + \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ij4} + s_{i4}; \qquad x_{i4}^{-} \leq x_{i4} \leq w_{i3}; \qquad y_{i4}, s_{i4}, x_{ij4} \geq 0$$

The theorem 7.1 and 7.2 also hold for these models. The optimal producer price in period t is defined as:

$$p_{it} = \lambda_{it}$$

with λ_{it} the optimal value of the Lagrange multiplier of the equilibrium constraint for period t in the model for period t. The results of these models will almost be the same as the *Equilibrium properties* 7.1 to 7.4 in Section 7.2. *Equilibrium property* 7.2 remains the same. The other properties change in:

Equilibrium property 9.1: For region i and period t we can derive that:
a) In the optimal solution $\pi_{it}(y_{it}) \leq p_{it}(x_{it}) + \hat{\alpha}$.
b) If in the solution $\pi_{it}(y_{it}) < p_{it}(x_{it}) + \hat{\alpha}$, then $y_{it} = 0$.
c) If in the optimal solution, supply and demand are both positive, $x_{it} > 0$ and $y_{it} > 0$, then the prices satisfy necessarily $\pi_{it}(y_{it}) = p_{it}(x_{it}) + \hat{\alpha}$.

Equilibrium property 9.2: In the solution, let $x_{ijt} > 0$, with $i,j = 1,\ldots,n$, $j \neq i$, $t = 1,\ldots,n$, then:

a) $x_{sit} = 0$, for $s = 1,\ldots,n$, $s \neq i$.
b) $x_{jst} = 0$, for $s = 1,\ldots,n$, $s \neq j$.
c) $x_{it} > 0$ or $s_{i,t-1} > 0$.
d) $y_{jt} > 0$ or $s_{jt} > 0$

Equilibrium property 9.3:
a) In the optimal solution $\pi_{jt}(y_{jt}) \leq p_{it}(x_{it}) + \hat{\alpha} + \tau_{ijt}$.
b) If in the solution $\pi_{jt}(y_{jt}) < p_{it}(x_{it}) + \hat{\alpha} + \tau_{ijt}$, then $x_{ijt} = 0$ or $y_{jt} = 0$.
c) If in the optimal solution supplies in region i, transport between region i and j, and demand in region j are positive, $x_{it} > 0$, $x_{ijt} > 0$, and $y_{jt} > 0$, then the optimal prices satsify necessarily $\pi_{jt}(y_{jt}) = p_{it}(x_{it}) + \hat{\alpha} + \tau_{ijt}$.

Equilibrium property 9.4:

a) If in the optimal solution $\hat{\delta}\hat{\sigma}\left(E\Pi_{i,t+1} - \hat{\alpha}\right) < p_{it}(x_{it}) + k_{it}$, then $s_{it} = 0$ or $r_{i,t+1}^{k} = 0$.

b) If in the optimal solution $\hat{\delta}\hat{\sigma}\left(E\Pi_{i,t+1} - \hat{\alpha}\right) \geq p_{it}(x_{it}) + k_{it}$, storage in period t, and planned sales in period t+1 are positive, $s_{it} > 0$ or $r_{i,t+1}^{k} > 0$ for all $k \in \{1,\ldots,K\}$, then an optimal solution exists satisfying $q_{it} = x_{it}$ or $r_{i,t+1}^{k} = y_{i,t+1}$ for at least one $k \in \{1,\ldots,K\}$. For $\hat{\delta}\hat{\sigma}\left(E\Pi_{i,t+1} - \hat{\alpha}\right) = p_{it}(x_{it}) + k_{it}$, an optimal solution is not unique.

10 Discussion of model results

Solving model (9.1), (9.2), (9.13), with demand functions given in (8.14) Section 8.1.3, and supply functions determined by model (8.36) in Section 8.2.1, gives us results which roughly reflect reality in Burkina Faso. We shortly discuss some of the main results.

Prices determined by the model are generally in line with the observed cereal prices given in Table 8.3. The values of the consumer and producer prices do have more or less the correct order and reflect seasonality - see Table 10.1. Price volatility is somewhat higher than the average observed in Table 8.3. This was expected as we deal with a specific year instead of the average for a number of years. The results show that prices are lowest in the high production areas, from which many cereals are transported, and highest in the low production and shortage areas.

Estimated transport flows are in line with the flows observed in reality. Most goods are transported from the largest surplus zone Mouhoun towards the region Centre with the capital Ouagadougou (see Figure 10.1). Also the shortage regions Sahel and Nord receive a large part of the surplus from the regions Mouhoun and Est. Transport towards these regions is highest during the lean period, from July to September, when farmers' stocks get depleted. In the period October – December the region Hauts Bassins has a relatively low price and a large surplus that is transferred to Ouagadougou, where the prices are relatively high. However, from april onwards, Hauts Bassins becomes a deficit region that imports from the Mouhoun and the Centre-Ouest. It should be noted that many cereals from the region Mouhoun, Sud Ouest and Comoe are transferred to the north and center via traders based in Bobo Dioulasso, which is one of the most important redistribution markets of the country. About 1/3 of the marketable surplus (see Table 8.1 and 9.8) is transported towards other regions.

About 10% of the annual sales are stored by traders for at least one period. These cereals are generally stored in the surplus zones. Traders are not involved in intertemporal storage in the third and fourth period, since farmers prefer to execute this function themselves. In the supply model producers expect to receive the highest prices by selling during the lean season, and therefore preserve a large part of their annual sales for the last period. This result, corresponds to observations made by Bassolet (2000) in Burkina Faso, Lutz (1996) in Benin, and Armah (1989) in Ghana, that most goods are stored by the producers and that only a few traders store cereals for a longer period.

Table 10.1: Results of the multi-period, spatial equilibrium model.

a. Consumer price levels and supply per person.

	Consumer price level (FCFA/kg)					Supply per person (kg per person)				
	Oct-Dec	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**Average**	**Oct-Dec**	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**Total**
Centre	103	108	115	119	111	4.2	17.5	1.3	2.3	26.0
Centre Nord	107	110	116	123	114	2.6	4.5	5.8	2.5	16.0
Centre Ouest	102	102	108	121	108	6.6	9.4	16.3	7.1	41.0
Centre Sud	100	102	109	113	106	8.2	11.7	2.6	26.0	50.9
Sahel	109	110	123	131	118	6.0	6.0	1.5	1.2	15.0
Mouhoun	92	98	104	108	101	10.9	14.9	11.9	56.0	99.0
Est	100	99	106	119	106	6.9	9.9	20.7	3.9	43.0
Centre Est	106	104	111	114	109	5.8	8.3	1.8	18.4	36.0
Nord	101	106	113	118	109	2.2	5.8	4.4	1.1	14.0
Sud Ouest	100	98	101	134	108	8.8	12.7	20.8	10.5	55.0
Hauts Bassins	92	107	111	123	108	61.2	15.0	12.0	10.0	100.0
Comoe	97	102	114	118	108	10.1	13.8	13.9	15.1	55.2
Average price	101	104	111	120	109					
Av. supply						11.1	10.8	9.4	12.8	44.1

b. Cereal demand and consumption per rural and urban consumer

	Cereal demand per rural consumer (kg/person)					Cereal consumption per urban consumer (kg/person)				
	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sept	annual consump.[1]	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sept	annual consump
Centre	3.2	2.9	4.1	6.5	122	31.4	29.7	29.9	29.7	121
Centre Nord	2.8	2.7	3.9	6.3	163	26.2	26.0	26.2	26.3	105
Centre Ouest	2.9	3.1	4.5	6.6	179	26.4	27.1	27.5	26.4	107
Centre Sud	3.1	3.1	4.6	6.4	238	27.2	27.4	27.9	26.1	109
Sahel	4.9	4.9	6.9	10.6	162	26.0	26.1	25.8	25.4	103
Mouhoun	3.5	4.8	7.4	8.7	208	27.4	27.5	27.9	28.3	111
Est	2.7	2.8	4.2	6.4	188	26.4	27.1	27.7	26.8	108
Centre Est	2.9	3.1	4.3	6.8	163	26.4	27.2	26.9	26.8	107
Nord	2.9	2.8	4.1	6.5	143	26.7	26.5	26.8	26.6	107
Sud Ouest	5.2	7.3	11.5	10.5	253	27.0	27.5	28.2	25.2	108
Hauts Bassins	3.8	4.4	6.8	7.3	206	32.9	29.8	30.7	29.2	123
Comoe	3.6	4.4	6.6	7.8	231	27.6	26.6	26.6	26.9	108
Average	3.5	3.9	5.8	7.5	182	27.6	27.4	27.7	27.0	110

Note: 1) Annual consumption equals production per rural inhabitant (Table 8.1) + annual demand – annual supply per person (Table 9.8).

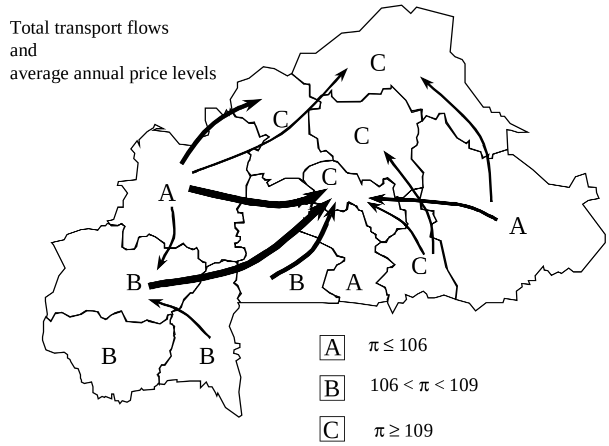
c. Transported and stored quantities.

Quantity transported (in 1000 tonnes)						
From	To	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sept	Total
Centre Ouest	Centre	0	0	10.8	0	10.8
Centre Ouest	Sud Ouest	0	0	0	0.46	0.46
Centre Sud	Centre	0	3.43	0.74	3.95	8.12
Centre Sud	Centre Nord	0	0	0	5.33	5.33
Mouhoun	Centre	0	0.35	10.01	23.31	33.67
Mouhoun	Centre Ouest	0	0	0	3.08	3.08
Mouhoun	Sahel	0	0	0	7.93	7.93
Mouhoun	Nord	3.35	0	2.52	7.82	13.69
Mouhoun	Hauts Bassins	0	0	0	8.83	8.83
Est	Centre	0	7.48	6.28	0	13.76
Est	Sahel	0	0	4.89	0	4.89
Est	Centre Est	0.61	0	3.66	0	4.26
Centre Est	Centre	0	0	0	1.73	1.73
Centre Est	Est	0	0	0	3.98	3.98

Quantity stored (in 1000 tonnes)		
	Oct-Dec	Jan-Mar
Centre	0	0
Centre Nord	0	0
Centre Ouest	0.32	3.11
Centre Sud	2.04	2.35
Sahel	0	0
Mouhoun	2.28	10.55
Est	2.06	0
Centre Est	0	1
Nord	0	0
Sud Ouest	1.36	3.35
Hauts Bassins	2.41	0
Comoe	0	0
Total	10.48	20.36

In Period 3 and 4 traders store no cereals cont. Table 10.1.c

Sud Ouest	Hauts Bassins	0	0.24	7.58	0	7.82
Hauts Bassins	Centre	25.48	0	0	0	25.48
Hauts Bassins	Centre Nord	1.96	0	0	0	1.96
Hauts Bassins	Comoe	0.29	0	0	0	0.29
Comoe	Hauts Bassins	0	0.62	0	0	0.62
Total		31.68	12.12	46.48	66.44	156.72



Figuur 10.1: Consumer prices and transport flows in Burkina Faso

Transport costs

The main objective of this paper was to analyse the direct impact of transport costs on cereal trade. We recall that the total annual supply is given (see Chapter 9), however, the distribution of supply over the year changes. The model shows that if transport costs decrease, quarterly cereal prices in surplus regions increase, while cereal prices in deficit regions decrease – see Table 10.2. The changes are, however, small. Halving the transport costs causes average prices in the Sahel to decrease by 3.9% - see Table 10.2. This causes an increased demand of 4.3% and an increased cereal consumption of only 0.7%.[32]

[32] The impact on total consumption is smaller than the impact on market demand, as only a minor part of total consumption is purchased on the market (see Table 10.1.b).

Likewise, average prices in the region Centre, with the capital Ouagadougou, decrease with 1.7%. As a consequence, urban consumption increases with 1.1%, and rural consumption with only 0.2% (rural demand increases with 1.4%). A drawback from decreased prices in a region, is that producers earn less from their supplies (since the margin between producer and consumer prices is assumed to be fixed). Total transported quantities increase with 7.8% if transport costs decrease with 50%. Stored quantities decrease with 19%.

Looking at prices, it can be concluded that on average the price increase in the periods 2 and 3 is somewhat lower, whereas, the price increase in the last period is sharper than in the situation with high transport costs. Consumers in the main deficit area Sahel and producers in the main surplus area Mouhoun profit most from the reduction in transport costs. For producers and consumers in other markets, the effects are less striking. These results show that despite the large (50%) decrease in transport costs, the direct effects on prices and consumption are small. This result contradicts popular claims that transport costs are a major barrier for a more equal distribution of cereal production over the country. The model indicates that the bad income position of the Sahelian population is more likely to be responsible for the low demand. We note that the income position may increase in the long run as a result of infrastructural improvements. It is, however, not our intention to analyse the spin-off effects of infrastructural improvements on other sectors in the economy. Our partial economic approach is not suitable to analyse these indirect effects. We simply questioned the popular claim that high transport costs are a major barrier for cereal marketing. The research results show that this claim needs to be nuanced.

Table 10.2 Percentage change with base results if transport costs decrease with 50%.

a. Changes in consumer price levels and quantity stored.

	Changes in consumer price (%)				
	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sept	average
Centre	-1.8%	-2.7%	-3.2%	0.5%	-1.7%
Centre Nord	-3.5%	-2.9%	-2.6%	-1.1%	-2.5%
Centre Ouest	-1.9%	-1.4%	-1.0%	-0.1%	-1.1%
Centre Sud	-2.7%	-0.8%	2.2%	2.4%	0.4%
Sahel	-2.4%	-2.6%	-6.2%	-4.2%	-3.9%
Mouhoun	4.0%	4.0%	0.5%	4.9%	3.3%
Est	0.0%	0.6%	-0.2%	0.5%	0.2%
Centre Est	-2.9%	-1.9%	-1.8%	2.0%	-1.1%
Nord	0.2%	0.2%	-2.6%	1.1%	-0.3%
Sud Ouest	-0.8%	-0.8%	0.8%	-4.5%	-1.6%
Hauts Bassins	2.9%	-4.2%	-2.6%	-0.9%	-1.3%
Comoe	1.0%	-2.8%	-3.7%	0.0%	-1.4%
average price	-0.8%	-1.3%	-1.8%	-0.1%	-1.0%

Quantity stored (in 1000 tonnes)		
	Oct-Dec	Jan-Mar
Centre	0	0
Centre Nord	0	0
Centre Ouest	0.24	2.97
Centre Sud	1.94	0
Sahel	0	0
Mouhoun	0	8.95
Est	2.66	0
Centre Est	0	0.6
Nord	0	0
Sud Ouest	1.34	1.7
Hauts Bassins	1.28	0
Comoe	0	0
Total stored quantity	7.46	14.22

b. Change in cereal demand per rural and urban consumer

	Change in demand per rural consumer (%)					Change in demand per urban consumer (%)				
	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sept	total	Oct-Dec	Jan-Mar	Apr-Jun	Jul-Sept	total
Centre	1.6%	2.8%	3.4%	-0.5%	1.4%	1.1%	1.6%	2.0%	-0.3%	1.1%
Centre Nord	3.6%	3.0%	2.8%	1.1%	2.3%	1.7%	1.4%	1.3%	0.5%	1.2%
Centre Ouest	2.1%	1.3%	1.1%	0.2%	0.9%	0.9%	0.7%	0.5%	0.0%	0.5%
Centre Sud	2.6%	1.0%	-2.2%	-2.3%	-0.8%	1.4%	0.4%	-1.1%	-1.1%	-0.1%
Sahel	2.5%	2.6%	6.6%	4.4%	4.3%	1.1%	1.2%	3.0%	2.0%	1.8%
Mouhoun	-4.0%	-4.0%	-0.5%	-4.6%	-3.1%	-1.9%	-1.9%	-0.3%	-2.4%	-1.6%
Est	0.0%	-0.4%	0.2%	-0.5%	-0.2%	0.0%	-0.3%	0.1%	-0.2%	-0.1%
Centre Est	3.1%	1.9%	1.9%	-1.9%	0.5%	1.4%	1.0%	0.9%	-1.0%	0.6%
Nord	0.0%	-0.4%	2.7%	-1.1%	0.2%	-0.1%	-0.1%	1.3%	-0.5%	0.1%
Sud Ouest	0.8%	0.8%	-0.8%	4.7%	1.4%	0.4%	0.4%	-0.4%	2.1%	0.6%
Hauts Bassins	-2.9%	4.4%	2.8%	1.0%	1.5%	-1.8%	2.7%	1.7%	0.5%	0.7%
Comoe	-1.1%	3.0%	3.9%	0.0%	1.5%	-0.5%	1.3%	1.8%	0.0%	0.7%
Average demand	0.6%	1.0%	1.7%	0.3%	0.9%	0.3%	0.7%	0.9%	0.0%	0.5%

c. Transported quantities.

Quantity transported (in 1000 tonnes)						
From	**To**	**Oct-Dec**	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**Total**
Cen.Ouest	Centre			13.91		13.91
Cen. Sud	Centre		5.67		6.33	12
Cen. Sud	Cen. Nord				1.65	1.65
Cen. Sud	Cen. Est	0.05				0.05
Mouhoun	Centre			8.37	20.55	28.92
Mouhoun	Cen.Ouest				5.85	5.85
Mouhoun	Sahel	2.5			8.3	10.8
Mouhoun	Nord	3.34		2.65	7.74	13.73
Mouhoun	Sud Ouest				0.14	0.14
Mouhoun	H. Bassins				8.93	8.93
Est	Centre		8.09	3.7		11.79
Est	Cen. Nord			4.08		4.08
Est	Sahel			2.78		2.78
Est	Cen. Est			4.12		4.12

From	**To**	**Oct-Dec**	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**Total**
Cen. Est	Centre		0.32		2.01	2.34
Cen. Est	Est				3.82	3.82
Sud Ouest	Centre			0.2		0.2
Sud Ouest	H. Bass		1.83	7.91		9.74
Sud Ouest	Comoe			0.91		0.91
H. Bass	Centre	25.83				25.83
H. Bass	Cen.Nord	2.08				2.08
H. Bass	Sahel	0.32				0.32
H. Bass	Cen. Est	0.65				0.65
H. Bass	Comoe	0.27				0.27
Comoe	S. Ouest				3.51	3.51
Comoe	H. Bass		0.55			0.55
Total		35.04	16.47	48.65	68.83	168.99
% increase		10.6%	35.9%	4.7%	3.6%	7.8%

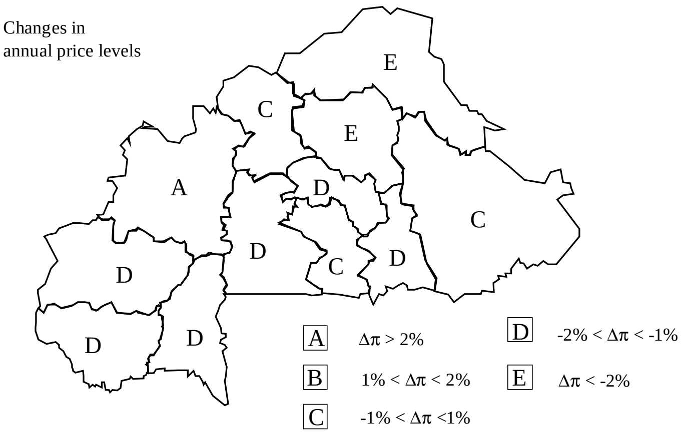


Figure 10.2: Changes in cereal prices if transport costs decrease with 50%.

Storage costs

The influence of storage costs on prices, supply, and demand is also weak. If storage costs decrease with 50%, prices increase on average with 2.2% (see Table 10.3). Prices in the first three periods increase on average, whereas prices in the fourth period remain the same in most regions, but decrease sharply in the regions Centre Ouest and Sud Ouest. This price increase is mainly due to changes in the producers' supply schedule. The general price increase is caused by the 'fixed' price expectations. Price expectations for period t+1 do not depend on prices in period t. So, if storage costs decrease but price expectations remain the same, traders expect to make higher profits from storage. Consequently, their demand increases, causing producer prices (and consequently also consumer prices) to increase. Stored quantities in the first and second period increase sharply (respectively with 63% and 29%). Even when storage costs decrease with 50%, traders do not store in the third and fourth period. Storage is expected not to be profitable for traders, while farmers expect to earn high profits if they sell in the lean season. Consumer demand and transported quantities decrease in this case due to the increased prices.

If consumer price expectations for the next period do depend on the current producer price, it is not clear whether consumer and producer prices will increase or decrease. But it can be expected that average changes will be smaller than in the current situation.

Table 10.3 Percentage change with base results if storage costs decrease with 50%.

a. Changes in consumer price levels and quantity stored.

	Changes in consumer price (%)				
	Oct-Dec	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**average**
Centre	3.4%	5.8%	4.0%	0.0%	3.2%
Centre Nord	1.5%	1.3%	6.1%	0.0%	2.2%
Centre Ouest	3.7%	5.4%	4.3%	-7.6%	1.1%
Centre Sud	5.9%	6.1%	4.3%	0.0%	4.0%
Sahel	1.5%	3.2%	3.8%	0.0%	2.1%
Mouhoun	3.7%	6.4%	4.4%	0.0%	3.5%
Est	4.0%	6.3%	4.4%	0.0%	3.5%
Centre Est	3.7%	5.2%	4.2%	0.0%	3.2%
Nord	3.4%	2.9%	4.1%	0.0%	2.5%
Sud Ouest	3.5%	6.3%	4.7%	-13.0%	-0.7%
Hauts Bassins	3.8%	-3.7%	4.3%	0.0%	1.0%
Comoe	3.6%	-1.9%	-0.3%	0.0%	0.3%
average price	3.4%	3.5%	4.0%	-1.8%	2.2%

Quantity stored (in 1000 tonnes)		
	Oct-Dec	**Jan-Mar**
Centre	0	0
Centre Nord	0	0
Centre Ouest	0.47	3.48
Centre Sud	2.14	2.39
Sahel	2.64	0.17
Mouhoun	2.6	9.87
Est	2.79	5.08
Centre Est	0	0.8
Nord	0	0
Sud Ouest	1.47	3.93
Hauts Bassins	4.97	0
Comoe	0	0.57
Total stored quantity	17.09	26.3

b. Change in cereal demand per rural and urban consumer

	Change in demand per rural consumer (%)					**Change in demand per urban consumer (%)**				
	Oct-Dec	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**total**	**Oct-Dec**	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**total**
Centre	-3.5%	-5.2%	-3.7%	0.0%	-2.5%	-2.0%	-3.3%	-2.3%	0.0%	-1.9%
Centre Nord	-1.4%	-1.1%	-5.9%	0.0%	-1.9%	-0.7%	-0.6%	-2.7%	0.0%	-1.0%
Centre Ouest	-3.4%	-5.2%	-4.2%	8.4%	0.6%	-1.6%	-2.5%	-2.0%	3.9%	-0.6%
Centre Sud	-5.8%	-5.8%	-4.1%	0.0%	-3.1%	-2.7%	-2.8%	-2.0%	0.0%	-1.9%
Sahel	-1.4%	-3.0%	-3.6%	0.0%	-1.8%	-0.7%	-1.4%	-1.7%	0.0%	-1.0%
Mouhoun	-3.7%	-6.1%	-4.2%	0.0%	-3.0%	-1.7%	-3.0%	-2.1%	0.0%	-1.7%
Est	-4.0%	-6.0%	-4.0%	0.0%	-2.8%	-1.8%	-2.9%	-2.1%	0.0%	-1.7%
Centre Est	-3.5%	-5.2%	-3.9%	0.0%	-2.5%	-1.6%	-2.4%	-1.9%	0.0%	-1.5%
Nord	-3.2%	-2.8%	-3.9%	0.0%	-2.0%	-1.5%	-1.3%	-1.9%	0.0%	-1.2%
Sud Ouest	-3.5%	-6.0%	-4.4%	14.9%	1.2%	-1.6%	-2.9%	-2.2%	6.6%	-0.2%
Hauts Bassins	-3.7%	3.7%	-4.1%	0.0%	-1.1%	-2.3%	2.3%	-2.6%	0.0%	-0.7%
Comoe	-3.6%	2.1%	0.3%	0.0%	-0.1%	-1.7%	0.9%	0.1%	0.0%	-0.2%
Average demand	-3.5%	-3.4%	-3.8%	2.4%	-1.4%	-1.7%	-1.6%	-2.0%	0.9%	-1.1%

c. Transported quantities.

Quantity transported (in 1000 tonnes)						
From	**To**	**Oct-Dec**	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**Total**
Cen.Ouest	Centre			4.65	0.87	5.52
Cen.Ouest	Cen. Nord				2.46	2.46
Cen. Sud	Centre		3.6	0.88		4.48
Cen. Sud	Cen. Nord				4.05	4.05
Mouhoun	Centre		1.75	10.08	26.4	38.23
Mouhoun	Sahel				7.93	7.93
Mouhoun	Nord	3.22		2.21	7.82	13.25
Mouhoun	H.Bassins				8.83	8.83
Est	Centre		3.33	11.48		14.8
Est	Cen. Nord			0.3		0.3
Est	Sahel			4.52		4.52
Est	Cen. Est			3.67		3.67
Cen. Est	Centre		0.37		1.73	2.1
Cen. Est	Est				3.98	3.98
Sud Ouest	H. Bass			7.08		7.08
H. Bass	Centre	24.85	1.24			26.09
H. Bass	Cen. Est	0.49				0.49
H. Bass	Comoe	0.22				0.22
Total		28.77	10.29	44.86	64.06	147.99
%decrease		9.2%	15.1%	3.5%	3.6%	5.6%

Sensitivity analysis

A brief sensitivity analysis shows that price expectations, total production, and consumer income are the parameters having the largest influence on the solutions. Changes in their values have large consequences. The other model parameters have only a marginal influence on the model results.

11 Final discussion

In this report we pursued three objectives. The first was to develop a model to analyse cereal arbitrage in space and time. The second was to to analyse the interaction between the various actors on the cereal market in Burkina Faso. The third was to apply this model to analyse the direct impact of transport and storage costs on the distribution of cereals and on cereal prices in different regions of Burkina Faso. Much emphasis was put on adapting standard micro-economic equilibrium theory to the specific situation of cereal trade in West Africa. The two most challenging issues of the approach were 1) to model behaviour of burkinabè farmers and traders, and 2) to take into account the uncertain character of cereal prices.

What have we learned from this modelling approach? Firstly, by developing step by step the equilibrium models we identified the limits and possibilities of spatial equilibrium theory. By simply adopting the properties of a Spatial Price Equilibrium as discussed in the introduction of Chapter 5, the existence of *price uncertainty* is neglected, as well as other market situations deviating from a ‘perfect market’. By introducing explicitly trader behaviour, the influence of stochastic future prices can be analysed. The model elaborated in Chapter 5 to 7, is also useful to analyse other market imperfections. For example, a lack of credit facilities, and the existence of oligopolistic market power.

Secondly, in the Chapters 5 to 7, we discussed the strategies of the market actors involved in cereal trade, and developed a stochastic, multi-period, spatial equilibrium model. We recall that we proved that the welfare optimizing results of the equilibrium model are in line with the optimal strategies of the actors operating on the cereal market: producers, traders and consumers. We have shown that prices on a market are formed by the joint action of the market actors, who follow each their individual optimal strategy. This means that actors do not trade if they lose money (or expect to lose, in the case of storage). As long as each actor follows this principle (and market entry of new actors are free and capital availability is not constraining), equilibrium prices will be realized satisfying the equilibrium conditions discussed above. This explanation is more comprehensible and satisfactory, then the often cited 'invisible hand', directing the market towards a price level for which supply and demand are in equilibrium. The inclusion of trader behaviour in the model makes the functioning of the 'invisible hand' explicit.

Thirdly, as for the practical results of the models, the results demonstrated that the influence of transport and storage costs on cereal trade are limited. The direct effects of lower transport costs on the food situation of the poorer, rural regions are small. Furthermore, it was confirmed that long term storage is more often a task for producers than for traders. It can not be said whether producers or consumers do profit the most from decreasing transport or storage costs. The deficit regions do profit from decreased transport costs, however, the influence on cereal consumption is only marginal. Consumer income, total annual production, and price expectations are the factors which determine the equilibrium prices and the timing of supplies and demand. The results of the model provide useful elements for the discussions on improving the functioning of the cereal market in Burkina Faso.

Although the use of the model can be criticised because of unrealistic assumptions, unreliable estimates, and incomplete treatment of actors' strategies, it is still a useful tool to simulate the effects of market liberalization policies and agricultural development. The subject of market functioning is very complex, as many factors are interrelated: price formation results from the joint action of all market actors. With a descriptive or statistical analysis these factors can not be analysed in their coherence. Furthermore, the modelling approach forces the researcher to structure the descriptive analysis. For example, the choice of a cereal demand function of a certain type, indicates which parameters have to be estimated, and accordingly the elements which have to analysed. A descriptive analysis on its turn identifies the factors which are important to include in the model. A descriptive analysis of traders' strategies in Burkina Faso, for example, revealed that information on present and future prices are factors constraining traders' strategies. Therefore, we introduced stochastic prices in our equilibrium models. Another example concerns the timing of cereal sales and purchases within a year, which is important for poor households' survival strategies. A review of many surveys on farmers' strategies revealed that a widespread belief, that most farmers sell their largest quantity in the months following the harvest, had to be nuanced. This stressed once more the importance of including the timing of sales and purchases in the equilibrium models. Summarizing, a modelling approach 'can structure the discussions and the understanding of the issues considerably' (Schweigman, 1994).

The model results nuanced a widespread belief that transport costs are a major barrier for cereal trade. We do not want to claim that transport costs are not important for the development of the agricultural sector, but the direct impact of lower transport costs, as the direct impact of other price measures, are likely to be small. The reasons are clear. Cereal demand elasticities are low, and annual cereal supply is unlikely to change a lot if cereal prices change. Probably, a changed price will not rigourously alter the demand and supply behaviour of consumers and producers. Whether demand is price inelastic because other food commodities are hardly available on the market, or whether other food commodities are not available because cereal demand is inelastic, is another question, which is not addressed in this research. Similarly, whether annual cereal supply is price inelastic because producers do not sell their produce on the market because producer prices are considered to be too low, or because farmers are not able to produce more with the limited resources available, is a question which is not addressed here. The results subscribe to the more and more common view that improving single market constraining elements (like e.g. transport prices) is fruitless if not more complementary measures, or comprehensive packages of policy measures, are implemented to relax constraining elements of food markets and food production (deJanvry et al., 1997; Thorbecke, 2000). For example, the impact of an improved infrastructure will be larger, if this policy is complemented by proper agricultural research and extension services. The models set up in this paper can be used to learn about which measures are likely to yield the largest benefits.

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Appendix 1: Proofs of Properties and Theorems in the Chapters 5, 6 and 7

- **Properties of Section 5.1:**

Trader property 5.1: For each region $i \in \{1,...,n\}$*:*

a) If $\pi_i < p_i$, then any optimal solution of (5.12) satisfies: $q_i = 0$ or $r_i = 0$.

b) If $\pi_i \geq p_i$, then an optimal solution of (5.12) exists which satisfies the condition $q_i = x_i$ or $r_i = y_i$. Nota bene: for $\pi_i > p_i$, any optimal solution of (5.12) has to satisfy this condition; for $\pi_i = p_i$, other optimal solutions may exist not satisfying this condition.

Proof:

a) Let $\pi_i < p_i$. If in the optimal solution of (5.12) $r_i > 0$ and $q_i > 0$, then necessarily, see (5.13), $\pi_i \geq \lambda_i$ and $\lambda_i \geq p_i$, which contradicts $\pi_i < p_i$. So necessarily $r_i = 0$ or $q_i = 0$.

b) An optimal solution of the linear programming model (5.12) exists, since its feasible region is bounded. We make a distinction between $\pi_i > p_i$ and $\pi_i = p_i$.

* Let $\pi_i > p_i$. If in the optimal solution $q_i < x_i$ and $r_i < y_i$, then necessarily, see (5.13), $p_i \geq \lambda_i$ and $\lambda_i \geq \pi_i$, which contradicts $\pi_i > p_i$. So necessarily, $q_i = x_i$ or $r_i = y_i$.

* Let $\pi_i = p_i$. Consider an optimal solution q_s, r_s, q_{sj}, $s \neq j$, $s,j = 1,\ldots,n$, in which optimal purchases, sales and transports in region i, satisfy $q_i < x_i$ and $r_i < y_i$. Call $\Delta_i = \min(x_i - q_i; y_i - r_i)$. The solution q_s', r_s', q_{sj}', defined by $q_i' = q_i + \Delta_i$, $r_i' = r_i + \Delta_i$, $q_{ij}' = q_{ij}$, $q_s' = q_s$, $r_s' = r_s$, and $q_{sj}' = q_{sj}$, for $s \neq j$, is optimal as well, as can be seen easily by inspecting (5.12). Moreover, $q_i' = x_i$ or $r_i' = y_i$.

Trader property 5.2: Let q_i, r_j, q_{ij}, $j \neq i$, $i,j = 1,\ldots,n$, be an optimal solution of (5.12). Let a trader transport from a region i to a region j, so $q_{ij} > 0$, for $i,j \in (1,\ldots,n\}$, $i \neq j$, then:

a) no goods are transported from a region $s = 1,\ldots,n$, $s \neq i$, to region i, $q_{si} = 0$

b) no goods are transported from region j to a region $s = 1,\ldots,n$, $s \neq j$, $q_{js} = 0$.

c) purchases in region i are positive, $q_i > 0$

d) sales in region j are positive, $r_j > 0$.

Proof:

a) Let $q_{ij} > 0$. Suppose that $q_{si} > 0$. In that case, see (5.13), $\lambda_j = \lambda_i + \tau_{ij}$ and $\lambda_i = \lambda_s + \tau_{si}$. This implies that $\lambda_j = \lambda_s + \tau_{si} + \tau_{ij}$. Due to (5.13) we know that $\lambda_j \leq \lambda_s + \tau_{sj}$. It would follow that $\tau_{sj} \geq \lambda_j - \lambda_s = \tau_{si} + \tau_{ij}$. This is in contradiction with the analogue of property (5.11), which says that $\tau_{sj} < \tau_{si} + \tau_{ij}$. So, necessarily $q_{si} = 0$.

b) The proof of the second property is similar to the proof under a).

c) If $q_{ij} > 0$, then making use of a), the equilibrium condition (5.7) leads to a contradiction if $q_i = 0$. Consequently, $q_i > 0$.

d) In a similar way it can be shown that necessarily $r_j > 0$, if $q_{ij} > 0$.

Trader property 5.3: For the regions i and j, $i,j \in \{1,...,n\}$, $i \neq j$:

a) If $\pi_j < p_i + \tau_{ij}$, then any optimal solution of (5.12) satisfies $q_{ij} = 0$.

b) If $\pi_j \geq p_i + \tau_{ij}$ and $q_{ij} > 0$, then an optimal solution of (5.12) exists satisfying $q_i = x_i$ or $r_j = y_j$; for $\pi_j = p_i + \tau_{ij}$ and $q_{ij} > 0$ an optimal solution of (5.12) is not necessarily unique.

Proof:

a) Let $\pi_j < p_i + \tau_{ij}$. If $q_{ij} > 0$, then necessarily, see *Trader property* 5.2, $q_i > 0$ and $r_j > 0$. As a consequence, see (5.13), $\pi_j \geq \lambda_j$, $\lambda_j = \lambda_i + \tau_{ij}$, and $\lambda_i \geq p_i$, which contradicts $\pi_j < p_i + \tau_{ij}$. So necessarily, $q_{ij} = 0$.

b) An optimal solution of the linear programming model (5.12) exists, since its feasible region is bounded. We make a distinction between $\pi_j > p_i + \tau_{ij}$ and $\pi_j = p_i + \tau_{ij}$:

* Let $\pi_j > p_i + \tau_{ij}$ and $q_{ij} > 0$. According to (5.13) $\lambda_j = \lambda_i + \tau_{ij}$. If $q_i < x_i$ and $r_j < y_j$, then necessarily, see (5.13), $\pi_j \leq \lambda_j$ and $\lambda_i \leq p_i$, which contradicts $\pi_j > p_i + \tau_{ij}$. So necessarily, $q_i = x_i$ or $r_j = y_j$.

* Let $\pi_j = p_i + \tau_{ij}$ and $q_{ij} > 0$. Consider the optimal solution q_s, r_s, q_{sv}, $s,v = 1,\ldots,n$, $s \neq v$, and assume that for region i and j: $q_i < x_i$ and $r_j < y_j$. Call $\Delta_{ij} = \min(x_i - q_i; y_j - r_j)$.

The solution q'_s, r'_s, q'_{sv} defined by $q'_i = q_i + \Delta_{ij}$, $r'_j = r_j + \Delta_{ij}$, $q'_{ij} = q_{ij} + \Delta_{ij}$, $q'_s = q_s$, $r'_v = r_v$, $q'_{sv} = q_{sv}$, for $s \neq i$, $v \neq j$, is optimal as well, as can be seen easily by inspecting (5.12). Moreover, $q'_i = x_i$ or $r'_j = y_j$.

- **Properties and Theorem of Section 5.2:**

<u>**Equilibrium property 5.1:**</u> For region $i \in \{1,\ldots,n\}$:

a) In the optimal solution of (5.19), $\pi_i(y_i) \leq p_i(x_i)$.

b) If in the optimal solution of (5.19), $\pi_i(y_i) < p_i(x_i)$, then $x_i = 0$ or $y_i = 0$.

c) If in the optimal solution of (5.19), supply and demand in region i are both positive, so $x_i > 0$ and $y_i > 0$, then necessarily $p_i(x_i) = \pi_i(y_i)$.

<u>Proof:</u>

a) Since the solution of (5.19) has to satisfy $\pi_i(y_i) \leq \lambda_i$ and $\lambda_i \leq p_i(x_i)$, see (5.21) – (5.24), it is only possible that $\pi_i(y_i) \leq p_i(x_i)$.

b) Using (5.21) and (5.23), the proof is similar to the proof of *Trader property* 5.1 a).

c) This follows from a) and b).

<u>**Equilibrium property 5.2:**</u> In the optimal solution of (5.19), let transport take place from market i to market j, i.e. $x_{ij} > 0$, with $i, j \in \{1,2, ...,n\}$, $j \neq i$,, then:

a) no cereals are transferred from other regions into market i, i.e. $x_{si} = 0$, for all $s \neq i$

b) no cereals are transported from market j to other regions, i.e. $x_{js} = 0$, for all $s \neq j$

c) the producer supply x_i in region i satisfies $x_i > 0$,

d) the consumer demand y_j in region j satisfies $y_j > 0$,

<u>Proof</u>*:*

The proof is similar to the proof of *Trader property* 5.2.

<u>**Equilibrium property 5.3:**</u> For region i and j, $i,j \in \{1,\ldots,n\}$, $i \neq j$:

a) In the optimal solution of (5.19), $\pi_j(y_j) \leq p_i(x_i) + \tau_{ij}$.

b) If in the optimal solution of (5.19), $\pi_j(y_j) < p_i(x_i) + \tau_{ij}$, then $x_{ij} = 0$.

c) If in the optimal solution of (5.19), transport between region i and j is positive, $x_{ij} > 0$, then the optimal prices satisfy necessarily $\pi_j(y_j) = p_i(x_i) + \tau_{ij}$.

Proof:

a) Since the solution of (5.19) always has to satisfy $\pi_j(y_j) \leq \lambda_j$, $\lambda_j \leq \lambda_i + \tau_{ij}$ and $\lambda_i \leq p_i(x_i)$, see (5.21) – (5.26), it is only possible that $\pi_j(y_j) \leq p_i(x_i) + \tau_{ij}$.

b) Using (5.21), (5.23) and (5.25), the proof is similar to the proof of *Trader property* 5.3 a).

c) This follows from a) and b).

Theorem 5.1:

Let x_i, y_i, x_{ij}, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of the equilibrium model (5.19). Let $\pi_i = \pi_i(y_i)$, $p_i = p_i(x_i)$. The solution:

(5.27) $$q_i = x_i \;\; ; \;\; r_i = y_i \;\; ; \;\; q_{ij} = x_{ij} \qquad \text{for } i,j \in \{1,\ldots,n\},\ i \neq j$$

is an optimal solution of trader decision problem (5.12). The value of the objective function is equal to 0, meaning that the trader makes no profits or losses.

Proof:

The proof will consist of three parts: a) (5.27) is a feasible solution of (5.12), b) the solution (5.27) results in a value of the objective function of (5.12) which is equal to 0; c) the solution (5.27) is an optimal solution of (5.12).

a) See (5.27), (5.16), (5.7), and (5.12).

b) Due to (5.21) – (5.26), $\pi_i \cdot y_i = \lambda_i \cdot y_i$, $p_i \cdot x_i = \lambda_i \cdot x_i$, and $\tau_{ij} \cdot x_{ij} = (\lambda_j - \lambda_i) \cdot x_{ij}$, with λ_i the optimal value of the Lagrange multiplier of the equilibrium condition in model (5.16) – (5.21). So, for the solution (5.27) the objective function of model (5.12) can be written as follows:

$$\sum_{i=1}^{n}\left[\lambda_i y_i - \lambda_i x_i - \sum_{\substack{j=1\\ j\neq i}}^{n}\left(\lambda_j - \lambda_i\right)x_{ij}\right] = \sum_{i=1}^{n}\left[\lambda_i\left(y_i - x_i - \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ji} + \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}\right)\right] = 0$$

due to the equilibrium condition (5.7).

c) Consider any feasible solution q_i, r_i, q_{ij} of (5.12), and market prices $\pi_i = \pi_i(y_i)$ and $p_i = p_i(x_i)$, with y_i and x_i the optimal demanded and supplied quantities of equilibrium model (5.19), (5.16), and (5.17). Due to *Equilibrium properties* 5.1 – 5.3 $\pi_i \leq \lambda_i \leq p_i$ and $\pi_j \leq \lambda_j \leq \lambda_i + \tau_{ij} \leq p_i + \tau_{ij}$, for all $i,j = 1,\ldots,n$, $i \neq j$, with λ_i the Lagrange multiplier of the equilibrium condition in model (5.19), (5.16), and (5.17). The objective function of model (5.12) can be written as:

$$\sum_{i=1}^{n}\left[\pi_i r_i - p_i q_i - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ij} q_{ij}\right] \leq \sum_{i=1}^{n}\left[\lambda_i r_i - \lambda_i q_i - \sum_{\substack{j=1\\ j\neq i}}^{n}\left(\lambda_j - \lambda_i\right)q_{ij}\right]$$

$$= \sum_{i=1}^{n}\left[\lambda_i\left(r_i - q_i - \sum_{\substack{j=1\\ j\neq i}}^{n} q_{ji} + \sum_{\substack{j=1\\ j\neq i}}^{n} q_{ij}\right)\right] = 0$$

So, for market prices $\pi_i = \pi_i(y_i)$ and $p_i = p_i(x_i)$, the objective function always has a value ≤ 0. The objective function of model (5.12) reaches a maximum for the solution (5.27). Consequently, (5.27) is an optimal solution of (5.12).

- **Properties of Section 6.1:**

<u>Trader property 6.1:</u> For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:

a) If $\pi_{it} < p_{it}$, then any optimal solution of (6.11) satisfies $q_{it} = 0$ or $r_{it} = 0$

b) If $\pi_{it} \geq p_{it}$, then an optimal solution of (6.11) exists satisfying the condition $q_{it} = x_{it}$ or $r_{it} = y_{it}$; for $\pi_{it} = p_{it}$, other optimal solutions of (6.11) may exist, not satisfying this condition.

Proof: Using (6.11), the proof is similar to the proof of *Trader property* 5.1 in Section 5.1.

Trader property 6.2: Let q_{it}, r_{jt}, q_{ijt}, v_{it}, $j \neq i$, $i,j = 1,\ldots,n$, $t = 1,\ldots,T$, be an optimal solution of (6.11). Let a trader transport in a period t from a region i to a region j, so $q_{ijt} > 0$, for $i,j \in \{1,\ldots,n\}$, $j \neq i$, $t \in \{1,\ldots,T\}$, then:

a) in period t, no goods are transported from a region $s = 1,\ldots,n$, $s \neq i$, to region i, $q_{sit} = 0$

b) in period t, no goods are transported from region j to a region $s = 1,\ldots,n$, $s \neq j$, $q_{jst} = 0$.

c) in period t, purchases in region i are positive, $q_{it} > 0$, or the stock remaining from the previous period is positive, $v_{i,t-1} > 0$.

d) in period t, sales in region j are positive, $r_{jt} > 0$, or the stock at the end of period t in region j is positive, $v_{jt} > 0$.

Proof: Using (6.9) and (6.11), the proof is similar to the proof of *Trader property* 5.2 in Section 5.1.

Trader property 6.3: For region $i,j \in \{1,\ldots,n\}$, $j \neq i$, and period $t \in \{1,\ldots,T\}$:

a) If $\pi_{jt} < p_{it} + \tau_{ijt}$, then any optimal solution of (6.11) has to satisfy $q_{it} = 0$ or $q_{ijt} = 0$ or $r_{jt} = 0$.

b) If $\pi_{jt} \geq p_{it} + \tau_{ijt}$, and $q_{it} > 0$, $q_{ijt} > 0$ and $r_{jt} > 0$, then an optimal solution of (6.11) exists satisfying $q_{it} = x_{it}$ or $r_{jt} = y_{jt}$; for $\pi_{jt} = p_{it} + \tau_{ijt}$, an optimal solution of (6.11) is not unique.

Proof:

a) Let $\pi_{jt} < p_{it} + \tau_{ijt}$. If $q_{it} > 0$, $q_{ijt} > 0$ and $r_{jt} > 0$, then, see (6.11), $\lambda_{it} \geq p_{it}$, $\lambda_{jt} = \lambda_{it} + \tau_{ijt}$ and $\lambda_{jt} \leq \pi_{jt}$, which contradicts $\pi_{jt} < p_{it} + \tau_{ijt}$. So necessarily, $q_{it} = 0$, $q_{ijt} = 0$ or $r_{jt} = 0$.

b) An optimal solution of problem (6.11) exists, since its feasible region is bounded. We make a distinction between $\pi_{jt} > p_{it} + \tau_{ijt}$ and $\pi_{jt} = p_{it} + \tau_{ijt}$:

* Let $\pi_{jt} > p_{it} + \tau_{ijt}$ and $q_{it} > 0$, $q_{ijt} > 0$ and $r_{jt} > 0$. If the optimal solution satisfies $q_{it} < x_{it}$ and $r_{jt} < y_{jt}$, then, see (6.11), $\lambda_{it} = p_{it}$, $\lambda_{jt} = \lambda_{it} + \tau_{ijt}$ and $\lambda_{jt} = \pi_{jt}$. It would follow that $\pi_{jt} = p_{it} + \tau_{ijt}$, which is in contradiction with $\pi_{jt} > p_{it} + \tau_{ijt}$. So necessarily, $q_{it} = x_{it}$ or $r_{jt} = y_{jt}$.

* Let $\pi_{jt} = p_{it} + \tau_{ijt}$. Consider the optimal solution $q_{s\tau}$, $q_{sv\tau}$, $r_{v\tau}$, for $s,v = 1,\ldots,n$, and $\tau = 1,\ldots,T$, in which purchases, transports and sales for the regions i and j, and for the period t satisfy $0 < q_{it} < x_{it}$, $q_{ijt} > 0$ and $0 < r_{jt} < y_{jt}$. Call $\Delta_{ijt} = \min(x_{it}-q_{it}; y_{jt}-r_{jt})$. The solution $q'_{s\tau}, r'_{v\tau}, q'_{sv\tau}$ defined by $q'_{it} = q_{it} + \Delta_{ijt}$, $r'_{jt} = r_{jt} + \Delta_{ijt}$, $q'_{ijt} = q_{ijt} + \Delta_{ijt}$, $q'_{s\tau} = q_{s\tau}$, $r'_{v\tau} = r_{v\tau}$, $q'_{sv\tau} = q_{sv\tau}$, for $s \neq i$, $v \neq j$ and $\tau \neq t$, is optimal as well, as can be seen easily by inspecting (6.10). Moreover, $q'_{it} = x_{it}$ or $r'_{jt} = y_{jt}$.

Trader property 6.4: For region $i \in \{1,\ldots,n\}$, $j \neq i$, period $t \in \{1,\ldots,T\text{-}1\}$:

a) If $\pi_{i,t+1} < p_{it} + k_{it}$, then any optimal solution of (6.11) has to satisfy $q_{it} = 0$ or $v_{it} = 0$ or $r_{i,t+1} = 0$.

b) Analogously for $\tau \in \{t+1,\ldots,T\}$: if $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$, then any optimal solution of (6.11) has to satisfy $q_{it} = 0$ or $v_{it} = 0$, or $\ldots$, or $v_{i,\tau-1} = 0$, or $r_{i\tau} = 0$.

c) If $\pi_{i,t+1} \geq p_{it} + k_{it}$, and $q_{it} > 0$, $v_{it} > 0$ and $r_{i,t+1} > 0$, then an optimal solution of (6.11) exists which satisfies the condition $q_{it} = x_{it}$ or $r_{i,t+1} = y_{i,t+1}$. Nota bene: for $\pi_{i,t+1} > p_{it} + k_{it}$, any optimal solution of (6.11) has to satisfy this condition; for $\pi_{i,t+1} = p_{it} + k_{it}$, an optimal solution is not unique.

d) Analogously for $\tau \in \{t+1,\ldots,T\}$: if $\pi_{i\tau} \geq p_{it} + \kappa_{it\tau}$, and $q_{it} > 0$, $v_{it} > 0,\ldots,$ $v_{i,\tau-1} > 0$ and $r_{i\tau} > 0$, then an optimal solution of (6.11) exists satisfying the condition $q_{it} = x_{it}$ or $r_{i\tau} = y_{i\tau}$. For $\pi_{i\tau} = p_{it} + \kappa_{it\tau}$, an optimal solution of (6.11) is not unique.

Proof:

a) Let $\pi_{i,t+1} < p_{it} + k_{it}$. If $q_{it} > 0$, $v_{it} > 0$ and $r_{i,t+1} > 0$, then, see (6.11), $\lambda_{it} \geq p_{it}$, $\lambda_{i,t+1} = \lambda_{it} + k_{it}$ and $\lambda_{i,t+1} \leq \pi_{i,t+1}$, which contradicts $\pi_{i,t+1} < p_{it} + k_{it}$. So necessarily, $q_{it} = 0$, $v_{it} = 0$ or $r_{i,t+1} = 0$.

b) Let $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$ for $\tau \in \{t+1,\ldots,T\}$. If $q_{it} > 0$, $v_{it} > 0,\ldots, v_{i,\tau-1} > 0$ and $r_{i\tau} > 0$, then, see (6.11), $\lambda_{it} \geq p_{it}$, $\lambda_{i,t+1} = \lambda_{it} + k_{it}, \ldots, \lambda_{i\tau} = \lambda_{i,\tau-1} + k_{i,\tau-1}$ and $\lambda_{i\tau} \leq \pi_{i\tau}$, which contradicts $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$ - see (6.12). So necessarily, $q_{it} = 0$ or $v_{it} = 0,\ldots$ or $v_{i,\tau-1} = 0$ or $r_{i\tau} = 0$.

c) An optimal solution of problem (6.11) exists, since its feasible region is bounded. For this proof we make a distinction between $\pi_{i,t+1} > p_{it} + k_{it}$ and $\pi_{i,t+1} = p_{it} + k_{it}$:

* Let $\pi_{i,t+1} > p_{it} + k_{it}$ and $q_{it} > 0$, $v_{it} > 0$ and $r_{i,t+1} > 0$. If the optimal solution satisfies $q_{it} < x_{it}$ and $r_{i,t+1} < y_{i,t+1}$, then, see (6.11), $\lambda_{it} = p_{it}$, $\lambda_{i,t+1} = \lambda_{it} + k_{it}$ and $\lambda_{i,t+1} = \pi_{i,t+1}$. It would follow that $\pi_{i,t+1} = p_{it} + k_{it}$, which is in contradiction with $\pi_{i,t+1} > p_{it} + k_{it}$. So necessarily, $q_{it} = x_{it}$ or $r_{i,t+1} = y_{i,t+1}$.

* Let $\pi_{i,t+1} = p_{it} + k_{it}$. Consider the optimal solution $q_{s\tau}$, $v_{s\tau}$, $r_{s\tau}$ for $s = 1,\ldots,n$ and $\tau = 1,\ldots,T$, for which in region i and period t and t+1: $0 < q_{it} < x_{it}$, $v_{it} > 0$ and $0 < r_{i,t+1} < y_{i,t+1}$. Call $\Delta_{it} = \min(x_{it}\text{-}q_{it};\, y_{i,t+1}\text{-}r_{i,t+1})$. The solution $q'_{s\tau}, r'_{s\tau}, v'_{s\tau}$ defined by $q'_{it} = q_{it} + \Delta_{it}$, $r'_{i,t+1} = r_{i,t+1} + \Delta_{it}$, $v'_{it} = v_{it} + \Delta_{it}$, $q'_{s\tau} = q_{s\tau}$, $r'_{s\zeta} = r_{s\zeta}$, $v'_{s\tau} = v_{s\tau}$, for $s \neq i$, $\tau \neq t$, and $\zeta \neq t+1$, is optimal as well, as can be seen easily by inspecting (6.10). Moreover, $q'_{it} = x_{it}$ or $r'_{i,t+1} = y_{i,t+1}$.

d) The proof of this property is similar to the proof under c). The main difference is that, due to $v_{it} > 0,\ldots, v_{i,\tau-1} > 0$, $\lambda_{i\tau} = \lambda_{it} + \kappa_{it\tau}$, see (6.11) and (6.12).

- **Properties and Theorem of Section 6.2:**

Equilibrium property 6.1: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:

a) In the optimal solution of (6.14) – (6.16) $\pi_{it}(y_{it}) \leq p_{it}(x_{it})$.

b) If in the optimal solution of (6.14) – (6.16) $\pi_{it}(y_{it}) < p_{it}(x_{it})$, then $x_{it} = 0$ or $y_{it} = 0$.

c) If in the optimal solution of (6.14) – (6.16), supply and demand are both positive, $x_{it} > 0$ and $y_{it} > 0$, then the prices necessarily satisfy $p_{it}(x_{it}) = \pi_{it}(y_{it})$.

Proof:
Using (6.17) – (6.24), the proof is similar to the proof of *Equilibrium property* 5.1 in Section 5.2.

Equilibrium property 6.2: Let in the optimal solution of (6.14) – (6.16) transport in period t take place from a market i to a market j, so $x_{ijt} > 0$, with $i,j \in \{1,2, ...,n\}$, $j \neq i$, $t \in \{1,\ldots,T\}$, then:
a) in period t, no cereals are transferred from other regions into market i, i.e. $x_{sit} = 0$, for all $s \neq i$
b) in period t, no cereals are transported from market j to other regions, i.e. $x_{jst} = 0$, for all $s \neq j$
c) in period t, the producer supply in region i in period t satisfies, $x_{it} > 0$, or the stock remaining from the previous period is positive, $s_{i,t-1} > 0$.
d) in period t, the consumer demand in region j in period t satisfies $y_{jt} > 0$, or the quantity put in stock in region j is positive, $s_{jt} > 0$.

Proof*:*
The proof is similar to the proof of *Equilibrium property* 5.2 in Section 5.2.

Equilibrium property 6.3: For region i and j, $i,j \in \{1,\ldots,n\}$, $i \neq j$, and period $t \in \{1,\ldots,T\}$:
a) In the solution of (6.14) – (6.16) $\pi_{jt}(y_{jt}) \leq p_{it}(x_{it}) + \tau_{ijt}$.
b) If in the optimal solution of (6.14) – (6.16) $\pi_{jt}(y_{jt}) < p_{it}(x_{it}) + \tau_{ijt}$, then $x_{it} = 0$ or $x_{ijt} = 0$ or $y_{jt} = 0$.
c) If in the optimal solution of (6.14) – (6.16) supplies in region i, transport between region i and j, and demand in region j are positive, $x_{it} > 0$ and $x_{ijt} > 0$ and $y_{jt} > 0$, then the optimal prices necessarily satisfy $\pi_{jt}(y_{jt}) = p_{it}(x_{it}) + \tau_{ijt}$.

Proof:

Using (6.17) – (6.2), the proof is similar to the proof of *Equilibrium property* 5.3 in Section 5.2.

Equilibrium property 6.4: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\text{-}1\}$:

a) In the optimal solution of (6.14) – (6.16) $\pi_{i,t+1}(y_{i,t+1}) \leq p_{it}(x_{it}) + k_{it}$. Analogously, for $\tau \in \{t+1,\ldots,T\}$: $\pi_{i\tau}(y_{i\tau}) \leq p_{it}(x_{it}) + \kappa_{it\tau}$.

b) If in the optimal solution of (6.14) – (6.16) $\pi_{i,t+1}(y_{i,t+1}) < p_{it}(x_{it}) + k_{it}$, then $x_{it} = 0$ or $s_{it} = 0$ or $y_{i,t+1} = 0$. Analogously, for $\tau \in \{t+1,\ldots,T\}$ – see also (6.12): if $\pi_{i\tau} < p_{it} + \kappa_{it\tau}$, then any optimal solution of (6.14) – (6.16) has to satisfy $x_{it} = 0$ or $s_{it} = 0$ or $s_{i,t+1} = 0$ … or $s_{i,\tau\text{-}1} = 0$ or $y_{i\tau} = 0$.

c) If in the optimal solution of (6.14) – (6.16) supplies in period t, stock levels at the end of period t, and demand in period t+1 are positive, $x_{it} > 0$ and $s_{it} > 0$ and $y_{i,t+1} > 0$, then the optimal prices necessarily satisfy $\pi_{i,t+1}(y_{i,t+1}) = p_{it}(x_{it}) + k_{it}$.

d) If in the optimal solution of (6.14) – (6.16), supplies in period t, storage from period t to the end of period τ-1, and demand in period τ are positive, $x_{it} > 0$, $s_{it} > 0$, $s_{i,t+1} > 0,\ldots,$ $s_{i,\tau\text{-}1} > 0$ and $y_{i\tau} > 0$, then the optimal prices satisfy $\pi_{i\tau}(y_{i\tau}) = p_{it}(x_{it}) + \kappa_{it\tau}$, for $\tau \in \{t+1,\ldots,T\}$.

Proof:

a) Since the solution always has to satisfy $\pi_{i,t+1}(y_{i,t+1}) \leq \lambda_{i,t+1}$, $\lambda_{i,t+1} \leq \lambda_{it} + k_{it}$, and $\lambda_{it} \leq p_{it}(x_{it})$ – see (6.17) – (6.24), $\pi_{i,t+1}(y_{i,t+1}) \leq p_{it}(x_{it}) + k_{it}$. Analogously, for $\tau \in \{t+1,\ldots,T\}$, the solution has to satisfy $\pi_{i\tau}(y_{i\tau}) \leq \lambda_{i\tau}$, $\lambda_{i\tau} \leq \lambda_{i,\tau-1} + k_{i\tau},\ldots,$ $\lambda_{i,t+1} \leq \lambda_{it} + k_{it}$, and $\lambda_{it} \leq p_{it}(x_{it})$. Consequently, $\pi_{i\tau} \leq p_{it} + \kappa_{it\tau}$

b) Using (6.17), (6.19) and (6.23), the proof is similar to the proof of *Trader property* 6.4 a) and b) in Section 6.1.

c) This follows from a) and b).

d) This follows from a) and b) for $\tau \in \{t+1,\ldots,T\}$.

Theorem 6.1:

Let x_{it}, y_{it}, x_{ijt}, s_{it}, $i,j \in \{1,\ldots,n\}$, $i \neq j$, $t \in \{1,\ldots,T\}$, be an optimal solution of the equilibrium model (6.14) – (6.16). Let $\pi_{it} = \pi_{it}(y_{it})$, $p_{it} = p_{it}(x_{it})$. The solution:

$$q_{it} = x_{it}\ ;\ r_{it} = y_{it}\ ;\ q_{ijt} = x_{ijt}\ ;\ v_{it} = s_{it} \qquad \text{for } i,j \in \{1,\ldots,n\},\ i \neq j,\ t \in \{1,\ldots,T\}$$

is an optimal solution of trader decision problem (6.10). The value of the objective function is equal to 0, meaning that the trader makes no profits or losses.

Proof:

The proof will consist of three parts: a) (6.25) is a feasible solution of (6.10), b) the solution (6.25) results in a value of the objective function of (6.10) which is equal to 0; c) the solution (6.25) is an optimal solution of (6.10).

a) See (6.25), (6.15), (6.10), and (6.9).

b) Due to (6.17) - (6.24), the objective function of model (6.10) for the solution (6.25) can be written as follows, with λ_{it} the optimal value of the Lagrange multiplier of model (6.14) – (6.16):

$$\sum_{t=1}^{T}\sum_{i=1}^{n}\left[\lambda_{it}y_{it} - \lambda_{it}x_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}\left(\lambda_{jt} - \lambda_{it}\right)x_{ijt} - \left(\lambda_{i,t+1} - \lambda_{it}\right)s_{it}\right] =$$

$$\sum_{t=1}^{T}\sum_{i=1}^{n}\left[\lambda_{it}\left(y_{it} - x_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}x_{jit} + \sum_{\substack{j=1\\ j\neq i}}^{n}x_{ijt} - s_{i,t-1} + s_{it}\right)\right] = 0$$

due to the equilibrium condition (6.15).

c) Consider any feasible solution q_{it}, r_{it}, q_{ijt}, s_{it} of (6.10), and market prices $\pi_{it} = \pi_{it}(y_{it})$ and $p_{it} = p_{it}(x_{it})$, with y_{it} and x_{it} the optimal demanded and supplied quantities of equilibrium model (6.14) – (6.16). Due to *Equilibrium property* 6.1 – 6.4 and (6.17) – (6.24) $\pi_{it} \leq \lambda_{it} \leq p_{it}$ and $\pi_{jt} \leq \lambda_{jt} \leq \lambda_{it} + \tau_{ijt} \leq p_{it} + \tau_{ijt}$ and $\pi_{i,t+1}$

$\leq \lambda_{i,t+1} \leq \lambda_{it} + k_{it} \leq p_{it} + k_{it}$, for all $i = 1,\ldots,n$, $i \neq j$, $t = 1,\ldots,T$, with λ_{it} the Lagrange multiplier of model (6.14) – (6.16). The objective function of model (6.10) can be written as:

$$\sum_{t=1}^{T}\sum_{i=1}^{n}\left[\pi_{it}r_{it} - p_{it}q_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ijt}q_{ijt} - k_{it}v_{it}\right]$$

$$\leq \sum_{t=1}^{T}\sum_{i=1}^{n}\left[\lambda_{it}r_{it} - \lambda_{it}q_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}\left(\lambda_{jt} - \lambda_{it}\right)q_{ijt} - \left(\lambda_{i,t+1} - \lambda_{it}\right)v_{it}\right]$$

$$= \sum_{t=1}^{T}\sum_{i=1}^{n}\left[\lambda_{it}\left(r_{it} - q_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}q_{jit} + \sum_{\substack{j=1\\ j\neq i}}^{n}q_{ijt} - v_{i,t-1} + v_{it}\right)\right] = 0$$

So, the objective function is always ≤ 0 for market prices $\pi_{it} = \pi_{it}(y_{it})$ and $p_{it} = p_{it}(x_{it})$. The objective function of model (6.10) reaches a maximum for the solution (6.25), see b). Consequently, (6.25) is an optimal solution of (6.10).

- **Properties of Section 7.1:**

<u>Trader property 7.1:</u> For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:

a) If $\pi_{it} < p_{it}$, then any optimal solution of (7.27) satisfies $q_{it} = 0$ or $r_{it} = 0$.

b) If $\pi_{it} \geq p_{it}$, then an optimal solution of (7.27) exists, satisfying the condition $q_{it} = x_{it}$ or $r_{it} = y_{it}$. For $\pi_{it} = p_{it}$, other optimal solutions of (7.27) may exist, not satisfying this condition.

<u>Proof:</u> Using (7.29) and (7.30), the proof is similar to the proof of *Trader property* 5.1 in Section 5.1 and *Trader property* 6.1 in Section 6.1.

Trader property 7.2: Let q_{it}, r_{jt}, q_{ijt}, v_{it}, $j \neq i$, $i,j = 1,\ldots,n$, $t = 1,\ldots,T$, be an optimal solution of (7.27). Let a trader transport in a period t from region i to j, so $q_{ijt} > 0$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, $t \in \{1,\ldots,T\}$, then:

a) no goods are transported from a region $s = 1,\ldots,n, s \neq i$, to region i, $q_{sit} = 0$

b) no goods are transported from region j to a region $s = 1,\ldots,n, ,s \neq j$, $q_{jst} = 0$.

c) purchases in region i are positive, $q_{it} > 0$, or the stock remaining from the previous period is positive, $v_{i,t-1} > 0$.

d) sales in region j are positive, $r_{jt} > 0$, or the quantity put in stock in region j is positive, $v_{jt} > 0$.

Proof: Using (7.29) - (7.31), the proof is similar to the proof of *Trader property* 5.2 in Section 5.1 and *Trader property* 6.2 in Section 6.1.

Trader property 7.3: For region $i,j \in \{1,\ldots,n\}$, $i \neq j$, and period $t \in \{1,\ldots,T\}$:

a) If $\pi_{jt} < p_{it} + \tau_{ijt}$, then any optimal solution of (7.27) has to satisfy $q_{it} = 0$ or $q_{ijt} = 0$ or $r_{jt} = 0$.

b) If $\pi_{jt} \geq p_{it} + \tau_{ijt}$, and $q_{it} > 0$, $q_{ijt} > 0$ and $r_{jt} > 0$, then an optimal solution of (7.27) exists satisfying $q_{it} = x_{it}$ or $r_{jt} = y_{it}$. For $\pi_{jt} = p_{it} + \tau_{ijt}$, an optimal solution of (7.27) is not unique.

Proof: Using (7.29) - (7.31), the proof is similar to the proof of *Trader property* 6.3 in Section 6.1.

Trader property 7.4: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\text{-}1\}$:

a) If $E\pi_{i,t+1} < p_{it} + k_{it}$, then any optimal solution of (7.27) has to satisfy $q_{it} = 0$ or $v_{it} = 0$ or $r^k_{i,t+1} = 0$ for at least one $k \in \{1,...,K\}$.

b) Analogously, if $E\pi_{i,t+2} < p_{it} + \kappa_{it,t+2}$, see also (6.12), then any optimal solution of (7.27) has to satisfy $q_{it} = 0$ or $v_{it} = 0$ or $v^k_{i,t+1} = 0$ or $r^{k,l}_{i,t+2} = 0$ for at least one k, $l \in \{1,...,K\}$. Analogous properties can be derived for storage until the periods $\tau = t+3,...,T$ if $E\pi_{i\tau} < p_{it} + \kappa_{it\tau}$.

c) If $E\pi_{i,t+1} \geq p_{it} + k_{it}$ and $q_{it} > 0$ and $s_{it} > 0$ and $r^k_{i,t+1} > 0$ for all $k = 1,\ldots,K$, then an optimal solution of (7.27) exists satisfying $q_{it} = x_{it}$ or $r^k_{i,t+1} = y^k_{i,t+1}$ for at least one $k \in \{1,...,K\}$. For $E\pi_{i,t+1} = p_{it} + k_{it}$, an optimal solution of (7.27) is not unique.
d) Analogously: if $E\pi_{i,t+2} \geq p_{it} + \kappa_{it,t+2}$ and $q_{it} > 0$ and $v_{it} > 0$, $v^k_{i,t+1} > 0$ and $r^{k,l}_{i,t+2} > 0$, for all k, $l \in \{1,\ldots,K\}$, then a solution of (7.27) which satisfies $q_{it} = x_{it}$ or $r^{k,l}_{i,t+2} = y^{k,l}_{i,t+2}$, for at least one k, $l \in \{1,...,K\}$, is an optimal solution. For $E\pi_{i,t+2} = p_{it} + \kappa_{it,t+2}$, an optimal solution of (7.27) is not unique. Analogous properties can be derived for storage until the periods $\tau = t+3,...,T$ if $E\pi_{i\tau} \geq p_{it} + \kappa_{it\tau}$.

Proof:
a) Let $E\pi_{i,t+1} < p_{it} + k_{it}$. If $q_{it} > 0$, $v_{it} > 0$ and $r^k_{i,t+1} > 0$ for all $k \in \{1,...,K\}$, then, see (7.29), (7.30) and (7.32), $\lambda_{it} \geq p_{it}$, $\sum_{k=1}^K \lambda^k_{i,t+1} = \lambda_{it} + k_{it}$ and $\lambda^k_{i,t+1} \leq g^k_{t+1}\pi^k_{i,t+1}$, which contradicts, see (7.25), $E\pi_{i,t+1} < p_{it} + k_{it}$. So necessarily, $q_{it} = 0$, $v_{it} = 0$ or $r^k_{i,t+1} = 0$, for at least one $k \in \{1,...,K\}$.
b) Let $E\pi_{i,t+2} < p_{it} + \kappa_{it,t+2}$. If $q_{it} > 0$, $v_{it} > 0$, $v^k_{i,t+1} > 0$ and $r^{k,l}_{i,t+2} > 0$ for all k, $l \in \{1,...,K\}$, then, see (7.29), (7.30) and (7.32), $\lambda_{it} \geq p_{it}$, $\sum_{k=1}^K \lambda^k_{i,t+1} = \lambda_{it} + k_{it}$, $\sum_{l=1}^K \lambda^{k,l}_{i,t+2} = \lambda^k_{i,t+1} + g^k_{t+1}k_{i,t+1}$ and $\lambda^{k,l}_{i,t+2} \leq g^k_{t+1}g^l_{t+2}\pi^l_{i,t+2}$, which contradicts, see (7.25), $E\pi_{i,t+2} < p_{it} + \kappa_{i,t,t+2}$. So necessarily, $q_{it} = 0$, $v_{it} = 0$, $v^k_{i,t+1} = 0$ or $r^{k,l}_{i,t+2} = 0$ for at least one k, $l \in \{1,...,K\}$.
c) An optimal solution of problem (7.27) exists, since its feasible region is bounded. For this proof we make a distinction between $E\pi_{i,t+1} > p_{it} + k_{it}$ and $E\pi_{i,t+1} = p_{it} + k_{it}$:
* Let $E\pi_{i,t+1} > p_{it} + k_{it}$ and $q_{it} > 0$, $v_{it} > 0$ and $r^k_{i,t+1} > 0$ for all $k \in \{1,\ldots,K\}$. If the optimal solution satisfies $q_{it} < x_{it}$ and $r^k_{i,t+1} < y^k_{i,t+1}$ for all $k \in \{1,\ldots,K\}$, then, see (7.29), (7.30) and (7.32), $\lambda_{it} = p_{it}$, $\sum_{k=1}^K \lambda^k_{i,t+1} = \lambda_{it} + k_{it}$ and $\lambda^k_{i,t+1} = g^k_{t+1}\pi^k_{i,t+1}$. It

would follow that $E\pi_{i,t+1} = p_{it} + k_{it}$, which is in contradiction with $E\pi_{i,t+1} > p_{it} + k_{it}$. So necessarily, $q_{it} = x_{it}$ or $r^k_{i,t+1} = y^k_{i,t+1}$ for at least one $k \in \{1,\ldots,K\}$.

* Let $E\pi_{i,t+1} = p_{it} + k_{it}$. Consider the optimal solution q_{st}, v_{st}, $r^k_{s,t+1}$, $s = 1,\ldots,n$, with purchases, sales and storage for region i satisfying $0 < q_{it} < x_{it}$, $v_{it} > 0$ and $0 < r^k_{i,t+1} < y^k_{i,t+1}$, $k = 1,\ldots,K$. Call $\Delta_{it} = \min(x_{it}\text{-}q_{it};\ y^1_{i,t+1} - r^1_{i,t+1};\ldots;y^K_{i,t+1} - r^K_{i,t+1})$. The solution $q'_{st}, r'^k_{s,t+1}, v'_{st}$ defined by $q'_{it} = q_{it} + \Delta_{it}$, $r'^k_{i,t+1} = r^k_{i,t+1} + \Delta_{it}$, $v'_{it} = v_{it} + \Delta_{it}$, $q'_{st} = q_{st}$, $r'^k_{s,t+1} = r^k_{s,t+1}$, $v'_{st} = v_{st}$, for $s \neq i$, is optimal as well, as can be seen easily by inspecting (7.27) and (7.28). Moreover, $q'_{it} = x_{it}$ or $r'^k_{i,t+1} = y^k_{i,t+1}$ for at least one $k \in \{1,\ldots,K\}$.

d) The proof of this property is analogous to the proof under c).

- **Properties and Theorem of Section 7.2:**

Optimal equilibrium prices and quantities for period *T* – see model (7.44)

<u>Theorem 7.1a:</u>

Let in the optimal solution of the equilibrium model (7.44) for period T, $\hat{x}_{iT}$ be the optimal supply level and λ_{iT} be the corresponding optimal value of the Lagrange multiplier, for $i = 1,\ldots,n$. If the producer price in period T in region i is equal to:

(7.48) $p_{iT} = \lambda_{iT}$

then $x_{iT} = \hat{x}_{iT}$ is an optimal solution of model (7.10), the producer supply model for period T. In other words, the optimal equilibrium supply level is a supply level which gives the producers optimal profits in period T. Since the value of λ_{iT}, depends on the value of the equilibrium supply level, we write $p_{iT}(x_{iT}) = \lambda_{iT}$.

Proof:

It is trivial that $\hat{x}_{iT}$ is a feasible solution of (7.10) for $p_{iT}(\hat{x}_{iT}) = \lambda_{iT}$. After all, it follows from (7.44) that $0 \leq \hat{x}_{iT} \leq w_{i,T\text{-}1}$. To prove that $\hat{x}_{iT}$ is an optimal solution of (7.10), consider the following cases:

- if $\hat{x}_{iT} = 0$, then $p_{iT}(\hat{x}_{iT}) \leq c_{iT}$, see (7.45) and (7.46). For $p_{iT} \leq c_{iT}$ the producer supply level $x_{iT} = 0$ is an optimal solution of model (7.10) – see (7.11).
- if $0 < \hat{x}_{iT} < w_{i,T\text{-}1}$, then $p_{iT}(\hat{x}_{iT}) = c_{iT}$, see (7.45) and (7.46). For $p_{iT} = c_{iT}$ the optimal producer supply level determined by model (7.10) is not unique. Also supplying a level $x_{iT} = \hat{x}_{iT}$ will be optimal for the producer – see (7.11).

if $\hat{x}_{iT} = w_{i,T\text{-}1}$, then $p_{iT}(\hat{x}_{iT}) \geq c_{iT}$, see (7.45) and (7.46). For $p_{iT} \geq c_{iT}$ the producer supply level $x_{iT} = w_{i,T\text{-}1}$ is an optimal solution of model (7.10) – see (7.11).

Theorem 7.2a:

Let x_{iT}, y_{iT}, x_{ijT}, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of equilibrium model (7.44) for period T. Let $\pi_{iT} = \pi_{iT}(y_{iT})$, $p_{iT} = p_{iT}(x_{iT}) = \lambda_{iT}$. The solution:

(7.49) $\quad q_{iT} = x_{iT}\ ;\ r_{iT} = y_{iT}\ ;\ q_{ijT} = x_{ijT} \qquad$ for $i,j \in \{1,\ldots,n\}$, $i \neq j$

is an optimal solution of the trader decision problem (7.27) for period T.

Proof:

The proof will consist of three parts: a) (7.49) is a feasible solution of (7.27), b) solution (7.49) results in a certain value of the objective function of (7.27); c) the solution (7.49) is an optimal solution of (7.27), because any feasible solution of (7.27) has an objective value less than the objective value for solution (7.49).

a) See (7.49), (7.44), and (7.27).
b) Due to (7.45) - (7.47) and (7.48), the objective function of model (7.27) for the solution (7.49) can be written as, with λ_{iT} the optimal value of the Lagrange multiplier of model (7.44):

$$\sum_{i=1}^{n}\left[\pi_{iT}r_{iT}-p_{iT}q_{iT}-\sum_{\substack{j=1\\j\neq i}}^{n}\tau_{ijT}q_{ijT}\right]=\sum_{i=1}^{n}\left[\lambda_{iT}y_{iT}-\lambda_{iT}x_{iT}-\sum_{\substack{j=1\\j\neq i}}^{n}\left(\lambda_{jT}-\lambda_{iT}\right)x_{ijT}\right]$$

$$=\sum_{i=1}^{n}\lambda_{iT}\left(y_{iT}-x_{iT}-\sum_{\substack{j=1\\j\neq i}}^{n}x_{jiT}+\sum_{\substack{j=1\\j\neq i}}^{n}x_{ijT}\right)=\sum_{i=1}^{n}\lambda_{iT}s_{i,T-1}$$

due to the equilibrium condition of (7.44).

c) Consider any feasible solution q_{iT}, r_{iT}, q_{ijT} of (7.27), and market prices $\pi_{iT} = \pi_{iT}(y_{iT})$ and $p_{iT} = p_{iT}(x_{iT})$, with y_{iT} and x_{iT} the optimal demanded and supplied quantities of equilibrium model (7.44). Due to (7.45) - (7.47) and (7.48) $\pi_{iT} \leq \lambda_{iT} = p_{iT}$ and $\lambda_{jT} \leq \lambda_{iT} + \tau_{ijT}$, for all $i = 1,\ldots,n$, $i \neq j$, with λ_{iT} the optimal value of the Lagrange multiplier of model (7.44). The objective function of model (7.27) can be written as:

$$\sum_{i=1}^{n}\left[\pi_{iT}r_{iT}-p_{iT}q_{iT}-\sum_{\substack{j=1\\j\neq i}}^{n}\tau_{ijT}q_{ijT}\right]\leq\sum_{i=1}^{n}\left[\lambda_{iT}r_{iT}-\lambda_{iT}q_{iT}-\sum_{\substack{j=1\\j\neq i}}^{n}\left(\lambda_{jT}-\lambda_{iT}\right)q_{ijT}\right]$$

$$=\sum_{i=1}^{n}\lambda_{iT}s_{i,T-1}$$

due to the equilibrium condition of (7.27). So, (7.49) is an optimal solution of model (7.27).

Optimal equilibrium prices and quantities for period *T*-1 – see model (7.50)

The Kuhn-Tucker conditions of model (7.50) result in the following expressions – see (3.34) and (3.38) – (3.40):

$$\text{(A1.1)}\quad \begin{cases} \text{if } x_{i,T-1} = 0 & \text{then } -c_{i,T-1} + \lambda_{i,T-1} - \sum_{k=1}^{K} \gamma_i^k \le 0 \\ \text{if } x_{i,T-1} \ge 0 & \text{then } -c_{i,T-1} + \lambda_{i,T-1} - \sum_{k=1}^{K} \gamma_i^k = 0 \end{cases}$$

$$\text{(A1.2)}\quad \begin{cases} \text{if } y_{i,T-1} = 0 & \text{then } \pi_{i,T-1}(0) - \lambda_{i,T-1} \le 0 \\ \text{if } y_{i,T-1} > 0 & \text{then } \pi_{i,T-1}(y_{i,T-1}) - \lambda_{i,T-1} = 0 \end{cases}$$

$$\text{(A1.3)}\quad \begin{cases} \text{if } x_{ij,T-1} = 0 & \text{then } -\tau_{ij,T-1} - \lambda_{i,T-1} + \lambda_{j,T-1} \le 0 \\ \text{if } x_{ij,T-1} > 0 & \text{then } -\tau_{ij,T-1} - \lambda_{i,T-1} + \lambda_{j,T-1} = 0 \end{cases}$$

$$\text{(A1.4)}\quad \begin{cases} \text{if } s_{i,T-1} = 0 & \text{then } -k_{i,T-1} - \lambda_{i,T-1} + \sum_{k=1}^{K} \lambda_{iT}^k \le 0 \\ \text{if } \mathrm{s}_{i,T-1} > 0 & \text{then } -k_{i,T-1} - \lambda_{i,T-1} + \sum_{k=1}^{K} \lambda_{iT}^k = 0 \end{cases}$$

$$\text{(A1.5)}\quad \begin{cases} \text{if } x_{iT}^k = 0 & \text{then } f_{iT}^k \cdot \left(p_{iT}^k - c_{iT}\right) - \gamma_i^k \le 0 \\ \text{if } x_{iT}^k > 0 & \text{then } f_{iT}^k \cdot \left(p_{iT}^k - c_{iT}\right) - \gamma_i^k = 0 \end{cases}$$

$$\text{(A1.6)}\quad \begin{cases} \text{if } r_{iT}^k = 0 & \text{then } g_T^k \cdot \pi_{iT}^k - \lambda_{iT}^k - \mu_{iT}^k \le 0 \\ \text{if } r_{iT}^k > 0 & \text{then } g_T^k \cdot \pi_{iT}^k - \lambda_{iT}^k - \mu_{iT}^k = 0 \end{cases}$$

$$\text{(A1.7)}\quad \begin{cases} \text{if } q_{iT}^k = 0 & \text{then } -g_T^k p_{iT}^k + \lambda_{iT}^k - \vartheta_{iT}^k \le 0 \\ \text{if } q_{iT}^k > 0 & \text{then } -g_T^k p_{iT}^k + \lambda_{iT}^k - \vartheta_{iT}^k = 0 \end{cases}$$

$$\text{(A1.8)}\quad \begin{cases} \text{if } q_{ijT}^k = 0 & \text{then } -g_t^k \tau_{ijT} + \lambda_{jT}^k - \lambda_{iT}^k \le 0 \\ \text{if } q_{ijT}^k > 0 & \text{then } -g_t^k \tau_{ijT} + \lambda_{jT}^k - \lambda_{iT}^k = 0 \end{cases}$$

$$\text{(A1.9)}\quad \begin{cases} \mu_{iT}^k \left(\bar{y}_{iT}^k - r_{iT}^k\right) = 0 \\ \vartheta_{iT}^k \left(\bar{x}_{iT}^k - q_{iT}^k\right) = 0 \\ \gamma_i^k \left(w_{i,T-2} - x_{i,T-1} - x_{iT}^k\right) = 0 \end{cases}$$

Using these conditions, the following theorem can be derived.

Theorem 7.1b:

Let in the optimal solution of the equilibrium model (7.50), $\hat{x}_{i,T-1}$ and $\hat{x}_{iT}^{k}$ be the optimal supply levels for period T-1 and T, respectively, and let $\lambda_{i,T\text{-}1}$, λ_{iT}^{k} and γ_{i}^{k} be the corresponding optimal values of the Lagrange multipliers, for i = 1,…,n and k = 1,…,K. If the producer price in period T-1 in region i is equal to:

(7.52) $p_{i,T\text{-}1} = \lambda_{i,T\text{-}1}$

then $x_{i,T\text{-}1} = \hat{x}_{i,T-1}$, and $x_{iT}^{k} = \hat{x}_{iT}^{k}$ are optimal solutions of model (7.14), the producer supply model for period T-1. In other words, the optimal equilibrium supply levels give the producers optimal profits in period T-1. Since the value of $\lambda_{i,T\text{-}1}$, depends on the value of the equilibrium supply level, we write $p_{i,T\text{-}1}(x_{i,T\text{-}1}) = \lambda_{i,T\text{-}1}$.

Proof:

It is trivial that the solution $\hat{x}_{i,T-1}$, $\hat{x}_{iT}^{k}$ is a feasible solution of (7.14) for $p_{i,T\text{-}1}(\hat{x}_{i,T-1}) = \lambda_{i,T\text{-}1}$. After all, it follows from (7.50) that $0 \leq \hat{x}_{i,T-1} + \hat{x}_{iT}^{k} \leq w_{i,T\text{-}2}$. To prove that $\hat{x}_{i,T-1}$ and $\hat{x}_{iT}^{k}$ are optimal solutions of (7.14), first determine the value of the objective function of (7.14) for the solution $\hat{x}_{i,T-1}$ and $\hat{x}_{iT}^{k}$, and then show that this value is optimal:

- Due to (A1.1), (A1.5), (A1.9) and (7.52), the objective function of (7.14) can be written as:

  $$\left(p_{i,T-1} - c_{i,T-1}\right)\hat{x}_{i,T-1} + \sum_{k=1}^{K} f_{iT}^{k}\left(p_{iT}^{k} - c_{iT}\right)\hat{x}_{iT}^{k} = \sum_{k=1}^{K}\gamma_{i}^{k}\left(\hat{x}_{i,T-1} + \hat{x}_{iT}^{k}\right) = \sum_{k=1}^{K}\gamma_{i}^{k} w_{i,T-2}$$

- Consider any feasible solution of model (7.14), $x_{i,T\text{-}1}$, x_{iT}^{k} for k = 1,…,K, satisfying $x_{i,T\text{-}1} + x_{iT}^{k} \leq w_{i,T\text{-}2}$. Consider the optimal equilibrium market price $p_{i,T\text{-}1}$ and Lagrange multiplier γ_{i}^{k} of equilibrium model (7.50), satisfying: $p_{i,T\text{-}1} - c_{i,T\text{-}1}$

$-\sum_{k=1}^{K}\gamma_i^k \le 0$, and $f_{iT}^k\left(p_{iT}^k - c_{iT}\right) - \gamma_i^k \le 0$ for $k = 1,\ldots,K$ – see (A1.1) and (A1.5).

The objective function of (7.14) can be written as:

$$\left(p_{i,T-1} - c_{i,T-1}\right)x_{i,T-1} + \sum_{k=1}^{K} f_{iT}^k\left(p_{iT}^k - c_{iT}\right)x_{iT}^k \le \sum_{k=1}^{K}\gamma_i^k\left(x_{i,T-1} + x_{iT}^k\right) \le \sum_{k=1}^{K}\gamma_i^k w_{i,T-2}$$

So, $\hat{x}_{i,T-1}$, $\hat{x}_{iT}^k$ is an optimal solution of model (7.14) if the producer price satisfies (7.52).

Theorem 7.2b:

Let $y_{i,T\text{-}1}$, $x_{i,T\text{-}1}$, $x_{ij,T\text{-}1}$, $s_{i,T\text{-}1}$, $\hat{r}_{iT}^k$, $\hat{q}_{iT}^k$, and $\hat{q}_{ijT}^k$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of equilibrium model (7.50). Let $\pi_{i,T\text{-}1} = \pi_{i,T\text{-}1}(y_{i,T\text{-}1})$, $p_{i,T\text{-}1} = p_{i,T\text{-}1}(x_{i,T\text{-}1}) = \lambda_{i,T\text{-}1}$. The solution:

(7.53) $q_{i,T\text{-}1} = x_{i,T\text{-}1}$; $r_{i,T\text{-}1} = y_{i,T\text{-}1}$; $q_{ij,T\text{-}1} = x_{ij,T\text{-}1}$; $v_{i,T\text{-}1} = s_{i,T\text{-}1}$;
$r_{iT}^k = \hat{r}_{iT}^k$; $q_{iT}^k = \hat{q}_{iT}^k$; $q_{ijT}^k = \hat{q}_{ijT}^k$

for $k \in \{1,\ldots,K\}$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, is an optimal solution of the trader decision problem (7.27) for period T-1.

Proof:

The proof will consist of three parts: a) (7.53) is a feasible solution of (7.27), b) solution (7.53) results in a certain value of the objective function of (7.27); c) (7.53) is an optimal solution of (7.27), because any feasible solution of (7.27) has an objective value less than the objective value for (7.53).

a) See (7.53), (7.50), and (7.27).
b) Due to (A1.1) – (A1.9) and (7.52), the objective function of model (7.27) for the solution (7.53) can be written as:

$$\sum_{i=1}^{n}\left[\pi_{i,T-1}r_{i,T-1}-p_{i,T-1}q_{i,T-1}-\sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ij,T-1}q_{ij,T-1}-k_{i,T-1}v_{i,T-1}+EZ_T^{tr}\left(\Pi_{iT},P_{iT},v_{i,T-1}\right)\right]$$

$$=\sum_{i=1}^{n}\left[\lambda_{i,T-1}y_{i,T-1}-\lambda_{i,T-1}x_{i,T-1}-\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\lambda_{j,T-1}-\lambda_{i,T-1}\right)x_{ij,T-1}-\left(\sum_{k=1}^{K}\lambda_{iT}^{k}-\lambda_{i,T-1}\right)s_{i,T-1}\right.$$

$$\left.\left.+\sum_{k=1}^{K}\left[\left(\lambda_{iT}^{k}+\mu_{iT}^{k}\right)r_{iT}^{k}-\left(\lambda_{iT}^{k}-\vartheta_{iT}^{k}\right)q_{iT}^{k}-\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\lambda_{jT}^{k}-\lambda_{iT}^{k}\right)q_{ijT}^{k}\right]\right]\right]$$

$$=\sum_{i=1}^{n}\left[\lambda_{i,T-1}s_{i,T-2}-\sum_{k=1}^{K}\lambda_{iT}^{k}s_{i,T-1}+\sum_{k=1}^{K}\left[\lambda_{iT}^{k}s_{i,T-1}+\mu_{iT}^{k}r_{iT}^{k}+\vartheta_{iT}^{k}q_{iT}^{k}\right]\right]$$

$$=\sum_{i=1}^{n}\left[\lambda_{i,T-1}s_{i,T-2}+\sum_{k=1}^{K}\left[\mu_{iT}^{k}\bar{y}_{iT}^{k}+\vartheta_{iT}^{k}\bar{x}_{iT}^{k}\right]\right]$$

due to the equilibrium conditions of (7.50) and (A1.9).

c) Consider any feasible solution $q_{i,T-1}$, $r_{i,T-1}$, $q_{ij,T-1}$, $v_{i,T-1}$, r_{iT}^{k}, q_{iT}^{k}, q_{ijT}^{k} of (7.27), and market prices $\pi_{i,T-1} = \pi_{i,T-1}(y_{i,T-1})$ and $p_{i,T-1} = p_{i,T-1}(x_{i,T-1})$, with $y_{i,T-1}$ and $x_{i,T-1}$ the optimal demanded and supplied quantities of equilibrium model (7.50). Due to (A1.1) – (A1.9) and (7.52):

$$\pi_{i,T-1} \le \lambda_{i,T-1} = p_{i,T-1}\ ; \quad \lambda_{j,T-1} \le \lambda_{i,T-1} + \tau_{ij,T-1}; \quad -k_{i,T-1} - \lambda_{i,T-1} + \sum\nolimits_{k=1}^{K}\lambda_{iT}^{k} \le 0$$

$$g_T^k\pi_{iT}^k-\lambda_{iT}^k-\mu_{iT}^k\le 0\,;\quad -g_T^k p_{iT}^k+\lambda_{iT}^k-\vartheta_{iT}^k\le 0\,;\ -g_t^k\tau_{ijT}+\lambda_{jT}^k-\lambda_{iT}^k\le 0$$

$$\bar{y}_{iT}^k-r_{iT}^k\ge 0\,;\qquad \bar{x}_{iT}^k-q_{iT}^k\ge 0\,;\qquad \mu_{iT}^k,\vartheta_{iT}^k\ge 0$$

for all $i = 1,\dots,n$, $i \neq j$, with $\lambda_{i,T-1}$, $\lambda_{iT}^{k},\mu_{iT}^{k},\vartheta_{iT}^{k}$ the optimal values of the Lagrange multipliers of model (7.50). Analogous to b), for the objective function of model (7.27) follows:

$$\sum_{i=1}^{n}\left[\pi_{it}r_{it} - p_{it}q_{it} - \sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ijt}q_{ijt} - k_{it}v_{it} + EZ_{t+1}^{tr}\left(\Pi_{i,t+1}, P_{i,t+1}, v_{it}\right)\right] \leq$$

$$\sum_{i=1}^{n}\left[\lambda_{i,T-1}s_{i,T-2} + \sum_{k=1}^{K}\left[\mu_i^k \bar{y}_{iT}^k + \vartheta_i^k \bar{x}_{iT}^k\right]\right]$$

due to the equilibrium condition of (7.27). So, (7.53) is an optimal solution of model (7.27).

Optimal equilibrium prices and quantities for period t $\in$ {1,…,T-2} – see model (7.54):

A part of the Kuhn Tucker conditions of model (7.54) are as follows:

$$\text{(A1.10)}\begin{cases}\text{if } x_{it}=0 & \text{then } -c_{it}+\lambda_{it}-\sum_{k_1=1}^{K}\cdots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}} \leq 0\\ \text{if } x_{it}>0 & \text{then } -c_{it}+\lambda_{it}-\sum_{k_1=1}^{K}\cdots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}} = 0\end{cases}$$

$$\text{(A1.11)}\begin{cases}\text{if } y_{it}=0 & \text{then } \pi_{it}(0)-\lambda_{it}\leq 0\\ \text{if } y_{it}>0 & \text{then } \pi_{it}(y_{it})-\lambda_{it}=0\end{cases}$$

$$\text{(A1.12)}\begin{cases}\text{if } x_{ijt}=0 & \text{then } -\tau_{ijt}-\lambda_{it}+\lambda_{jt}\leq 0\\ \text{if } x_{ijt}>0 & \text{then } -\tau_{ijt}-\lambda_{it}+\lambda_{jt}=0\end{cases}$$

$$\text{(A1.13)}\begin{cases}\text{if } s_{it}=0 & \text{then } -k_{it}-\lambda_{it}+\sum_{k_1=1}^{K}\lambda_{i,t+1}^{k_1}\leq 0\\ \text{if } \mathrm{s}_{it}>0 & \text{then } -k_{it}-\lambda_{it}+\sum_{k_1=1}^{K}\lambda_{i,t+1}^{k_1}=0\end{cases}$$

$$
\text{(A1.14)}\begin{cases}\text{if } x_{i,t+\tau}^{k_1,\dots,k_\tau}=0 & \text{then } f_{i,t+1}^{k_1}\cdot\ldots\cdot f_{i,t+\tau}^{k_1}\left(p_{i,t+\tau}^{k_\tau}-c_{i,t+\tau}\right)-\sum_{k_{\tau+1}=1}^{K}\cdots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}\le 0\\ \text{if } x_{i,t+\tau}^{k_1,\dots,k_\tau}>0 & \text{then } f_{i,t+1}^{k_1}\cdot\ldots\cdot f_{i,t+\tau}^{k_1}\left(p_{i,t+\tau}^{k_\tau}-c_{i,t+\tau}\right)-\sum_{k_{\tau+1}=1}^{K}\cdots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}=0\end{cases}
$$

for $\tau = 1,\ldots,T\text{-}t$

$$
\text{(A1.15)}\begin{cases}\text{if } r_{i,t+\tau}^{k_1,\dots,k_\tau}=0 & \text{then } g_{t+1}^{k_1}\cdot\ldots\cdot g_{t+\tau}^{k_\tau}\pi_{i,t+\tau}^{k_\tau}-\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}-\mu_{i,t+\tau}^{k_1,\dots,k_\tau}\le 0\\ \text{if } r_{i,t+\tau}^{k_1,\dots,k_\tau}>0 & \text{then } g_{t+1}^{k_1}\cdot\ldots\cdot g_{t+\tau}^{k_\tau}\pi_{i,t+\tau}^{k_\tau}-\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}-\mu_{i,t+\tau}^{k_1,\dots,k_\tau}=0\end{cases}
$$

for $\tau = 1,\ldots,T\text{-}t$

$$
\text{(A1.16)}\begin{cases}\text{if } q_{i,t+\tau}^{k_1,\dots,k_\tau}=0 & \text{then } -g_{t+1}^{k_1}\cdot\ldots\cdot g_{t+\tau}^{k_\tau}p_{i,t+\tau}^{k_\tau}+\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}-\vartheta_{i,t+\tau}^{k_1,\dots,k_\tau}\le 0\\ \text{if } q_{i,t+\tau}^{k_1,\dots,k_\tau}>0 & \text{then } -g_{t+1}^{k_1}\cdot\ldots\cdot g_{t+\tau}^{k_\tau}p_{i,t+\tau}^{k_\tau}+\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}-\vartheta_{i,t+\tau}^{k_1,\dots,k_\tau}=0\end{cases}
$$

for $\tau = 1,\ldots,T\text{-}t$

$$
\text{(A1.17)}\begin{cases}\text{if } q_{ij,t+\tau}^{k_1,\dots,k_\tau}=0 & \text{then } -g_{i,t+1}^{k_1}\cdot\ldots\cdot g_{i,t+\tau}^{k_\tau}\tau_{ij,t+1}+\lambda_{j,t+\tau}^{k_1,\dots,k_\tau}-\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}\le 0\\ \text{if } q_{ij,t+\tau}^{k_1,\dots,k_\tau}>0 & \text{then } -g_{i,t+1}^{k_1}\cdot\ldots\cdot g_{i,t+\tau}^{k_\tau}\tau_{ij,t+1}+\lambda_{j,t+\tau}^{k_1,\dots,k_\tau}-\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}=0\end{cases}
$$

for $\tau = 1,\ldots,T\text{-}t$

$$
\text{(A1.18)}\begin{cases}\text{if } v_{i,t+\tau}^{k_1,\dots,k_\tau}=0 & \text{then } -g_{i,t+1}^{k_1}\cdot\ldots\cdot g_{i,t+\tau}^{k_\tau}k_{i,t+\tau}-\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}+\sum_{k_{\tau+1}=1}^{K}\lambda_{i,t+\tau+1}^{k_1,\dots,k_{\tau+1}}\le 0\\ \text{if } v_{i,t+\tau}^{k_1,\dots,k_\tau}>0 & \text{then } -g_{i,t+1}^{k_1}\cdot\ldots\cdot g_{i,t+\tau}^{k_\tau}k_{i,t+\tau}-\lambda_{i,t+\tau}^{k_1,\dots,k_\tau}+\sum_{k_{\tau+1}=1}^{K}\lambda_{i,t+\tau+1}^{k_1,\dots,k_{\tau+1}}=0\end{cases}
$$

for $\tau = 1,\ldots,T\text{-}t\text{-}1$

$$
\text{(A1.19)}\begin{cases}\mu_{i,t+\tau}^{k_1,\dots,k_\tau}\left(\bar{y}_{i,t+\tau}^{k_1,\dots,k_\tau}-r_{i,t+\tau}^{k_1,\dots,k_\tau}\right)=0\\ \vartheta_{i,t+\tau}^{k_1,\dots,k_\tau}\left(\bar{x}_{i,t+\tau}^{k_1,\dots,k_\tau}-q_{i,t+\tau}^{k_1,\dots,k_\tau}\right)=0\\ \gamma_i^{k_1,\dots,k_{T-t}}\left(w_{i,t-1}-x_{it}-x_{i,t+1}^{k_1}-x_{i,t+2}^{k_1,k_2}-\ldots-x_{iT}^{k_1,\dots,k_{T-t}}\right)=0\end{cases}\qquad \text{for } \tau = 1,\ldots,T\text{-}t\text{-}1
$$

Theorem 7.1c:

Let in the optimal solution of the equilibrium model (7.54) for period t, $t = 1,\ldots,T$-2, $\hat{x}_t, \hat{x}_{t+1}^{k_1}, \hat{x}_{t+2}^{k_1,k_2}, \ \ldots, \hat{x}_T^{k_1,\ldots,k_{T-t}}$ be the optimal supply level and λ_{it} be the corresponding optimal value of the Lagrange multiplier of the equilibrium condition for period t, for $i = 1,\ldots,n$, $k_1,\ldots,k_{T-t} = 1,\ldots,K$. If the producer price in period t in region i is equal to:

(7.55) $p_{it} = \lambda_{it}$

then $x_{it} = \hat{x}_{it}$, $x_{t+1}^{k_1} = \hat{x}_{t+1}^{k_1}, \ldots, x_T^{k_1,\ldots,k_{T-t}} = \hat{x}_T^{k_1,\ldots,k_{T-t}}$, for $k_1,\ldots,k_{T-t} = 1,\ldots,K$, is an optimal solution of supply model (7.14), the producer supply model for the periods 1 to T-2. In other words, the optimal equilibrium supply levels give the producers optimal profits. Since the value of λ_{it}, depends on the value of the equilibrium supply level, we write $p_{it}(x_{it}) = \lambda_{it}$.

Proof:

Consider the optimal solution of model (7.54) for period t: $\hat{x}_t, \hat{x}_{t+1}^{k_1}, \hat{x}_{t+2}^{k_1,k_2}, \ldots, \hat{x}_T^{k_1,\ldots,k_{T-t}}$ for the periods t to T, at producer prices equal to $p_t, p_{t+1}^{k_1}, p_{t+2}^{k_2}, \ldots, p_T^{k_{T-t}}$, for $k_1,\ldots,k_{T-t} = 1,\ldots,K$. Is this solution an optimal solution of producer supply problem (7.14) for period t? It is trivial that the solution is a feasible solution of model (7.14). To prove that the solution is also optimal, first determine the value of the objective function of (7.14), and then show that this value is optimal:

- Due to (A1.10), (A1.14), (A1.19), and (7.55), the objective function of (7.14) can be written as:

$$\left(p_{it}-c_{it}\right)\hat{x}_{it}+EZ_{i,t+1}^{pr}\left(w_{i,t-1}-\hat{x}_{it};P_{i,t+1}\right)$$

$$=\left(p_{it}-c_{it}\right)\hat{x}_{it}+\sum_{k_1=1}^{K}f_{i,t+1}^{k_1}\left[\left(p_{i,t+1}^{k_1}-c_{i,t+1}\right)\hat{x}_{i,t+1}^{k_1}+\sum_{k_2=1}^{K}f_{i,t+2}^{k_2}\left[\left(p_{i,t+2}^{k_2}-c_{i,t+2}\right)\hat{x}_{i,t+2}^{k_1,k_2}+\dots\right.\right.$$

$$\left.\left.\dots+\sum_{k_{T-t-1}=1}^{K}f_{i,T-1}^{k_{T-t}}\left[\left(p_{i,T-1}^{k_{T-t-1}}-c_{i,T-1}\right)\hat{x}_{i,T-1}^{k_1,\dots,k_{T-t-1}}+\sum_{k_{T-t}=1}^{K}f_{iT}^{k_{T-t}}\left(p_{iT}^{k_{T-t}}-c_{iT}\right)\hat{x}_{iT}^{k_1,\dots,k_{T-t}}\right]\dots\right]\right]$$

$$=\sum_{k_1=1}^{K}\dots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}\hat{x}_{it}+\sum_{k_1=1}^{K}\left[\sum_{k_2=1}^{K}\dots\sum_{k_{T-t}}^{K}\gamma_i^{k_1,\dots,k_{T-t}}\hat{x}_{i,t+1}^{k_1}+\sum_{k_2=1}^{K}\left[\sum_{k_3=1}^{K}\dots\sum_{k_{T-t}}^{K}\gamma_i^{k_1,\dots,k_{T-t}}\hat{x}_{i,t+2}^{k_1,k_2}+\dots\right.\right.$$

$$\left.\left.\dots+\sum_{k_{T-t-1}=1}^{K}\left[\sum_{K_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}\hat{x}_{i,T-1}^{k_1,\dots,k_{T-t-t}}+\sum_{K_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}\hat{x}_{iT}^{k_1,\dots,k_{T-t}}\right]\dots\right]\right]$$

$$=\sum_{k_1=1}^{K}\dots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}w_{it}$$

- Consider any feasible solution of model (7.14): $x_t, x_{t+1}^{k_1}, x_{t+2}^{k_1,k_2}, \dots, x_T^{k_1,\dots,k_{T-t}}$ for the periods t to T, at producer prices equal to $p_t, p_{t+1}^{k_1}, p_{t+2}^{k_2}, \dots, p_T^{k_{T-t}}$, for $k_1,\dots,k_{T-t}$ = 1,…,K. Suppose that prices satisfy the following properties – see (A1.10), (A1.14), (A1.19), and (7.55):

$$p_{it}-c_{it}\leq\sum_{k_1=1}^{K}\dots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_1,\dots,k_{T-t}}$$

$$f_{i,t+1}^{k_1}\cdot\dots\cdot f_{i,t+\tau}^{k_1}\left(p_{i,t+\tau}^{k_\tau}-c_{i,t+\tau}\right)\leq\sum_{k_{\tau+1}=1}^{K}\dots\sum_{k_{T-t}=1}^{K}\gamma_i^{k_{1,\dots},\dots,k_{T-t}}$$

$$x_{it}+x_{i,t+1}^{k_1}+x_{i,t+2}^{k_1,k_2}+\dots+x_{iT}^{k_1,\dots,k_{T-t}}\leq w_{i,t-1}\qquad\gamma_i^{k_1,\dots,k_{T-t}}\geq 0$$

The value of the objective function of (7.14) for the feasible solution satisfies:

$$\left(p_{it}-c_{it}\right)x_{it}+EZ_{i,t+1}^{pr}\left(w_{i,t-1}-x_{it};P_{i,t+1}\right)$$

$$\leq \sum_{k_1=1}^{K} \cdots \sum_{k_{T-t}=1}^{K} \gamma_i^{k_1,\ldots,k_{T-t}} x_{it} + \sum_{k_1=1}^{K} \left[\sum_{k_2=1}^{K} \cdots \sum_{k_{T-t}}^{K} \gamma_i^{k_1,\ldots,k_{T-t}} x_{i,t+1}^{k_1} + \sum_{k_2=1}^{K} \left[\sum_{k_3=1}^{K} \cdots \sum_{k_{T-t}}^{K} \gamma_i^{k_1,\ldots,k_{T-t}} x_{i,t+2}^{k_1,k_2} + \ldots \right.\right.$$

$$\left.\left. \ldots + \sum_{k_{T-t-1}=1}^{K} \left[\sum_{K_{T-t}=1}^{K} \gamma_i^{k_1,\ldots,k_{T-t}} x_{i,T-1}^{k_1,\ldots,k_{T-t-t}} + \sum_{K_{T-t}=1}^{K} \gamma_i^{k_1,\ldots,k_{T-t}} x_{iT}^{k_1,\ldots,k_{T-t}} \right] \ldots \right] \right]$$

$$\leq \sum_{k_1=1}^{K} \cdots \sum_{k_{T-t}=1}^{K} \gamma_i^{k_1,\ldots,k_{T-t}} w_{it}$$

So, $\hat{x}_t, \hat{x}_{t+1}^{k_1}, \hat{x}_{t+2}^{k_1,k_2}, \ldots, \hat{x}_T^{k_1,\ldots,k_{T-t}}$ is an optimal solution of (7.14), if the producer price satisfies (7.55).

<u>Theorem 7.2c:</u>

Let $\hat{x}_{it}, \hat{y}_{it}, \hat{x}_{ijt}, \hat{s}_{it}, \hat{q}_{i,t+1}^{k_1}, \hat{r}_{i,t+1}^{k_1}, \hat{q}_{ij,t+1}^{k_1}, \hat{v}_{i,t+1}^{k_1}, \hat{q}_{i,t+\tau}^{k_1,\ldots,k_\tau}, \hat{r}_{i,t+\tau}^{k_1,\ldots,k_\tau}, \hat{q}_{ij,t+\tau}^{k_1,\ldots,k_\tau}, \hat{v}_{i,t+\tau}^{k_1,\ldots,k_\tau}$, $\tau = 2,\ldots,T\text{-}t$, $i,j \in \{1,\ldots,n\}$, $i \neq j$, be an optimal solution of equilibrium model (7.54). Let $\pi_{it} = \pi_{it}(\hat{y}_{it})$, $p_{it} = p_{it}(\hat{x}_{it}) = \lambda_{it}$. The solution:

$$\text{(7.56)} \quad \begin{aligned} & q_{it} = \hat{x}_{it};\ r_{it} = \hat{y}_{it};\ q_{ijt} = \hat{x}_{ijt};\ v_{it} = \hat{s}_{it}; \\ & q_{i,t+1}^{k_1} = \hat{q}_{i,t+1}^{k_1};\ r_{i,t+1}^{k_1} = \hat{r}_{i,t+1}^{k_1};\ q_{ij,t+1}^{k_1} = \hat{q}_{ij,t+1}^{k_1};\ v_{i,t+1}^{k_1} = \hat{v}_{i,t+1}^{k_1}; \\ & q_{i,t+\tau}^{k_1,\ldots,k_\tau} = \hat{q}_{i,t+\tau}^{k_1,\ldots,k_\tau};\ r_{i,t+\tau}^{k_1,\ldots,k_\tau} = \hat{r}_{i,t+\tau}^{k_1,\ldots,k_\tau};\ q_{ij,t+\tau}^{k_1,\ldots,k_\tau} = \hat{q}_{ij,t+\tau}^{k_1,\ldots,k_\tau};\ v_{i,t+\tau}^{k_1,\ldots,k_\tau} = \hat{v}_{i,t+\tau}^{k_1,\ldots,k_\tau}; \\ & \text{for } \tau \in \{2,\ldots,T-t\} \end{aligned}$$

for $i,j \in \{1,\ldots,n\}$, $i \neq j$, is an optimal solution of the trader decision problem (7.27).

<u>Proof:</u>

The proof will consist of three parts: a) (7.56) is a feasible solution of (7.27), b) solution (7.56) results in a certain value of the objective function of (7.27); c) (7.56) is an optimal solution of (7.27), because any feasible solution of (7.27) has an objective value less than the objective value for (7.56).

a) See (7.56), (7.54), and (7.27).

b) Due to (A1.10) – (A1.19) and (7.55), the objective function of model (7.27) for the solution (7.56) can be written as:

$$\sum_{i=1}^{n}\left[\pi_{it}r_{it}-p_{it}q_{it}-\sum_{\substack{j=1\\j\neq i}}^{n}\tau_{ijt}q_{ijt}-k_{it}v_{it}+EZ_{t+1}^{tr}\left(\Pi_{i,t+1},P_{i,t+1},v_{it}\right)\right]$$

$$=\sum_{i=1}^{n}\left[\lambda_{it}\hat{y}_{it}-\lambda_{it}\hat{x}_{it}-\sum_{\substack{j=1\\j\neq i}}^{n}\left(\lambda_{jt}-\lambda_{it}\right)\hat{x}_{ijt}-\left(\sum_{k_1=1}^{K}\lambda_{i,t+1}^{k_1}-\lambda_{it}\right)\hat{s}_{it}\right.$$

$$+\sum_{k_1=1}^{K}\left[\left(\lambda_{i,t+1}^{k_1}+\mu_{i,t+1}^{k_1}\right)\hat{r}_{i,t+1}^{k_1}-\left(\lambda_{i,t+1}^{k_1}-\vartheta_{i,t+1}^{k_1}\right)\hat{q}_{i,t+1}^{k_1}-\sum_{\substack{j=1\\j\neq i}}^{n}\left(\lambda_{j,t+1}^{k_1}-\lambda_{i,t+1}^{k_1}\right)\hat{q}_{ij,t+1}^{k_1}\right.$$

$$-\left(\sum_{k_2=1}^{K}\lambda_{i,t+2}^{k_1,k_2}-\lambda_{i,t+1}^{k_1}\right)\hat{v}_{i,t+1}^{k_1}+\ldots+\sum_{k_\tau=1}^{K}\left[\left(\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}+\mu_{i,t+\tau}^{k_1,\ldots,k_\tau}\right)\hat{r}_{i,t+\tau}^{k_1,\ldots,k_\tau}-\left(\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}-\vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau}\right)\hat{q}_{i,t+\tau}^{k_1,\ldots,k_\tau}\right.$$

$$-\sum_{\substack{j=1\\j\neq i}}^{n}\left(\lambda_{j,t+\tau}^{k_1,\ldots,k_\tau}-\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}\right)\hat{q}_{ij,t+\tau}^{k_1,\ldots,k_\tau}-\left(\sum_{k_{\tau+1}=1}^{K}\lambda_{i,t+\tau+1}^{k_1,\ldots,k_{\tau+1}}-\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}\right)\hat{v}_{i,t+\tau}^{k_1,\ldots,k_\tau}+\ldots+$$

$$\sum_{k_{T-t}=1}^{K}\left[\left(\lambda_{iT}^{k_1,\ldots,k_{T-t}}+\mu_{iT}^{k_1,\ldots,k_{T-t}}\right)\hat{r}_{iT}^{k_1,\ldots,k_{T-t}}-\left(\lambda_{iT}^{k_1,\ldots,k_{T-t}}-\vartheta_{iT}^{k_1,\ldots,k_{T-t}}\right)\hat{q}_{iT}^{k_1}\right.$$

$$\left.\left.\left.\left.-\sum_{\substack{j=1\\j\neq i}}^{n}\left(\lambda_{jT}^{k_1,\ldots,k_{T-t}}-\lambda_{iT}^{k_1,\ldots,k_{T-t}}\right)\hat{q}_{ijT}^{k_1,\ldots,k_{T-t}}\right]\ldots\right]\ldots\right]\right]$$

$$= \sum_{i=1}^{n} \left[\lambda_{it} \hat{s}_{i,t-1} - \sum_{k_1=1}^{K} \lambda_{i,t+1}^{k_1} \hat{s}_{it} + \sum_{k_1=1}^{K} \left[\lambda_{i,t+1}^{k_1} \hat{s}_{it} - \sum_{k_2=1}^{K} \lambda_{i,t+2}^{k_1,k_2} \hat{v}_{i,t+1}^{k_1} + \ldots \right.\right.$$

$$\left.\left. + \sum_{k_\tau=1}^{K} \left[\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau} \hat{v}_{i,t+\tau-1}^{k_1,\ldots,k_{\tau-1}} - \sum_{k_{\tau+1}=1}^{K} \lambda_{i,t+\tau+1}^{k_1,\ldots,k_{\tau+1}} \hat{v}_{i,t+\tau}^{k_1,\ldots,k_\tau} + \ldots + \sum_{k_{T-t}=1}^{K} \lambda_{iT}^{k_1,\ldots,k_{T-t}} \hat{v}_{i,T-1}^{k_1,\ldots,k_{T-t-1}} \right] \ldots \right]\right]$$

$$+ \sum_{i=1}^{n} \left[\sum_{k_1=1}^{K} \left[\mu_{i,t+1}^{k_1} \hat{r}_{i,t+1}^{k_1} + \vartheta_{i,t+1}^{k_1} \hat{q}_{i,t+1}^{k_1} + \sum_{k_2=1}^{K} \left[\mu_{i,t+1}^{k_1,k_2} \hat{r}_{i,t+1}^{k_1,k_2} + \vartheta_{i,t+1}^{k_1,k_2} \hat{q}_{i,t+1}^{k_1,k_2} + \ldots \right.\right.\right.$$

$$+ \sum_{k_\tau=1}^{K} \left[\mu_{i,t+\tau}^{k_1,\ldots,k_\tau} \hat{r}_{i,t+\tau}^{k_1,\ldots,k_\tau} + \vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau} \hat{q}_{i,t+\tau}^{k_1,\ldots,k_\tau} + \ldots + \right.$$

$$\left.\left.\left. + \sum_{k_{T-t}=1}^{K} \left[\mu_{iT}^{k_1,\ldots,k_{T-t}} \hat{r}_{iT}^{k_1,\ldots,k_{T-t}} + \vartheta_{iT}^{k_1,\ldots,k_{T-t}} \hat{q}_{iT}^{k_1,\ldots,k_{T-t}} \right] \ldots \right] \ldots \right]\right]$$

$$= \sum_{i=1}^{n} \left[\lambda_{it} \hat{s}_{i,t-1} \right] + \sum_{i=1}^{n} \left[\sum_{k_1=1}^{K} \left[\mu_{i,t+1}^{k_1} \overline{y}_{i,t+1}^{k_1} + \vartheta_{i,t+1}^{k_1} \overline{x}_{i,t+1}^{k_1} + \sum_{k_2=1}^{K} \left[\mu_{i,t+1}^{k_1,k_2} \overline{y}_{i,t+1}^{k_1,k_2} + \vartheta_{i,t+1}^{k_1,k_2} \overline{x}_{i,t+1}^{k_1,k_2} + \ldots \right.\right.\right.$$

$$+ \sum_{k_\tau=1}^{K} \left[\mu_{i,t+\tau}^{k_1,\ldots,k_\tau} \overline{y}_{i,t+\tau}^{k_1,\ldots,k_\tau} + \vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau} \overline{x}_{i,t+\tau}^{k_1,\ldots,k_\tau} + \ldots + \right.$$

$$\left.\left.\left. + \sum_{k_{T-t}=1}^{K} \left[\mu_{iT}^{k_1,\ldots,k_{T-t}} \overline{y}_{iT}^{k_1,\ldots,k_{T-t}} + \vartheta_{iT}^{k_1,\ldots,k_{T-t}} \overline{x}_{iT}^{k_1,\ldots,k_{T-t}} \right] \ldots \right] \ldots \right]\right]$$

due to the equilibrium conditions of (7.54) and (A1.19).

c) Consider any feasible solution q_{it}, r_{it}, q_{ijt}, v_{it}, $r_{i,t+1}^{k_1}, q_{i,t+1}^{k_1}, q_{ij,t+1}^{k_1}, v_{i,t+1}^{k_1}$, $r_{i,t+\tau}^{k_1,\ldots,k_\tau}, q_{i,t+\tau}^{k_1,\ldots,k_\tau}, q_{ij,t+\tau}^{k_1,\ldots,k_\tau}, v_{i,t+\tau}^{k_1,\ldots,k_\tau}$ of model (7.27), and market prices $\pi_{it} = \pi_{it}(\hat{y}_{it})$, $p_{it} = p_{it}(\hat{x}_{it}) = \lambda_{it}$. Due to (A1.10) – (A1.19) and (7.55):

$$\pi_{it} \le \lambda_{it} = p_{it} \qquad \lambda_{jt} \le \lambda_{it} + \tau_{ijt} \qquad -k_{i,t} - \lambda_{it} + \sum_{k_1=1}^{K} \lambda_{i,t+1}^{k_1} \le 0$$

$$g_{t+1}^{k_1} \cdot \pi_{i,t+1}^{k_1} - \lambda_{i,t+1}^{k_1} - \mu_{i,t+1}^{k_1} \le 0 \qquad g_{t+1}^{k_1} \cdot \ldots \cdot g_{t+\tau}^{k_\tau} \pi_{i,t+\tau}^{k_1,\ldots,k_\tau} - \lambda_{i,t+\tau}^{k_1,\ldots,k_\tau} - \mu_{i,t+\tau}^{k_1,\ldots,k_\tau} \le 0$$

$$-g_{t+1}^{k_1} p_{i,t+1}^{k_1} + \lambda_{i,t+1}^{k_1} - \vartheta_{i,t+1}^{k_1} \le 0 \qquad -g_{t+\tau}^{k_1} \cdot \ldots \cdot g_{t+\tau}^{k_\tau} p_{i,t+\tau}^{k_1,\ldots,k_\tau} + \lambda_{i,t+\tau}^{k_1,\ldots,k_\tau} - \vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau} \le 0$$

$$-g_{t+1}^{k_1}\tau_{ij,t+1}+\lambda_{j,t+1}^{k_1}-\lambda_{i,t+1}^{k_1}\leq 0 \qquad -g_{t+1}^{k_1}\cdot\ldots\cdot g_{t+\tau}^{k_\tau}\tau_{ij,t+\tau}+\lambda_{j,t+\tau}^{k_1,\ldots,k_\tau}-\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}\leq 0$$

$$-g_{t+1}^{k_1}k_{i,t+1}-\lambda_{i,t+1}^{k_1}+\sum\nolimits_{k_2=1}^{K}\lambda_{i,t+2}^{k_1,k_2}\leq 0$$

$$-g_{t+1}^{k_1}\cdot\ldots\cdot g_{t+\tau}^{k_\tau}k_{i,t+\tau}-\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau}+\sum\nolimits_{k_{t+\tau}=1}^{K}\lambda_{i,t+\tau+1}^{k_1,\ldots,k_{t+\tau+1}}\leq 0$$

$$\overline{y}_{i,t+1}^{k_1}-r_{i,t+1}^{k_1}\geq 0 \qquad \overline{y}_{i,t+\tau}^{k_1,\ldots,k_\tau}-r_{i,t+\tau}^{k_1,\ldots,k_\tau}\geq 0 \qquad \mu_{i,t+1}^{k_1},\vartheta_{i,t+1}^{k_1},\mu_{i,t+\tau}^{k_1,\ldots,k_\tau},\vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau}=0$$

$$\overline{x}_{i,t+1}^{k_1}-q_{i,t+1}^{k_1}\geq 0 \qquad \overline{x}_{i,t+\tau}^{k_1,\ldots,k_\tau}-q_{i,t+\tau}^{k_1,\ldots,k_\tau}\geq 0$$

for all $\tau = 2,\ldots,T\text{-}t$, $i = 1,\ldots,n$, $i \neq j$, with λ_{it}, $\lambda_{i,t+1}^{k_1},\lambda_{i,t+\tau}^{k_1,\ldots,k_\tau},\mu_{i,t+1}^{k_1}$, $\mu_{i,t+\tau}^{k_1,\ldots,k_\tau},\vartheta_{i,t+1}^{k_1},\vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau}$ the optimal values of the Lagrange multipliers of model (7.54). Analogous to b), for the objective function of model (7.27) follows:

$$\sum_{i=1}^{n}\left[\pi_{it}r_{it}-p_{it}q_{it}-\sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ijt}q_{ijt}-k_{it}v_{it}+EZ_{t+1}^{tr}\left(\Pi_{i,t+1},P_{i,t+1},v_{it}\right)\right]\leq$$

$$=\sum_{i=1}^{n}\left[\lambda_{it}\hat{s}_{i,t-1}\right]+\sum_{i=1}^{n}\left[\sum_{k_1=1}^{K}\left[\mu_{i,t+1}^{k_1}\overline{y}_{i,t+1}^{k_1}+\vartheta_{i,t+1}^{k_1}\overline{x}_{i,t+1}^{k_1}+\sum_{k_2=1}^{K}\left[\mu_{i,t+1}^{k_1,k_2}\overline{y}_{i,t+1}^{k_1,k_2}+\vartheta_{i,t+1}^{k_1,k_2}\overline{x}_{i,t+1}^{k_1,k_2}+\ldots\right.\right.\right.$$

$$+\sum_{k_\tau=1}^{K}\left[\mu_{i,t+\tau}^{k_1,\ldots,k_\tau}\overline{y}_{i,t+\tau}^{k_1,\ldots,k_\tau}+\vartheta_{i,t+\tau}^{k_1,\ldots,k_\tau}\overline{x}_{i,t+\tau}^{k_1,\ldots,k_\tau}+\ldots+\right.$$

$$\left.\left.\left.\left.+\sum_{k_{T-t}=1}^{K}\left[\mu_{iT}^{k_1,\ldots,k_{T-t}}\overline{y}_{iT}^{k_1,\ldots,k_{T-t}}+\vartheta_{iT}^{k_1,\ldots,k_{T-t}}\overline{x}_{iT}^{k_1,\ldots,k_{T-t}}\right]\ldots\right]\ldots\right]\right]\right]$$

due to the equilibrium condition of (7.29). So, (7.76) is an optimal solution of model (7.29).

Some equilibrium properties:

<u>**Equilibrium property 7.1:**</u> For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,T\}$:

a) In the optimal solution of (7.44) $\pi_{it}(y_{it}) \leq p_{it}(x_{it})$.

b) If in the optimal solution of (7.44) $\pi_{it}(y_{it}) < p_{it}(x_{it})$, then $y_{it} = 0$.
c) If in the optimal solution of (7.44), supply and demand are both positive, $x_{it} > 0$ and $y_{it} > 0$, then the prices satisfy necessarily $\pi_{it}(y_{it}) = p_{it}(x_{it})$.

Proof:
a) Since the solution always has to satisfy $\pi_{it}(y_{it}) \leq \lambda_{it} = p_{it}(x_{it})$, see for example (A1.2) and (7.52), $\pi_{it}(y_{it}) \leq p_{it}(x_{it})$.
b) Let $\pi_{it}(y_{it}) < p_{it}(x_{it})$. If in the optimal solution $y_{it} > 0$, then necessarily, see for example (A1.2) and (7.52), $\pi_{it}(y_{it}) = \lambda_{it} = p_{it}(x_{it})$, which contradicts $\pi_{it}(y_{it}) < p_{it}(x_{it})$. So, necessarily, $y_{it} = 0$.
c) This follows from a) and b).

Equilibrium property 7.2: In the optimal solution of (7.44), let transport take place from market i to market j in period t, i.e. $x_{ijt} > 0$, with $i,j \in \{1,\ldots,n\}$, $j \neq i$, $t \in \{1,\ldots,n\}$, then:
a) no goods are transported from a region $s = 1,\ldots,n$, to region i, $x_{sit} = 0$, for $s \neq i$.
b) no goods are transported from region j to a region $s = 1,\ldots,n$, $x_{jst} = 0$, for $s \neq j$.
c) the producer supply in region i satisfies, $x_{it} > 0$, or the stock remaining from the previous period is positive, $s_{i,t-1} > 0$.
d) the consumer demand in region j satisfies, $y_{jt} > 0$, or the quantity in stock at the end of period t in region j is positive, $s_{jt} > 0$ (this is equal to the statement that the quantity in stock at the end of period t, to be sold in period τ, is positive for at least one period τ, $\dot{s}_{jt\tau} > 0$, $\tau \in \{t+1,\ldots,T\}$).

Proof: Using for example (A1.3), the proof is similar to the proof of *Equilibrium property* 5.2.

Equilibrium property 7.3: For region i and j, $i,j \in \{1,\ldots,n\}$, $j \neq i$, and period $t \in \{1,\ldots,n\}$:
a) In the optimal solution of (7.44) $\pi_{jt}(y_{jt}) \leq p_{it}(x_{it}) + \tau_{ijt}$.

b) If in the optimal solution of (7.44) $\pi_{jt}(y_{jt}) < p_{it}(x_{it}) + \tau_{ijt}$, then $x_{ijt} = 0$ or $y_{jt} = 0$

c) If in the optimal solution of (7.44) supplies in region i, transport between region i and j, and demand in region j are positive, $x_{it} > 0$, $x_{ijt} > 0$ and $y_{jt} > 0$, then the optimal prices satisfy necessarily $\pi_{it}(y_{it}) = p_{it}(x_{it}) + \tau_{ijt}$.

Proof:

a) Since the solution always has to satisfy $\pi_{jt}(y_{jt}) \leq \lambda_{jt} \leq \lambda_{it} + \tau_{ijt} = p_{it}(x_{it}) + \tau_{ijt}$, see for example (A1.2), (A1.3) and (7.52), $\pi_{jt}(y_{jt}) \leq p_{it}(x_{it}) + \tau_{ijt}$.

b) Let $\pi_{it}(y_{it}) < p_{it}(x_{it}) + \tau_{ijt}$. If in the optimal solution $x_{ijt} > 0$ and $y_{it} > 0$, then necessarily, see for example (A1.2), (A1.3) and (7.52), $\pi_{jt}(y_{jt}) = \lambda_{jt} = \lambda_{it} + \tau_{ijt} = p_{it}(x_{it}) + \tau_{ijt}$, which contradicts $\pi_{it}(y_{it}) < p_{it}(x_{it}) + \tau_{ijt}$. So, necessarily, $x_{ijt} = 0$ or $y_{it} = 0$.

c) This follows from a) and b).

Equilibrium property 7.4: For region $i \in \{1,\ldots,n\}$, and period $t \in \{1,\ldots,n\}$, we can derive that:

a) If in the optimal solution of (7.44) $E\Pi_{i,t+1} < p_{it}(x_{it}) + k_{it}$, then $s_{it} = 0$ or $r_{i,t+1}^{k} = 0$ for at least one $k \in \{1,\ldots,K\}$.

b) If in the optimal solution of (7.44) $E\pi_{i,t+1} \geq p_{it} + k_{it}$, storage in period t, and planned sales in period t+1 are positive, $s_{it} > 0$, and $r_{i,t+1}^{k_1} > 0$ for all $k_1 \in \{1,\ldots,K\}$, then an optimal solution exists satisfying $q_{it} = x_{it}$ or $r_{i,t+1}^{k} = y_{i,t+1}^{k}$ for at least one $k \in \{1,...,K\}$. For $E\pi_{i,t+1} = p_{it} + k_{it}$, an optimal solution is not unique.

Proof:

a) Let $E\Pi_{i,t+1} < p_{it}(x_{it}) + k_{it}$. Suppose that $s_{it} > 0$ and $r_{i,t+1}^{k_1} = 0$ for all $k_1 = 1,\ldots,K$. Then, see for example (A1.2), (A1.4) and (7.52): $E\Pi_{i,t+1} \geq \lambda_{it} + k_{it} = p_{it}(x_{it}) + k_{it}$, which is in contradiction with $E\Pi_{i,t+1} < p_{it}(x_{it}) + k_{it}$.

b) The proof is similar to the proof of *Trader property* 7.4 in Section 7.1.

Appendix 2: The stochastic supply model

In Section 9.2, the cereal supply decision problems of cereal producers in Burkina Faso have been discussed for all periods t = 1,…,4. In this appendix we derive the optimal solutions of these models for each period. In Section 9.2, it has been supposed that when producers decide on their optimal supplies for period t, they know the stock level at the end of the previous period $w_{t\text{-}1}$, and the producer price level in period t, p_t. For the stochastic future prices, $P_{t+1},\dots,P_4$, we assumed that the stochastic price for period t+1 is independent of the price in period t, p_t. The stochastic prices for the other periods $\tau = t+2,\dots,4$, P_τ, however, depend on the stochastic price in the previous period, $P_{\tau-1}$. In the model for period t, for t = 1,2,3, future prices for the periods t+1 to 4 were written as – see (9.32), (9.37), and (9.41):

$$\text{(A2.1)} \quad P_{t+1} = \hat{p}_t + \left(\overline{p}_{t+1} - \overline{p}_t\right) + \Theta_{t+1}; \qquad P_\tau = P_{\tau-1} + \left(\overline{p}_\tau - \overline{p}_{\tau-1}\right) + \Theta_\tau$$
$$\text{for } \tau = t+2,\dots,4,$$

with $\hat{p}_t$ and $\overline{p}_\tau$ constants, and $\mathrm{E}\Theta_\upsilon = 0$, for $\upsilon = t+1,\dots,4$. Θ_{t+1} are random disturbances from the expected price in period t+1: $\mathrm{E}(P_{t+1})$; Θ_τ are random disturbances from the expected price in period τ, given the price in period τ-1: $\mathrm{E}(P_\tau | P_{\tau-1}=p_{\tau-1})$ for $\tau = t+2,\dots,4$. We assume that the random disturbances Θ_υ, for $\upsilon = t+1,\dots,4$, are independent, and have a discrete, empirical distribution with possible realisations θ^k, for $k = 1,\dots,K$, and:

$$\text{(A2.2)} \quad \Pr(\Theta_\upsilon = \theta^k) = f_\upsilon^k \qquad \text{for } \upsilon = t+1,\dots,4, \text{ and } k = 1,\dots,K$$

with $0 \le f_\upsilon^k \le 1$ and $\sum_{k=1}^K f_\upsilon^k = 1$. In the models for the periods 1, 2, and 3, possible prices and probability distributions are defined, for k,l,m = 1,...,K, as follows – see (9.33), (9.34), (9.38), (9.39), (9.42), and (9.43):

For the model for period 1, (9.29):

$$\text{(A2.3)}\quad \begin{aligned} p_2^k &= \hat{p}_1 + (\overline{p}_2 - \overline{p}_1) + \theta_2^k & \Pr\left(P_2 = p_2^k\right) &= f_2^k \\ p_3^{kl} &= p_2^k + (\overline{p}_3 - \overline{p}_2) + \theta_3^k & \Pr\left(P_3 = p_3^{kl} \mid P_2 = p_2^k\right) &= f_3^l \\ p_4^{klm} &= p_3^{kl} + (\overline{p}_4 - \overline{p}_3) + \theta_4^k & \Pr\left(P_4 = p_4^{klm} \mid P_2 = p_2^k, P_3 = p_3^{kl}\right) &= f_4^m \end{aligned}$$

For the model for period 2, (9.28):

$$\text{(A2.4)}\quad \begin{aligned} p_3^k &= \hat{p}_2 + (\overline{p}_3 - \overline{p}_2) + \theta_3^k & \Pr\left(P_3 = p_3^k\right) &= f_3^k \\ p_4^{kl} &= p_3^k + (\overline{p}_4 - \overline{p}_3) + \theta_4^l & \Pr\left(P_4 = p_4^{kl} \mid P_3 = p_3^k\right) &= f_4^l \end{aligned}$$

For the model for period 3, (9.27):

$$\text{(A2.5)}\quad p_4^k = \hat{p}_3 + (\overline{p}_4 - \overline{p}_3) + \theta_4^l \qquad \Pr\left(P_4 = p_4^k\right) = f_4^k$$

We start with the decision problem for period 4.

Optimal supply in period 4

When the producer decides on his supplies for period 4, he knows the quantities in stock at the end of the previous period, w_3, and the price in period 4, p_4. He solves the following problem – see (9.26):

$$\text{(A2.6)}\quad z_4(w_3; p_4) = \underset{x_4}{Max}\left\{(p_4 - c_4)x_4 \,\middle|\, x_4^- \le x_4 \le w_3\right\}$$

From this it follows directly that the producer will earn negative profits if $p_4 < c_4$. In that case, he will sell the smallest quantity possible, i.e. the minimum quantity x_4^-. If $p_4 \ge c_4$, the producer will earn positive profits if he supplies a positive quantity. In that case it is optimal for him to sell the largest quantity possible, i.e his stock w_3. If $p_4 = c_4$, the solution is not unique. In that case the producer will make neither losses nor profits if he sells. Supply for period 4 can be written as:

(A2.7) $$\begin{cases} \text{if } p_4 < c_4 & x_4 = x_4^- \\ \text{if } p_4 \geq c_4 & x_4 = w_3 \\ \text{if } p_4 = c_4 & \text{any solution } x_4 \text{ between } x_4^- \text{ and } w_3 \text{ is optimal} \end{cases}$$

Optimal supply in period 3

Consider period 3. At the moment when the producer decides on his supplies, he knows the stock at the end of period 2, w_2, and the producer price for period 3, p_3. The distribution of the stochastic price for period 4 has been defined in (A2.1), (A2.2) and (A2.5). Possible prices and the probability distribution have been defined in (A2.5), see also (9.33) and (9.34). The decision problem for period 3 is – see (9.36):

(A2.8) $$z_3(w_2; p_3) = \underset{x_3, x_4^k}{Max} \left\{ (p_3 - c_3)x_3 + \sigma \sum_{k=1}^{K} f_4^k \left(p_4^k - c_4\right) x_4^k \;\middle|\; x_3 + \frac{x_4^k}{\delta} \leq w_2, x_3 \geq x_3^-, x_4^k \geq x_4^-, \, k = 1, \ldots, K \right\}$$

The supplies for period 4, x_4^k, at producer price p_4^k, for k = 1,…,K, satsify (A2.7). Introduce the sets:

$$K_4^1 = \left\{ k = 1, \ldots, K \,\middle|\, p_4^k < c_4 \right\}; \qquad K_4^2 = \left\{ k = 1, \ldots, K \,\middle|\, p_4^k \geq c_4 \right\}$$

For $k \in K_4^1$, optimal supply in period 4 is $x_4^k = x_4^-$; for $k \in K_4^2$, optimal supply may be written as $x_4^k = w_3 = (w_2 - x_3) \cdot \delta$. For the k for which $p_4^k = c_4$, optimal supply is not unique. We can rewrite the objective of (A2.8) as – see (A2.7):

$$(A2.9)\quad \begin{aligned}&(p_3-c_3)x_3+\sigma\sum_{k\in K_4^1} f_4^k\left(p_4^k-c_4\right)x_4^- +\sigma\sum_{k\in K_4^2} f_4^k\left(p_4^k-c_4\right)(w_2-x_3)\delta\\ &=x_3\left(p_3-c_3-\sigma\sum_{k\in K_4^2} f_4^k\left(p_4^k-c_4\right)\delta\right)+A_3+B_3w_2\end{aligned}$$

with A_3 and B_3 constants. The first term is the expected surplus value of selling the quantity x_3 now in stead of in period 4. It is the difference between revenues from selling in period 3 and expected positive revenues from selling in period 4 or not selling at all. Expected revenues for period 4 are multiplied with the time preference indicator σ. This gives the discounted value of expected revenues in period 4. Define:

(A2.10) Ψ_4 the expected revenues of selling one unit of cereals in period 4, or not selling at all

$$(A2.11)\quad \Psi_4=\sigma\sum_{k=1}^{K} f_4^k\left(p_4^k-c_4\right)^+\delta=\sigma\sum_{k=1}^{K} f_4^k\left(\hat{p}_3+\left(\overline{p}_4-\overline{p}_3\right)+\theta_4^k-c_4\right)^+\delta$$

with $a^+ = \max(a;0)$. From (A2.9) and (A2.11), it follows that it is more profitable to sell in period 3, if $p_3 - c_3 - \Psi_4 < 0$. In that case the producer will supply the minimally required quantity $x_3 = x_3^-$, in period 3. If $p_3 - c_3 - \Psi_4 > 0$, it is expected to be more profitable to sell now in stead of in the next period. It will be optimal to sell in this period a quantity as large as possible, and in the next period only the minimally required quantity, x_4^-. The maximum possible quantity, taken into account the minimum supplies in period 4 are: $x_3 = w_2 - x_4^-/\delta$. If $p_3 - c_3 - \Psi_4 = 0$, the optimal solution is not unique. We can write the supply function for period 3 as:

$$\text{(A2.12)}\quad \begin{cases} \text{if } p_3 < c_3 + \Psi_4 & x_3 = x_3^- \\ \text{if } p_3 \geq c_3 + \Psi_4 & x_3 = w_2 - \dfrac{x_4^-}{\delta} \\ \text{if } p_3 = c_3 + \Psi_4 & \text{any solution } x_3 \text{ between } x_3^- \text{ and } w_2 \text{ is optimal} \end{cases}$$

Optimal supply in period 2

Consider period 2. When the producer decides on his optimal supplies for period 2, he knows the stock level at the end of the previous period, w_1, and the price in period 2, p_2. Possible price realisations for period 3 and 4, and the probability distribution of the stochastic prices have been defined in (A2.1), (A2.2), and (A2.4) – see also (9.38) and (9.39). The decision problem for period 2 can be written as – see (9.40):

$$z_2(w_1;p_2) =$$

$$\text{(A2.13)}\quad \underset{x_2,x_3^k,x_4^{kl}}{Max}\left\{(p_2-c_2)x_2+\sigma\sum_{k=1}^{K} f_3^k\left((p_3^k-c_3)x_3^k+\sigma\sum_{l=1}^{K} f_4^l\left(p_4^{kl}-c_4\right)x_4^{kl}\right)\right.$$

$$\left.\left|\; x_2+\frac{x_3^k}{\delta}+\frac{x_4^{kl}}{\delta^2}\leq w_1,\quad x_2\geq x_2^-,\ x_3^k\geq x_3^-,\ x_4^{kl}\geq x_4^-,\ k,l=1,\ldots,K\right.\right\}$$

Supplies in period 3, x_3^k, if the price in period 3 is p_3^k, satisfy (A2.12). Supplies in period 4, x_4^{kl}, if the prices in period 3 and period 4 are p_3^k and p_4^{kl}, respectively, satisfy (A2.7). Change definition (A2.11) in the following way:[33]

$$\text{(A2.14)}\quad \Psi_4\left(p_3^k\right)=\sigma\sum_{l=1}^{K} f_4^l\left(p_4^{kl}-c_4\right)^{+}\delta$$

Introduce the following sets:

[33] In Chapter 7, $\Psi_4\left(p_3^k\right)=\Psi_4$, because p_4^l is independent of p_3^k in Chapter 7.

$$K_3^1 = \left\{k = 1,\dots,K \,\middle|\, p_3^k < c_3 + \Psi_4\left(p_3^k\right)\right\}; \quad L_4^{1k} = \left\{l = 1,\dots,K \,\middle|\, p_4^{kl} < c_4\right\}$$

$$K_3^2 = \left\{k = 1,\dots,K \,\middle|\, p_3^k \geq c_3 + \Psi_4\left(p_3^k\right)\right\}; \quad L_4^{2k} = \left\{l = 1,\dots,K \,\middle|\, p_4^{kl} \geq c_4\right\}$$

$$\text{for } k \in \{1,\dots K\}$$

For $k \in K_3^1$, optimal supply in period 3 is $x_3^k = x_3^-$, and for $k \in K_3^2$, optimal supply may be written as $x_3^k = (w_1 - x_2)\delta - x_4^-/\delta$. For the k for which $p_3^k = c_3 + \Psi_4\left(p_3^k\right)$, optimal supply is not unique. For $l \in L_4^{1k}$, optimal supply in period 4 is x_4^-, and for $l \in L_4^{2k}$, optimal supply may be written as $x_4^{kl} = ((w_1 - x_2)\delta - x_3^k)\delta$. Note that for $k \in K_3^2$, supply in period 4 is equal to $x_4^{kl} = x_4^-$ for all $l = 1,\dots,K$. In that case it is expected in period 3, that it is more profitable to sell in period 3 a large quantity, and in period 4 only the minimally required quantity. We can write the objective function of (A2.13) as follows:

$$\begin{aligned}
&x_2\left(p_2 - c_2\right) + \sigma \sum_{k \in K_3^1} f_3^k \left(\left(p_3^k - c_3\right)x_3^- + \sigma \sum_{l \in L_4^1} f_4^l \left(p_4^{kl} - c_4\right)x_4^- + \right. \\
&\qquad \left. + \sigma \sum_{l \in L_4^2} f_4^l \left(p_4^{kl} - c_4\right)\left(\left(w_1 - x_2\right)\delta - x_3^-\right)\delta \right) \\
&\qquad + \sigma \sum_{k \in K_3^2} f_3^k \left(\left(p_3^k - c_3\right)\left(\left(w_1 - x_2\right)\delta - x_4^- / \delta\right) + \sigma \sum_{k=1}^{K} f_4^l \left(p_4^{kl} - c_4\right)x_4^- \right) = \\
&= x_2\left(p_2 - c_2 - \sigma\delta \sum_{k \in K_3^2} f_3^k \left(p_3^k - c_3\right) - \sigma^2\delta^2 \sum_{k \in K_3^1}\sum_{l \in L_4^2} f_3^k f_4^l \left(p_4^{kl} - c_4\right) \right) + A_2 + B_2 w_1
\end{aligned}$$

(A2.15)
$$\begin{aligned}
&= x_2\left(p_2 - c_2 - \sigma\delta \sum_{k \in K_3^2} f_3^k \left(p_3^k - c_3 - \sigma\delta \sum_{l=1}^{L} f_4^l \left(p_4^{kl} - c_4\right)^+ \right) \right. \\
&\qquad \left. - \sigma^2\delta^2 \sum_{k=1}^{K}\sum_{l=1}^{K} f_3^k f_4^l \left(p_4^{kl} - c_4\right)^+ \right) + A_2 + B_2 w_1
\end{aligned}$$

with A_2 and B_2 constants. The first term is the expected surplus value from selling in period 2 the quantity x_2, in stead of in period 3 or 4. It is the difference between revenues from selling in period 2 and expected revenues from selling in period 3 or 4, or not selling at all. Define:

(A2.16)
$$\begin{aligned}\Psi_3 &= \sigma\delta\sum_{k=1}^{K} f_3^k\left(p_3^k - c_3 - \sigma\delta\sum_{l=1}^{K} f_4^l\left(p_4^{kl} - c_4\right)^+\right)^+ \\ &\quad +\sigma^2\delta^2\sum_{k=1}^{K}\sum_{l=1}^{K} f_3^k f_4^l\left(p_4^{kl} - c_4\right)^+ \\ &= \sigma\delta\sum_{k=1}^{K} f_3^k\left(p_3^k - c_3 - \Psi_4\left(p_3^k\right)\right)^+ + \sigma\delta\sum_{k=1}^{K} f_3^k\Psi_4\left(p_3^k\right)\end{aligned}$$

Now (A2.15) can be written as: $x_2\ (p_2 - c_2 - \Psi_3) + A_2 + B_2 w_1$. From this it follows again, that the producer will only sell the minimally required quantity, if $p_2 - c_2 - \Psi_3 < 0$: $x_2 = x_2^-$. If $p_2 - c_2 - \Psi_3 > 0$, it turns out to be optimal to sell in period 2 in stead of in one of the later periods. The producer will sell the maximum quantity possible, i.e. the stock remaining from the first period, minus the quantities which have to be saved for future periods: $x_2 = w_1 - x_3^-/\delta - x_4^-/\delta^2$. If $p_2 - c_2 - \Psi_3 = 0$, the optimal solution is not unique. Optimal supply for period 2 can be written as:

(A2.17)
$$\begin{cases}\text{if } p_2 < c_2 + \Psi_3 & x_2 = x_2^- \\ \text{if } p_2 \geq c_2 + \Psi_3 & x_2 = w_1 - \dfrac{x_3^-}{\delta} - \dfrac{x_4^-}{\delta^2} \\ \text{if } p_2 < c_2 + \Psi_3 & \text{any solution } x_2 \text{ between } x_2^- \text{ and } w_1 \text{ is optimal}\end{cases}$$

Optimal supply in period 1:

Analogously, we can derive the supply function for period 1. When the producer decides on his optimal supplies for period 1, he knows the initial stock level, w_0, and the price in period 1, p_1. The possible price realisations, and probability distributions of the stochastic prices for the periods 2, 3 and 4 have been defined in (A2.1) - (A2.3) – see also (9.42) and (9.43). Consider the supply problem for period 1 – see (9.44):

$$\begin{aligned} &z_1(w_0; p_1) = \\ &\underset{x_1, x_2^k, x_3^{kl}, x_4^{klm}}{Max} \Bigg\{ (p_1 - c_1)x_1 + \sigma \sum_{k=1}^{K} f_2^k \Big[(p_2^k - c_2)x_2^k + \sigma \sum_{l=1}^{K} f_3^l \Big[(p_3^{kl} - c_3)x_3^{kl} \\ (\text{A2.18}) \quad & + \sigma \sum_{m=1}^{K} f_4^m (p_4^{klm} - c_4) x_4^{klm} \Big] \Big] \Bigg| \; x_1 + \frac{x_2^k}{\delta} + \frac{x_3^{kl}}{\delta^2} + \frac{x_4^{klm}}{\delta^3} \le w_0, \;\; x_1 \ge x_1^-, \\ & x_2^k \ge x_2^-, \; x_3^{kl} \ge x_3^-, \; x_4^{klm} \ge x_4^-; \; k, l, m = 1, \ldots, K \Bigg\} \end{aligned}$$

For producer price p_2^k, supply in period 2, x_2^k, satisfies (A2.17). For producer prices p_2^k and p_3^{kl}, supply in period 3, x_3^{kl}, satisfies (A2.12). For producer prices p_2^k, p_3^{kl}, and p_4^{klm}, supply in period 4, x_4^{klm}, satisfies (A2.7). Change the definitions of (A2.14) and (A2.16) into:

$$\Psi_4(p_3^{kl}, p_2^k) = \sigma\delta \sum_{m=1}^{K} f_4^m (p_4^{klm} - c_4)^+$$

$$\Psi_3(p_2^k) = \sigma\delta \sum_{l=1}^{K} f_3^l \left(p_3^{kl} - c_3 - \Psi_4(p_3^{kl}, p_2^k) \right)^+ + \sigma\delta \sum_{l=1}^{K} f_3^l \Psi_4(p_3^{kl}, p_2^k)$$

Introduce the following sets:

$$\begin{aligned} &K_2^1 = \left\{ k = 1, \ldots, K \,\middle|\, p_2^k < c_2 + \Psi_3(p_2^k) \right\} \\ &K_2^2 = \left\{ k = 1, \ldots, K \,\middle|\, p_2^k \ge c_2 + \Psi_3(p_2^k) \right\} \\ (\text{A2.19}) \; &L_3^{1k} = \left\{ l = 1, \ldots, K \,\middle|\, p_3^{kl} < c_3 + \Psi_4(p_3^{kl}, p_2^k) \right\} \end{aligned}$$

$$L_3^{2k} = \left\{ l = 1,\dots,K \,\middle|\, p_3^{kl} \geq c_3 + \Psi_4\left(p_3^{kl}, p_2^k\right)\right\}$$
$$M_4^{1kl} = \left\{ m = 1,\dots,K \,\middle|\, p_4^{klm} < c_4 \right\}$$
$$M_4^{2kl} = \left\{ m = 1,\dots,K \,\middle|\, p_4^{klm} \geq c_4 \right\} \text{ for } k,l \in \{1,\dots,K\}$$

Optimal supply in period 4 may be written as $x_4^{klm} = w_3 = \left(\left((w_0 - x_1)\delta - x_2^k\right)\delta - x_3^{kl}\right)\delta$, for $m \in M_4^{2kl}$, see (A2.7). For $l \in L_3^{2k}$, optimal supply in period 3 may be written as $x_3^{kl} = w_2 - x_4^- / \delta = \left((w_0 - x_1)\delta - x_2^k\right)\delta - x_4^- / \delta$, see (A2.12). For $k \in K_2^2$, the optimal supply in period 2 may be written as $x_2^k = (w_0 - x_1)\delta - x_3^- / \delta - x_4^- / \delta^2$. In that case supply in period 3 and 4 are the minimally required quantities, x_3^- and x_4^- even for $l \in L_3^{2k}$ and $m \in M_4^{2kl}$. Using (A2.7), (A2.12), (A2.17) and (A2.19), we can rewrite the objective of model (A2.18) as follows – see also (A2.15):

$$x_1\left(p_1 - c_1 - \sigma\delta \sum_{k \in K_2^2} f_2^k \left(p_2^k - c_2\right) - \sigma^2\delta^2 \sum_{k \in K_2^1} \sum_{l \in L_3^{2k}} f_2^k f_3^l \left(p_3^{kl} - c_3\right) \right.$$
$$\left. -\sigma^3\delta^3 \sum_{k \in K_2^1} \sum_{l \in L_3^{1k}} \sum_{m \in M_4^{2kl}} f_2^k f_3^l f_4^m \left(p_4^{klm} - c_4\right) \right) + A_1 + B_1 w_0 =$$

$$= x_1\Bigg(p_1 - c_1 - \sigma\delta \sum_{k\in K_2^2} f_2^k \left(p_2^k - c_2\right)$$

$$\sigma^2\delta^2 \left\{ \sum_{k\in K}\sum_{l\in L_3^{2k}} f_2^k f_3^l \left(p_3^{kl} - c_3\right) - \sum_{k\in K_2^2}\sum_{l\in L_3^{2k}} f_2^k f_3^l \left(p_3^{kl} - c_3\right) \right\}$$

$$-\sigma^3\delta^3 \Bigg\{ \sum_{k\in K}\sum_{l\in K}\sum_{m\in M_4^{2kl}} f_2^k f_3^l f_4^m \left(p_4^{klm} - c_4\right) - \sum_{k\in K_2^2}\sum_{l\in K}\sum_{m\in M_4^{2kl}} f_2^k f_3^l f_4^m \left(p_4^{klm} - c_4\right)$$

$$-\left\{ \sum_{k\in K}\sum_{l\in L_3^{2k}}\sum_{m\in M_4^{2kl}} f_2^k f_3^l f_4^m \left(p_4^{klm} - c_4\right) - \sum_{k\in K_2^2}\sum_{l\in L_3^{2k}}\sum_{m\in M_4^{2kl}} f_2^k f_3^l f_4^m \left(p_4^{klm} - c_4\right) \right\}\Bigg\}\Bigg)$$

$$+A_1 + B_1 w_0 =$$

$$= x_1\Bigg(p_1 - c_1 - \sigma\delta \sum_{k\in K_2^2} f_2^k \Bigg(p_2^k - c_2 - \sigma\delta \sum_{l=1}^{K} f_3^l \Big(p_3^{kl} - c_3 -$$

$$-\sigma\delta \sum_{m=1}^{K} f_4^m \left(p_4^{klm} - c_4\right)^+ \Big)^+ - \sigma^2\delta^2 \sum_{l=1}^{K}\sum_{m=1}^{K} f_3^l f_4^m \left(p_4^{klm} - c_4\right)^+ \Bigg)$$

$$-\sigma^2\delta^2 \sum_{k=1}^{K}\sum_{l=1}^{K} f_2^k f_3^l \left(p_3^{kl} - c_3 - \sigma\delta \sum_{m=1}^{K} f_4^m \left(p_4^{klm} - c_4\right)^+ \right)^+$$

$$-\sigma^3\delta^3 \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{m=1}^{K} f_2^k f_3^l f_4^m \left(p_4^{klm} - c_4\right)^+ \Bigg) + A_1 + B_1 w_0 =$$

(A2.20) $$= x_1\left(p_1 - c_1 - \sigma\delta \sum_{k=1}^{K} f_2^k \left(p_2^k - c_2 - \Psi_3\left(p_2^k\right)\right) - \sigma\delta \sum_{k=1}^{K} f_2^k \Psi_3\left(p_2^k\right) \right) + A_1 + B_1 w_0$$

with A_1 and B_1 constants. The term between brackets is the expected surplus value from selling now in stead of in one of the periods 2, 3 or 4. It is the difference between revenues from selling in period 1 and expected revenues from selling in period 2, 3 or 4, or not selling at all. Define:

(A2.21) $$\Psi_2 = \sigma\delta \sum_{k=1}^{K} f_2^k \left(p_2^k - c_2 - \Psi_3\left(p_2^k\right)\right)^+ + \sigma\delta \sum_{k=1}^{K} f_2^k \Psi_3\left(p_2^k\right)$$

(A2.20) can be written as $x_1(p_1 - c_1 - \Psi_2) + A_1 + B_1 w_0$. If $p_1 - c_1 - \Psi_2 > 0$, it is more profitable to sell now, then to store the goods and sell in one of the future periods. In that case the producer supplies the maximum quantity possible, i.e. the available supply w_0, minus the minimum quantities which have to sold in the future periods: $x_1 = w_0 - x_2^- / \delta - x_3^- / \delta^2 - x_4^- / \delta^3$. If $p_1 - c_1 - \Psi_2 < 0$, it is more profitable to sell in this period only the minimally required quantity x_1^-, and to sell the remainder in the future periods. If $p_1 - c_1 - \Psi_2 = 0$, the optimal solution is not unique. Optimal supply for period 1 can be written as:

$$\text{(A2.22)} \quad \begin{cases} \text{if } p_1 < c_1 + \Psi_2 & x_1 = x_1^- \\ \text{if } p_1 \geq c_1 + \Psi_2 & x_1 = w_0 - \dfrac{x_2^-}{\delta} - \dfrac{x_3^-}{\delta^2} - \dfrac{x_4^-}{\delta^3} \\ \text{if } p_1 \geq c_1 + \Psi_2 & \text{any solution } x_1 \text{ between } x_1^- \text{ and } w_0 \text{ is optimal} \end{cases}$$

We have shown that in each period $t = 1,...,4$, the optimal cereal supply, x_t, is:

- the minimally required quantity $x_t = x_t^-$, if the price is below a border level, $p_t < c_t + \Psi_{t+1}$
- the available stock minus quantities to be saved for minimally required future sales – see (A2.7), (A2.12), (A2.17) and (A2.22), if the price is above the border level, $p_t > c_t + \Psi_{t+1}$. Call this quantity x_t^+:
  $$x_t^+ = w_{t-1} - \sum_{\tau=t+1}^{4} \frac{x_\tau^-}{\delta^{\tau-t}}$$
- any solution between the minimum and maximum quantities is optimal, if the price is equal to the border level, $p_t = c_t + \Psi_{t+1}$.

Ψ_2 is defined in (A2.21), Ψ_3 is defined in (A2.16), Ψ_4 is given in (A2.11), and $\Psi_5 = 0$. The supply functions are presented schematically in Figure A2.1.

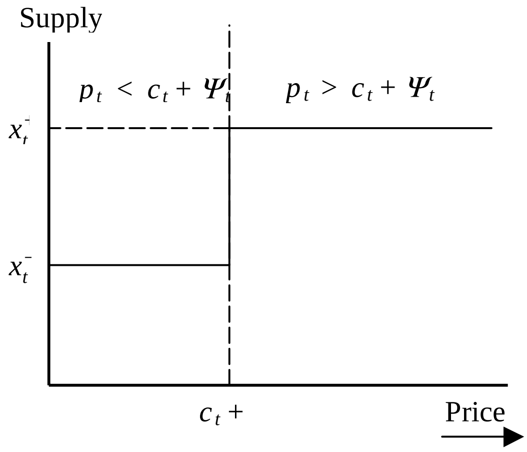


Figure A2.1: Schematic representation of the optimal supply for the periods 1, 2, 3 and 4.

Appendix 3: Parameter estimation

In Section 8.1 a survey is given of empirical evidence of cereal supply and demand, both in terms of quantities and timing. This survey is based on a review of a number of studies on cereal trade, supply and demand, which have been performed in Burkina Faso in the past. The most important results of these studies will be discussed in this Appendix. A comparison of the different studies is not given in this Appendix, but in Section 8.1. Results of the studies are used in Section 9.1 and 9.2 to estimate the parameters of cereal demand functions and of producer supply behaviour.

A3.1 Urban and rural population

In Section 8.1.1 the size of the urban and rural population is estimated for the year 2000. These estimates are used in Section 9.1 and 9.2 to estimate aggregate regional demand and supply. Estimates of the urban and rural population are based on the 1985 and 1996 census (1995a,b, 1998). Based on these data, expected growth rates of the urban and rural population can be calculated (see Table A3.1).

A remarkable observation is the annual growth of the urban population in Mouhoun of 18.78%. The reason for this is that the demographic surveys define an area as ‘urban’ if it has a certain minimal socio-economic and administrative infrastructure (administration services, schools, electricity, water), and if it houses more than 10,000 people (INSD, 1995a). In two of the three provinces in the CRPA Mouhoun (Mouhoun and Kossi) no areas were characterised as ‘urban’ in 1985, which had changed in 1996. Therefore, the urban population in these two provinces increased from zero in 1985 to 68,394 in 1996, resulting in a yearly increase of 18.78% for the entire CRPA. The same applies for the CRPA Centre Est, where the urban population in the province of Kouritenga increases from zero in 1985 to 53,339 in 1996. Since it is not realistic to assume that the yearly urbanisation rate continues to be that high, these rates for the CRPA Mouhoun and Centre Est are supposed to be the same as the

Table A3.1 Urban and rural population in 1985 and 1996, and population growht rates.

Population[1]	Population 1996			Population 1985			annual growth 1985-1996[2]		
CRPA	Total	Urban	Rural	Total	Urban	Rural	Total	Urban	Rural
Centre	1514048	709736	804312	959743	441514	518229	4.23%	4.41%	4.08%
Centre Nord	928321	51851	876470	729189	25814	703375	2.22%	6.55%	2.02%
Centre Ouest	989766	95024	894742	827419	51926	775493	1.64%	5.65%	1.31%
Centre Sud	530696	17146	513550	565227	14242	550985	-0.57%	1.70%	-0.64%
Sahel	708332	23768	684564	521911	10956	510955	2.82%	7.29%	2.69%
Mouhoun	1146689	83612	1063077	889803	12588	877215	2.33%	18.78%	1.76%
Est	934275	42920	891355	682246	20857	661389	2.90%	6.78%	2.75%
Centre Est	772530	84805	687725	600722	23331	577391	2.31%	12.45%	1.60%
Nord	955420	86982	868438	760408	53057	707351	2.10%	4.60%	1.88%
Sud Ouest	518343	16424	501919	456375	10657	445718	1.16%	4.01%	1.09%
Hauts Bassins	988988	326352	662636	721695	228668	493027	2.91%	3.29%	2.72%
Comoe	325201	62548	262653	249967	35319	214648	2.42%	5.33%	1.85%
Total	10312609	1601168	8711441	7964705	928929	7035776	2.38%	5.07%	1.96%

1) INSD, 1995a,b, 1998, 2) Annual population growth (in %) = $((\text{pop.1996}/\text{pop.1985})^{1/11}-1)*100\%$

national average: 5.07%. For the other CRPA's the growth rates are not exceptionally high. For all CRPA's it is supposed that the expected rural and urban growth rates after 1996 remain the same as presented in Table A3.1. The expected urban, rural and total population can now be estimated for the year 2000, see Table 8.1 in Section 8.1.1. These population figures are estimated as:

expected urban population 2000 = *urban population* 1996 * (1 + *urban growth rate*)^4
expected rural population 2000 = *rural population* 1996 * (1 + *rural growth rate*)^4
expected total population 2000 = *urban population* 2000 + *rural population* 2000.

A3.2 Cereal production

The annual cereal production per producer, is an important determinant of annual cereal supplies. Annual cereal supply, estimated in Section 9.2, is based on forecasted mean cereal production per producer. Production forecasts discussed in Section 8.1.2, are based on production, cultivated area and yield data for all produced crops, which are published each year by the 'Direction des Statistiques Agro-Pastorales' of the Ministry of Agriculture and Animal Resources. Aggregating the production data of millet, red sorghum, white sorghum and maize gives the total cereal production per year, and the average cereal production for the period 1984-1998 (see Table A3.2). The data presented in Table A3.2 to Table A3.5 and Figures A3.1 to A3.3 enable us to make the following observations:

1. Production, cultivated area and yield levels show a clear trend. Regression analysis shows that national production, cultivated area and yield levels increase significantly with a linear trend (at the 99% significance level). Production increases a bit faster than cultivated area, since this increase is caused by both area expansion and yield improvement.
2. Production increases per CRPA between 1984 and 1998 were also significant at the 95% level for most CRPA (except for Centre (significant at 80% level), Centre Ouest (significant al 90% level), Centre Sud (significant at 80% level), and Comoé (significant at 70% level)).

3. Area increases were significant at the 95% significance level for the CRPA Sahel, Centre Ouest, Mouhoun, Est, and Comoe, and at the 80% significance level for the CRPA Nord and Hauts Bassins. Some of the area increases were rather striking – see Table A3.3. Area increases in the CRPA Mouhoun may be logical because of land reclamation programmes along the Volta Noire river. It is, however, uncertain whether area increases as reported for the CRPA Sahel are lasting. This region is not very suitable for agriculture, and it is therefore, risky to suppose that the cultivated area continuous to increase as predicted by extrapolation of the trend line. Also differences between years are large. In some years acreage increases explosively, in other years, acreage decreases.
4. In 1991 average yields show a sharp increase (see Figure A3.1). Table A3.4 shows that compared to the period 1984-1990, the average cultivated area between 1991-1998 was 15% higher, average yield was 28% higher and average production even 48%. Although the pattern is the same for most CRPA, some CRPA show on average a decreased cultivated area during the last seven years. On the other hand, for example the CRPA Sahel shows an increased yield of 55% and an increased production of even 100%. It is not realistic to assume that such increases proceed for the years following this period. These high increases are probably partly caused by favourable rainfall during those years – see Table A3.5. Figure A3.1 also shows the trend lines if the period is cut in two: the period 1984-1990 and the period 1991-1998. Yield levels and production in the second period show a total different trend from the trend in the first period, they even slightly decrease. However, because of the few observations, the trend lines are not significant. Despite this, the figure shows that it is risky to assume that production increases yearly as presented in the 1984-1998 trend line.
5. Comparing the yearly expansion (decrease) of cultivated area with the rural population growth presented in Table A3.1 reveals that for some CRPA cultivated area increases faster than rural population, whereas in most CRPA population growth exceeds cultivated area growth (see Table A3.3). For the country as a whole, acreage expansion is lower than rural population growth. This shows that in total farmers cultivate less land per person every year.

6. Production, yield and acreage depend for a large part on rainfall. The tables and figures show that production, yields and area cultivated were lower than average in 1984, 1987, 1990 and 1997. These lower productions were mainly caused by low rainfall. Rainfall data of the National Meteorological Institute of Burkina Faso show that the 1984-'98 average yearly rainfall was 711 mm, while only 531 mm of rain fell in 1984, 601 mm in 1987, 577 mm in 1990, and 663 mm of rain fell in 1997 (see table Table A3.5; see also figure A3.3**)**. Linear regression analysis shows that production, yield and acreage depend significantly (at 99% significance level) on rainfall. This dependence is, however, not clear for all years. For example for the years 1992 and 1993 rainfall decreases with 13% compared to 1991, which is not reflected in lower cereal production. Production even increases slightly in 1992 and 1993. Regression results show that production forecasts depend for 58% on rainfall (R^2 = .58), yield for 47% and the area cultivated for 51%. Rainfall data per CRPA (aggregates for the rainfall stations in each CRPA) did not demonstrate the same dependance of production, yield and acreage on rainfall for all CRPA. Rainfall had a significant influence (90% significance level) on production in the CRPA Centre Nord, Centre Sud, Sahel, Mouhoun, Est and Nord, on cultivated area in the CRPA Sahel, Mouhoun, Centre Est, Est and Nord, and on yield in the CRPA Centre Nord, Centre Ouest, Centre Sud, Sahel, Est and Nord.

Table A3.2 Area cultivated with cereals (ha/year), cereal production (tonnes/year), and average cereal yields (kg/ha/year) for each CRPA.

Production	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
Centre															
Area	197910	171575	186629	109727	196623	258140	197000	189000	196300	196346	203580	177638	181580	201529	216342
production	64227	113791	126353	46576	134871	170745	110700	163300	158900	156542	120157	146922	128927	83912	160239
average yield[1]	325	663	677	424	686	661	562	864	809	797	590	827	710	416	741
Centre Nord															
Area	241556	247012	293209	250032	303680	317519	217000	201000	235600	293113	319035	250013	249264	268876	279946
production	88709	122436	158125	81666	216879	135118	107000	164900	205000	186034	201992	180514	168645	109013	231723
average yield	367	496	539	327	714	426	493	820	870	635	633	722	677	405	828
Centre Ouest															
Area	184065	236443	274106	289740	298990	302641	320800	297600	293100	332888	365908	287705	286859	286501	303941
production	102276	165127	176629	161211	229037	211612	165000	218400	131400	289408	269952	208404	228080	171668	208200
average yield	556	698	644	556	766	699	514	734	448	869	738	724	795	599	685
Centre Sud															
Area	168363	177319	199505	199758	193088	93791	179200	212500	207000	187150	201275	172568	162901	175038	195904
production	91075	136229	150079	112041	122314	71166	88000	203100	196100	169555	124066	145579	157873	137504	166518
average yield	541	768	752	561	633	759	491	956	947	906	616	844	969	786	850
Sahel															
Area	132367	200741	201440	145095	221519	298341	252000	268100	279200	286515	335981	252926	236564	236357	248207
production	56084	62621	63830	46251	113210	109522	94100	182700	195200	155680	213183	119872	105453	93309	184133
average yield	424	312	317	319	511	367	373	681	699	543	635	474	446	395	742

Cont. Table A3.2

Production	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**
Mouhoun															
Area	221322	270455	301380	327242	382496	388122	401000	478600	461500	480789	478129	419625	436827	502234	493242
production	172066	216453	253977	200907	297373	304094	244000	424600	456300	449722	379118	357454	464081	332820	393798
average yield	777	800	843	614	777	784	608	887	989	935	793	852	1062	663	798
Est															
Area	112047	194010	287390	267065	258562	257720	234000	276300	290700	311383	297152	312205	301974	301368	321799
production	77122	156138	226559	190170	198380	205413	149000	185200	216700	291796	263146	309639	285466	227489	318286
average yield	688	805	788	712	767	797	637	670	745	937	886	992	945	755	989
Centre Est															
area	143249	186338	190902	173044	218308	196549	62400	158800	168000	191673	197216	171353	188411	196424	183742
production	82965	126763	110108	114303	124823	153761	26300	119800	145300	188715	158743	202476	200341	141676	167278
Average yield	579	680	577	661	572	782	421	754	865	985	805	1182	1063	721	910
Nord															
area	139253	216939	156830	246040	275414	249011	214000	261700	271500	248850	243104	194625	227721	209791	268843
Production	72333	109694	90073	101005	190761	118703	63300	195500	160200	146924	169412	108319	176608	99046	230854
Average yield	519	506	574	411	693	477	296	747	590	590	697	557	776	472	859
Sud Ouest															
area	154069	150350	158814	195417	185626	188983	208000	192000	231000	173517	182467	175146	145803	160009	168165
Production	90083	109100	115504	114633	93696	112213	166000	173000	219000	160377	171893	156814	148596	162058	174906
Average yield	585	726	727	587	505	594	798	901	948	924	942	895	1019	1013	1040

Cont. Table A3.2

Hauts Bassins															
area	132437	141810	162370	178081	204413	227631	143000	156700	141800	202248	197391	187528	179121	190709	187208
Production	129248	143151	161915	172411	216960	235090	178000	254000	255800	203166	255428	218029	223040	275159	244848
Average yield	976	1009	997	968	1061	1033	1245	1621	1804	1005	1294	1163	1245	1443	1308
Comoe															
area	69205	67812	66295	74266	75007	81863	58000	65000	82400	62271	57810	49500	49058	52851	56119
Production	67183	72821	64021	67661	74356	69960	66000	94000	77000	82536	87080	58331	72049	79630	72550
Average yield	971	1074	966	911	991	855	1138	1446	934	1325	1506	1178	1469	1507	1293
Total															
Area	1895843	2260804	2478870	2455507	2813726	2860311	2486400	2757300	2858100	2966743	3079048	2650832	2646083	2781687	2923458
Production	1093371	1534324	1697173	1408835	2012660	1897397	1457400	2378500	2416900	2480455	2414170	2212353	2359159	1913284	2553333
Average yield	577	679	685	574	715	663	586	863	846	836	784	835	892	688	873

Notes: 1) Average yield levels are estimated by dividing total cereal production by total cultivated area. The ministry of agriculture estimates regional production by multiplying estimated yield (the average of a sample of measured yield levels) by estimated acreage for each crop. However, we consider here aggregate cereal production and acreage. These are estimated by adding up the production levels and cultivated areas of the different crops and provinces in a CRPA. Consequently, average ‘cereals’ yield levels can not be estimated on the basis of reported yield levels for each crop, but have to be based on total cereal production and total cultivated acreage.
Source: Ministère de l’agriculture et de l’élevage, 1984-1999.

Table A3.3 Average yearly growth of cultivated area between 1984 and 1998 and average yearly growth of the rural population.

	Growth of cultivated area[1]	Growth of rural population[2]
Centre	0.79%	4.08%
Centre Nord	0.25%	2.02%
Centre Ouest	1.66%	1.31%
Centre Sud	0.23%	-0.64%
Sahel	2.83%	2.69%
Mouhoun	4.25%	1.76%
Est	3.41%	2.75%

	Growth of cultivated area[1]	Growth of rural population[2]
Centre Est	0.68%	1.60%
Nord	1.44%	1.88%
Sud Ouest	-0.01%	1.09%
Hauts Bassins	1.39%	2.72%
Comoe	-2.23%	1.85%
Total	1.69%	1.96%

Notes: 1) Based on a regression of yearly cultivated area as a function of time, see Table A3.2 for data on cultivated area per CRPA. Growth of cultivated area = (predicted surface 1998/predicted surface 1984)$^{1/15}$ - 1; 2) Presented in Table A3.1.

Table A3.4 Average cultivated area (ha), production (tonnes) and yield (kg/ha) for the periods 1984-1990 and 1991-1998, and the percentage increase

	Average 84-90	Average 90-98	% increase
Centre			
Area	188229	195289	3.75%
production	109609	139862	27.60%
average yield	582	716	22.99%
Centre Nord			
Area	267144	262106	-1.89%
production	129990	180978	39.22%
average yield	487	690	41.90%
Centre Ouest			
Area	272398	306813	12.63%
production	172985	215689	24.69%
average yield	635	703	10.70%
Centre Sud			
Area	173003	189292	9.42%
production	110129	162537	47.59%
average yield	637	859	34.89%
Sahel			
Area	207358	267981	29.24%
Production	77945	156191	100.39%
average yield	376	583	55.05%
Mouhoun			
Area	327431	468868	43.20%
production	241267	407237	68.79%
average yield	737	869	17.87%
Est	0	0	0.00%
Area	230113	301610	31.07%
production	171826	262215	52.61%
average yield	747	869	16.43%

	Average 84-90	Average 90-98	% increase
Centre Est			
Area	167256	181952	8.79%
production	105575	165541	56.80%
average yield	631	910	44.13%
Nord			
Area	213927	240767	12.55%
production	106553	160858	50.97%
average yield	498	668	34.14%
Sud Ouest	0	0	0.00%
Area	177323	178513	0.67%
production	114461	170830	49.25%
average yield	645	957	48.25%
Hauts Bassins			
Area	169963	180338	6.10%
production	176682	241184	36.51%
average yield	1040	1337	28.65%
Comoe			
Area	70350	59376	-15.60%
production	68857	77897	13.13%
average yield	979	1312	34.04%
Total			
Area	2464494	2832906	14.95%
production	1585880	2341019	47.62%
average yield	643	826	28.42%

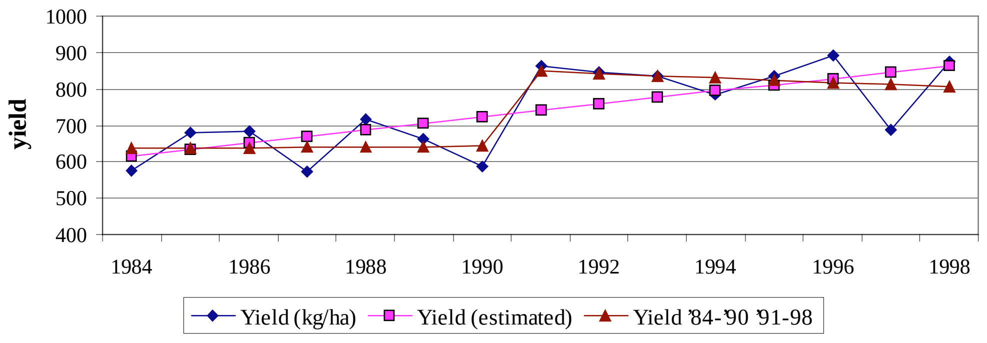
Table A3.5 Average yearly rainfall (in mm) and national cereal production (in 1000 tonnes) from 1984 to 1998.

	Average annual rainfall per CRPA[1]										Average rainfall Burkina Faso	Annual cereal prod.
CRPA	**Centre**	**Centre Nord**	**Centre Ouest**	**Centre Sud**	**Sahel**	**Mou-houn**	**Est**	**Centre Est**	**Nord**	**Hauts Bassins**		
Year	**(a)**	**(b)**	**(c)**	**(d)**	**(e)**	**(f)**	**(g)**	**(h)**	**(i)**	**(j)**	**(k)**	**(l)**
1984	511	473	602	622	301	676	522	532	443	789	531	1093
1985	617	437	764	764	345	782	534	628	501	1167	624	1534
1986	732	565	1021	1029	297	826	623	750	557	871	685	1697
1987	664	529	725	874	278	725	580	709	447	778	601	1409
1988	713	740	705	764	399	751	746	853	651	1060	713	2013
1989	658	579	772	1061	429	686	607	891	713	809	698	1897
1990	639	508	656	658	359	626	603	615	423	819	577	1457
1991	658	745	1037	1120	574	908	685	822	807	947	859	2379
1992	615	581	643	826	407	726	622	768	685	1028	697	2417
1993	715	587	776	962	266	823	658	592	548	793	668	2480
1994	718	588	760	826	543	1131	648	721	592	897	992	2414
1995	700	695	756	924	396	716	764	717	660	1278	779	2212
1996	677	558	826	1153	333	872	702	753	708	901	772	2359
1997	588	527	633	864	414	913	595	659	371	853	663	1913
1998	668	710	722	1068	594	990	830	803	782	1123	809	2553
Aver.[2]	658	588	760	901	396	810	648	721	592	941	711	1989

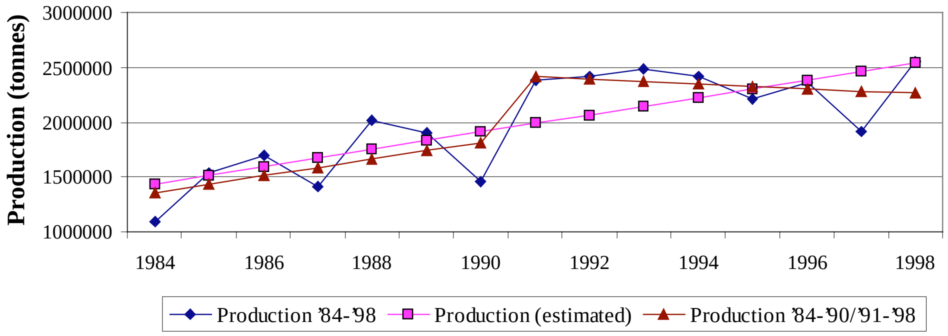
Notes: 1) Data are missing for the CRPA Sud Ouest and Comoe; 2) Average rainfall (column (a) – (k)) and average production (column (l)) for the period 1984 - 1998. a) – j) Averages of annual rainfall data for the following stations: a) Ouagadougou; b) Bam, Kaya; c) Koudougou, Leo; d) Po; e) Djibo, Dori, Arabinda; f) Boromo, Dedougou; g) Bogande, Diapaga, Fada N'Gourma, Kantchari; h)Tenkodogo, Zabre, Koupela; i) Ouahigouya, Yako; j) Hounde, Bobo Dioulasso. k) Average rainfall for Burkina Faso is the average over all stations. l) Annual cereal production is given in Table A3.2.
Source: Météo: National Meteorological Institute of Burkina Faso.

Figure A3.1: Average yield levels, cereal production and cultivated area.

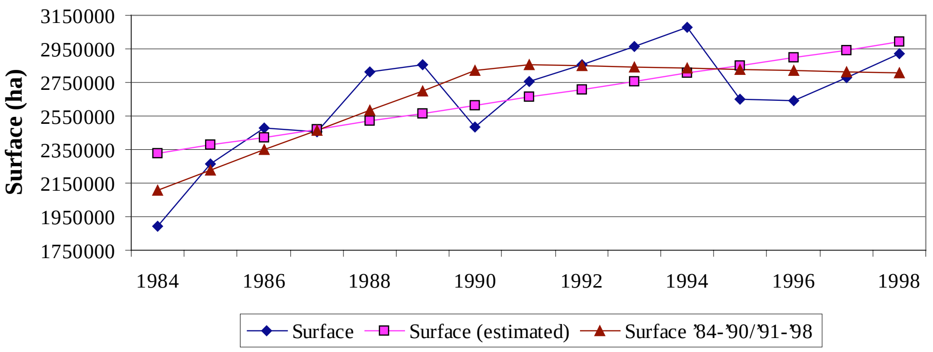
Average yield



Cereal production



Cultivated area



Source: Production, yield and surface data by DSAP of the Ministry of Agriculture.

Figure A3.2a: Average yield levels for each CRPA.

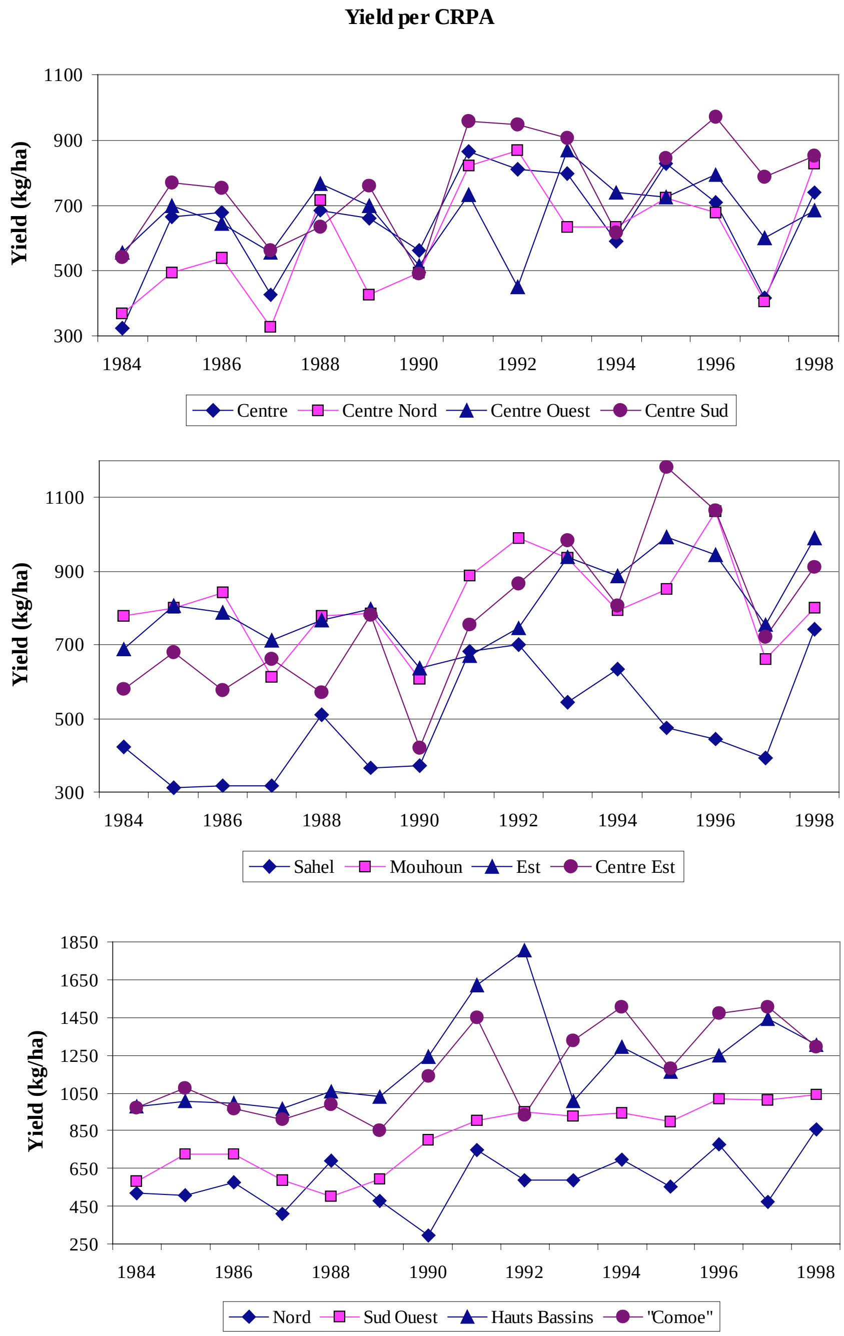
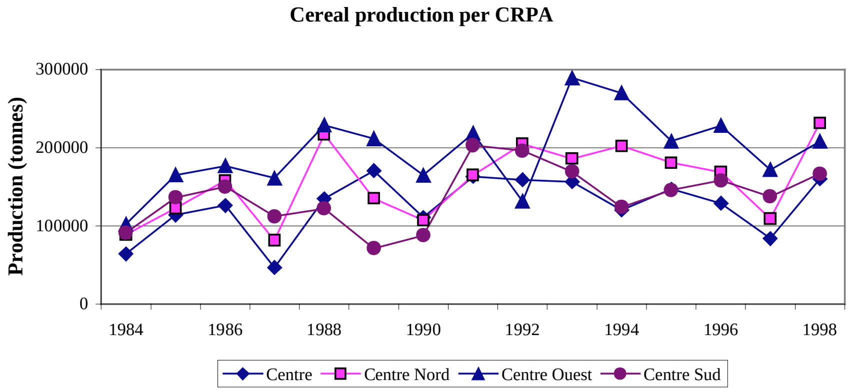
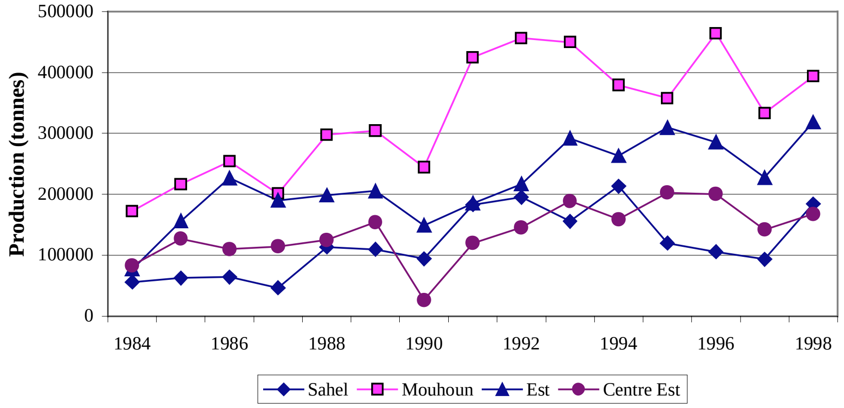


Figure A3.2b: Cereal production levels per CRPA





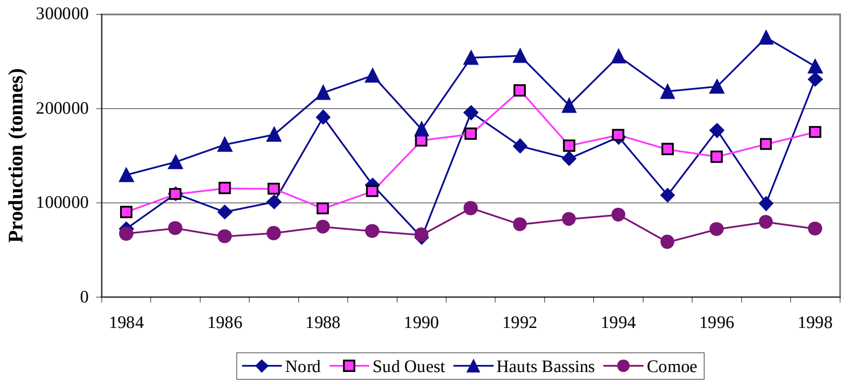


Figure A3.2c: Cultivated area per CRPA

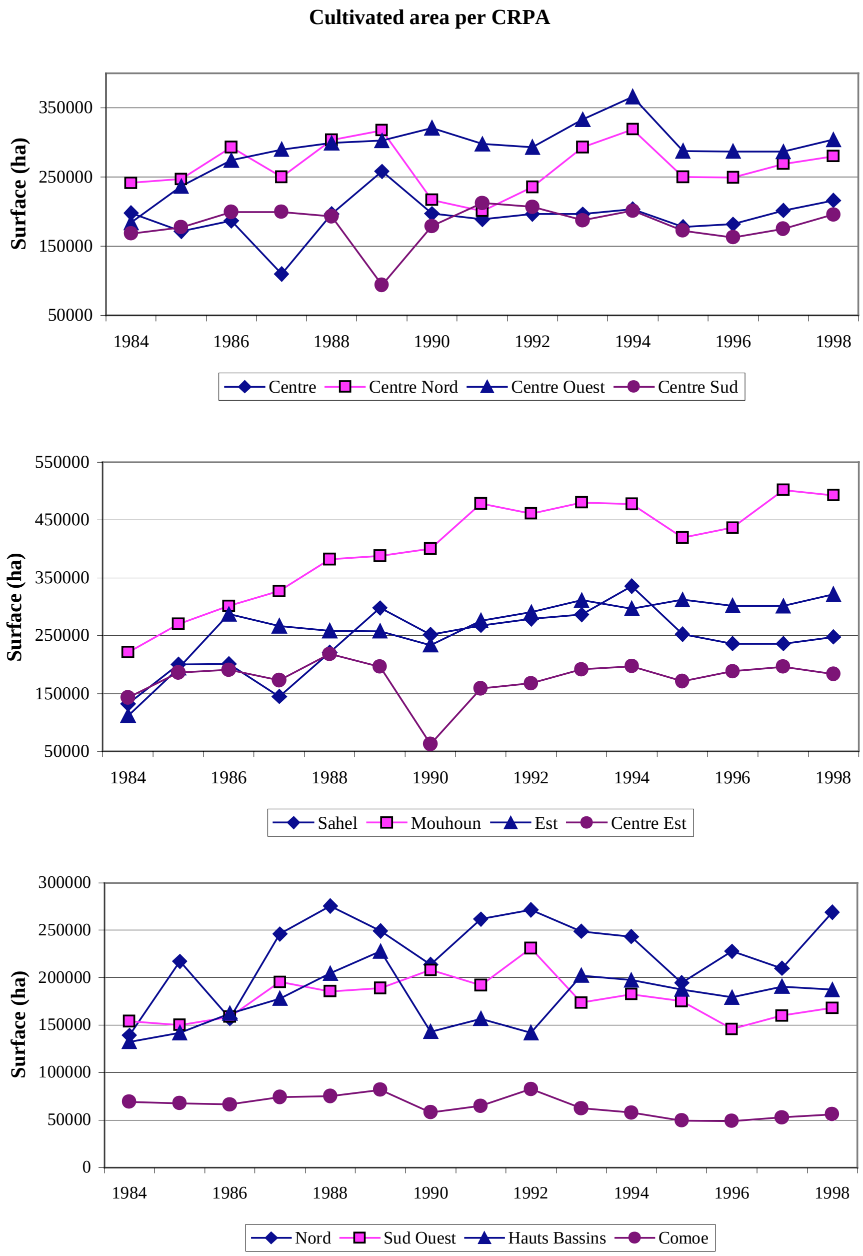
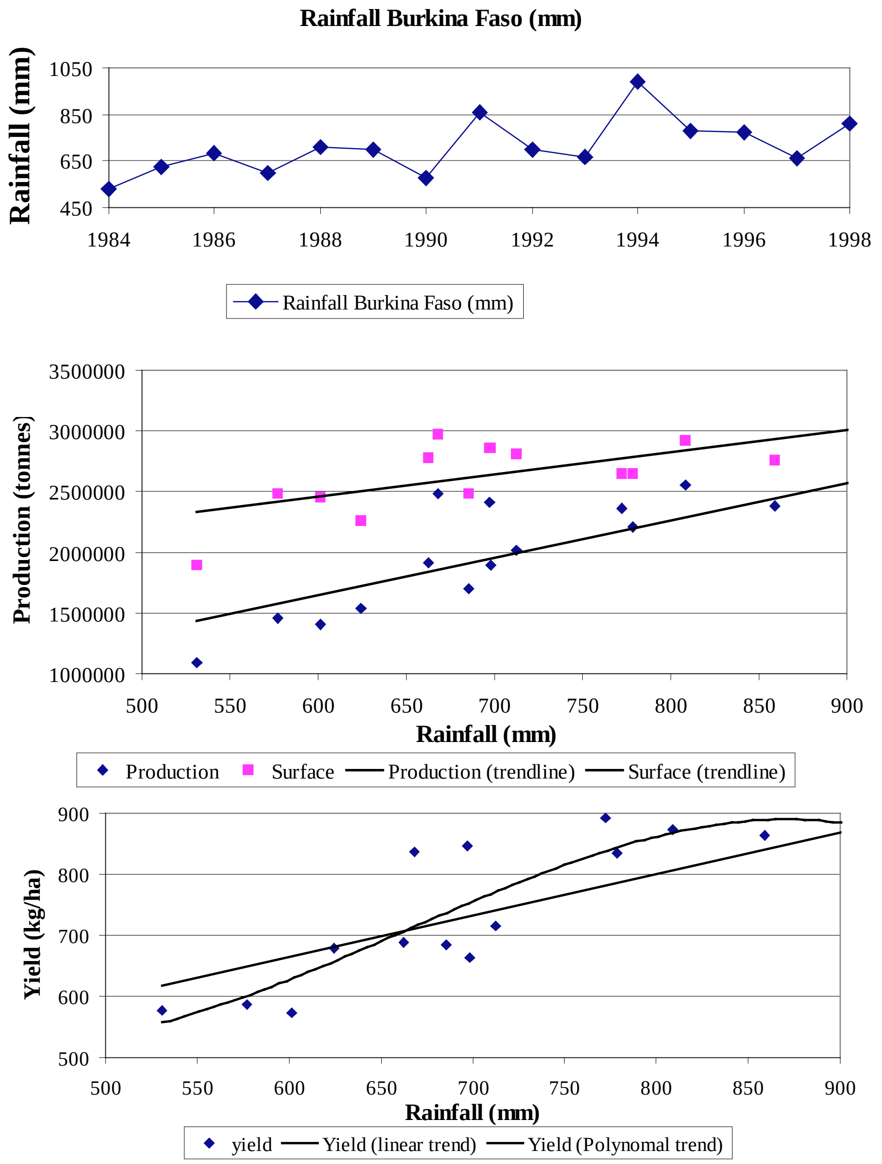


Figure A3.3: Production, yield and acreage as a function of rainfall



Source: Rainfall data by Météo, meteorological institute of Burkina Faso. Production, surface and yield data by DSAP of the Minstry of Agriculture and Animal Resources.

A3.3 Sales

In Section 8.1.3 factors influencing annual cereal sales and distribution of cereal sales over the year have been discussed. This discussion is based on a number of surveys performed in the past in Burkina Faso on cereal sales behaviour of households, which will be discussed below. The data presented are used in Section 9.2 to analyse producer supply behaviour.

Surveys by the Universities of Michigan and Wisconsin

The Universities of Michigan and Wisconsin executed in the unfavourable rainfall season 1983-‘84 a survey on the dynamics of grain marketing in Burkina Faso. Part of this survey concentrated on producer behaviour. In five villages across three ecological zones (the first five villages in Table A3.6) 224 households were sampled. Results of a study by SAFGRAD and Purdue University in the same year among 102 households in four villages were also used. Szarleta (1987) and Sherman et al. (1987) report on cereal sales in these nine villages (see Table A3.6). The difference between deficit and surplus villages is clear. In general, households in surplus villages sell a larger quantity of cereals and a larger part of their cereal harvest. Also the number of households selling cereals is larger in surplus villages than in deficit villages. Figures would be different in a normal rainfall year, but the pattern would probably remain similar.

ICRISAT surveys

ICRISAT carried out extensive farming systems studies in six villages in Burkina Faso from 1981 to 1985. They weekly surveyed 150 households from two villages in the province of Soum (Wouré and Silgey; average household size 10.2 people) in the north of Burkina Faso, two villages in the centre in the province of Passoré (Ouonon and Kolbila; average household size 12.2 persons) and two villages in the south-west in the province of Mouhoun (Sayero and Koho; average household size 12 people). Based on food production they classified households as surplus or shortage household. Shortage households were those which had a food production with a calo-

Table A3.6 Average cereal sales per household in different regions in the season 1983-84, reported by Szarleta (1987) and Sherman et al. (1987).

Province	Village[1]	Households selling cereals[2] (a)	Sales as % of total harvest (b)	Average cereal sales per selling household (kg) (c)	Average sales per sample household (kg) (d)	Average sales per sample household per AE (kg)[3] (e)
Yatenga	Menè (D)	7 (46)	3.1%	168	26	4.64
Passoré	Bougouré (D)	6 (42)	0.9%	13	2	0.3
Sourou	Tissi(S/D)	24 (40)	9.6%	85	51	8.4
Mouhoun	Dankui (S/D)	3 (42)	2.7%	169	12	2.5
Houet	Baré (S)	47 (50)	22.1%	638	600	84.1
Kossi	Dissankuy (S)	13 (27)	14.8%	374	180	39.5
Oubritenga	Nédogo (S)	28 (29)	20.7%	287	277	42.1
Zoundweogo	Poédogo (S)	18 (21)	24.9%	489	117	71.4
Gourma	Diapangou (S)	22 (25)	21.3%	444	391	71.5

Notes: 1) S = village with a surplus during the survey period, D = village in deficit during the survey period, S/D = village with a production which is more or less equal to the cereal consumption requirements of the village; 2) The total number of sample households between brackets; 3) Data for the first 5 villages are given in Consumer Equivalents (CE), and in Adult Equivalents (AE) for the last 4 villages. CE and AE are used to convert the lower consumption requirements and labour productivity of women and children in male consumption requirements and labour productivity units; (b) = total sample sales / total sample harvest; (c) = total sample sales / (a); (d) = total sample sales / total sample households; (e) = (d) / average household size in AE.
Source: Szarleta (1987), Sherman et al. (1987), Pardy (1987).

ric value below 80% of the WHO average yearly caloric requirement (requirements are 2,850 Kcal per adult equivalent; Reardon et al., 1988a). Reardon et al (1987) report on cereal sales per adult equivalent. These data do not exhibit a clear pattern between the regions. Sales by surplus households surpass sales of deficient households, though this difference is small in the province of Passoré. It has to be noted that the survey period comprised a period of severe drought with lower than usual production. This may cause the low production in the province of Mouhoun, and the abnormal feature that production in Soum exceeds production in the other, more fertile provinces. The sales pattern may not be representative for an average rainfall year.

Table A3.7 Cereals sales and production per adult equivalent between 1981-1985, by Reardon et al. (1987)

Province	Soum (shortage region)		Passoré (shortage region)		Mouhoun (equilibrium region)	
	Deficient household	Surplus household	Deficient household	Surplus household	Deficient household	Surplus household
Sales (kg)	3.27	15.2	6.8	8.6	6.7	19.0
Production (kg)	109	303	85	216	112	272
Sales (as % of production)	3%	5%	8%	4%	6%	7%

Source: Reardon et al. (1987)

Survey by J.T.Broekhuyse

Broekhuyse (1983, 1988, 1998) reports for 20 households in two villages in the province of Sanmatenga (Koalma and Basberikè) the sales by households applying manual labour (ML) or households using animal traction (AT) between 1979 and 1985. The average household size was 7.3 for ML housholds, and 10.2 people for AT households. He observes that on average ML households sell only 25.2 kg of cereals (3.8% of cereal production), and that AT households sell 58,5 kg of cereals (3.3% of cereal production).

Survey by O. Pieroni

Pieroni (1990) executed, under the authority of CILSS, a study of the behaviour of cereal producers. They interviewed between august 1986 and october 1987 114 households in 15 villages in the provinces of Houet (2 villages), Comoé (3 villages), Kénédougou (1 village), Kossi (3 villages), Sissili (3 villages) and Boulgou (3 villages). Rainfall in the 1986-'87 production season had been normal (see Table A3.5). He reports that in these villages 56.6% of the households sell more than 300 kg. He clearly observes a positive relation between production and the degree of market participation (see Table A3.8 and Table A3.9). In the survey year, the villages in Comoé have a shortage, and the villages in Sissili are either just in equilibrium or have a shortage. These villages lodge only a few households selling large quantities of cereals. In Sissili, households selling more than 1000 kg are absent. On the other hand, the villages in Houet, Kénédougou and Boulgou have a surplus and most households sell large quantities on the market. Hardly any household sells less than 100 kg. The province of Kossi holds an intermediate position, with two surplus villages and one shortage village. In both the shortage and surplus villages in this province, the number of households selling large quantities almost equals the households selling little (see Table A3.9).

The provinces of Houet, Kénédougou and Kossi are the cotton areas of the country, where the use of modern agricultural techniques is widespread. Despite the fact that much land and labour is allocated to cotton production, also cereal production is generally higher than in the other regions of the country. In Comoé sugar cane is produced on plantations using the most fertile soil and employing many of the young labourers. Cultivation of cereals is often not the main source of income, and is for a large part done by women and older farmers, without the use of modern techniques. In the provinces of Sissili and Boulgou modern agricultural techniques are not widespread, but soil fertility and rainfall are suitable. Trade conditions are also pretty favourable because of the presence of the Ghanian border. Pieroni clearly shows that sales are highly correlated with production. Although it does not apply for all households, it can be said that the more cereals a household produces, the more it sells. Those households are most of the time the larger households cultivating more land with more people. Other factors influencing household sales are capital needs, social relations (household composition and ethnic lineage) and the regional importance of the market (whether it is only a small local market or whether it is a larger market on which more products are traded).

Table A3.8 Cereal sales for some villages in 1986-87, by Pieroni (1990).

Villages/province[1]	N[2]	Average household size[3] (a)	Cultivated area per household (ha) (b)	Production per household (kg) (c)	Sales per household (kg) (d)	Sales per person (kg) (e)	Sales as % of production (f)
Zabré (S)	8	22.0	6.10	6340	2598	118.1	41.0%
Hono-bissa (S/D)	8	12.9	4.03	2488	1036	80.5	41.6%
Yoroko (S)	8	7.8	4.96	2660	1078	139.1	40.5%
Boulgou	24	14.2	5.03	3829	1571	110.6	41.0%
Fara Sissili (D)	8	15.8	3.42	2470	138	8.7	5.6%
Nabou (S/D)	8	15.3	4.87	2752	221	14.5	8.0%
Ton (S/D)	8	10.5	4.14	2060	232	22.1	11.3%
Sissili	24	13.8	4.14	2427	197	14.2	8.1%
Solenzo (S)	8	11.4	4.09	3139	1118	98.3	35.6%
Kié (D)	7	12.4	2.82	1936	541	43.5	28.0%
Lékoro (S)	6	10.8	7.56	3846	626	57.8	16.3%
Kossi	21	11.6	4.66	2940	785	67.9	26.7%
Dandé (S)	8	20.0	11.17	5148	1606	80.3	31.2%
Fara Kénédougou (S)	8	10.9	5.69	2994	1358	124.9	45.4%
Kouérédéni (S)	7	10.7	6.53	4443	2548	237.8	57.3%
Houet/Kénédougou	23	14.0	7.85	4184	1806	129.0	43.2%
Siniéna (D)	8	8.4	1.89	1127	155	18.5	13.7%
Diarabakoko (D)	7	13.6	3.88	1263	131	9.6	10.4%
Tangora (D)	7	13.0	2.71	1386	481	37.0	34.7%
Comoé	22	11.5	2.78	1253	251	21.8	20.0%

Note: 1) S = village in surplus during the survey period, D = village in deficit during the survey period, S/D = village with a production which is more or less equal to the cereal consumption requirements of the village; 2) The number of sample households in the sample villages; 3) Average number of household members. (e) = (d)/(a), (f) = (d)/(c)*100%. Source: Pieroni (1990).

Survey by E.P. Yonli

Yonli (1997) executed a survey from October 1991 to June 1993 in 4 villages in Yatenga (24 households; on average 14.6 people per household) and 4 villages in Sanmatenga (21 households; on average 11.1 people per household), see Table A3.10. In the 1991 agricultural season, rainfall was far above the 1970-’93 average in both provinces.[34] Production in the CRPA in which the villages are situated was above

[34] Yatenga: average rainfall: 550 mm, 1991 rainfall: 680 mm; Sanmatenga: average rainfall: 617 mm, 1991 rainfall: 821 mm. Data from National Meteorological Institute of Burkina Faso.

Table A3.9 Number of households (in %) selling less than 100 kg, between 100 kg and 1000 kg, or more than 1000 kg of cereals, for some villages in 1986-87, by Pieroni (1990).

Villages/Provinces[1)]	**N[2)]**	**< 100 kg**	**100-1000 kg**	**> 1000 kg**
Zabre (S)	8	12.5%	25.0%	62.5%
Hono-bissa (S/D)	8	0.0%	50.0%	50.0%
Yoroko (S)	8	0.0%	62.5%	37.5%
Boulgou	24	4.2%	45.8%	50.0%
Fara Sissili (D)	8	62.5%	37.5%	0.0%
Nabou (S/D)	7	42.9%	57.1%	0.0%
Ton (S/D)	8	37.5%	62.5%	0.0%
Sissili	23	47.8%	52.2%	0.0%
Solenzo (S)	8	50.0%	25.0%	25.0%
Kie (D)	7	28.6%	42.9%	28.6%
Lekoro (S)	6	33.3%	33.3%	33.3%
Kossi	21	38.1%	33.3%	28.6%
Dande (S)	8	0.0%	50.0%	50.0%
Fara Kenedougou (S)	8	0.0%	50.0%	50.0%
Koueredeni (S)	7	0.0%	0.0%	100.0%
Houet/Kenedougou	23	0.0%	34.8%	65.2%
Siniena (D)	8	50.0%	50.0%	0.0%
Diarabakoko (D)	7	71.4%	28.6%	0.0%
Tangora (D)	7	57.1%	14.3%	28.6%
Comoe	22	59.1%	31.8%	9.1%
Total		29.2%	39.8%	31.0%

Note: 1) See not 1 in Table A3.8; 2) Number of households in the sample. Source: Pieroni (1990).

average in both the '91-'92 and '92-'93 season, but nevertheless, both regions knew a shortage production, or were almost in equilibrium (see Table A3.2). For the sample villages, only the village of Noungou had a surplus production. It is remarkable to see that this is also the village with the smallest average household size. Both provinces produced mainly sorghum (Yatenga 61.7% and Sanmatenga 89.2% sorghum as percentage of cereal production). In the village of Noungou even 94.8% of cereal production consisted of sorghum. The data clearly show that, although a very small part of production was sold, sales were higher in the higher production province. The data per village show the same pattern. The differences between the villages are large, but even within a village differences are considerable. The standard deviation of the sold quantity was for most villages larger than the average sales.

Table A3.10 Cereal sales in some villages in Yatenga and Sanmatenga from October 1991 to September 1992, by Yonli (1997).

Villages/ Provinces[1]	**Number of households**	**Average household size (a)**	**Average sales per household (kg) (b)**	**Sales per person (kg) (c)**	**Production per person (kg) (d)**
Ramsa	3	14.3	5.0	0.35	128.9
Séguénéga	7	14.7	0.0	0.00	131
Kalsaka	7	15.6	0.0	0.00	111.0
Kossouka	7	13.4	3.2	0.24	132.4
Yatenga	**24**	**14.6**	**1.6**	**0.11**	**125.3**
Nessemtenga	1	12.0	0.0	0.00	n.a.[2]
Soubeira	5	14.2	26.8	1.89	172.6
Noungou	8	8.8	79.2	9.00	234.6
Singué	7	11.3	46.2	4.09	132.1
Sanmatenga	**21**	**11.1**	**57.6**	**5.24**	**179.7**

Notes: 1) The first four villages are in Yatenga, the last four villages are in Sanmatenga. (c) = (b)/(a). 2) n.a.: not available. Source: Yonli (1997)

Seasonal sales pattern

Some authors also report on the sales per season. Most authors observe, what has become a general characteristic of African agriculture, that farmers sell in the post-harvest, low-price season and buy in the pre-harvest, high-price season.

Surveys by Universities of Michigan and Wisconsin

The seasonal sales pattern is also observed by Sherman et al. (1987). Table A3.11 “reveals that the postharvest quarter is indeed the heaviest sales period for the largest number of households” (Ellsworth and Shapiro, 1989). On the other hand, the sales volume is more evenly distributed than is often thought. This indicates that many households have to sell small quantities immediately following the harvest. A smaller number of farmers can sell larger quantities later in the year. The sales pattern per village shows that especially in Baré, where by far the largest volume is sold (see Table A3.6), most households sell in the two post-harvest quarters from October to March. It has to be noted that cereal sales are only a part of the total amount of cereals that leave the farm. The quantity of cereals given to others via non-market transfers may be more than the amount sold. These gifts are often payments for agricultural work, and therefore a veiled form of sales (Ellsworth and Shapiro, 1989).

Table A3.11 Total cereal sales per quarter for five Burkina Faso villages in 1984, by Sherman et al. (1987).

Periods	**Jan-Mar**	**Apr-June**	**July-Oct 10**	**Oct 11-Dec**	**Total number of selling households[3]**
Sales (kg)[1]	9,520 (33%)	4,885 (17%)	6,347 (22%)	7,811 (27%)	
Number of households[2]					
Mené (Yatenga)	2	0	3	0	5 (46)
Bougouré (Passoré)	0	0	0	1	1 (42)
Tissi (Sourou)	1	1	1	9	12 (40)
Dankui (Mouhoun)	2	0	0	1	3 (42)
Baré (Houet)	16	4	4	23	47 (50)
Total	21	5	8	34	68 (220)

Notes: 1) Percent of annual total between brackets; 2) The data give the number of households in five villages with their largest volume of sales in a certain quarter. Only those households are considered who sold more than 25 kg; 3) Number of sample households between brackets. Source: Sherman et al (1987)

Pardy (1987) confirms the phenomenon of post-harvest sales, based on an analysis of the seasonal sales for the four surplus villages surveyed by SAFGRAD (see page A.284). He divides the year in four seasons: the harvest season from October to December, the dry season from January to March, the hot season from April to June and the rainy season from July to September. His data show that for three of the four villages sales were largest and most households were selling during the dry season (see Table A3.12). However, for two villages the largest sales per household are made during the rainy season when the prices are more favourable. Only in the village of Poédogo most cereals are sold during the harvest season. The large sales volume during the dry season in Dissankuy, which is in a cotton producing area, is striking.

These sales may be due to late payments of cotton sales, which compels households to earn money from other sources, for example cereal sales.

Table A3.12 Cereal sales per trimester for four villages from October 1983 to September 1984.

	Oct. – Dec.	Jan. – March	April - June	July - Sept.	Yearly total
Dissankuy (Kossi)	4[1] 790[2] 16.3%[3] 198[4]	9 1,640 33.8% 182	6 830 17.1% 138	9 1,599 32.9% 178	28 4,859 100% 174
Nédogo (Oubritenga)	20 1,749 21.8% 88	27 3,216 40.0% 119	18 1,420 17.7% 79	10 1,648 20.5% 165	75 8,033 100% 107
Poédogo (Zound-weogo)	17 4,204 47.8% 247	15 2,091 23.8% 139	11 868 9.9% 79	7 1,636 18.6% 234	50 8,799 100% 176
Diapangou (Gourma)	7 1,024 10.5% 146	20 5,196 53.2% 260	5 845 8.7% 169	7 2,700 27.7% 386	39 9,765 100% 250

Notes: 1) Number of households selling during that period; 2) total sales in kg; 3) percentage of yearly total; 4) sales per selling household in kg. Source: Pardy (1987).

Pardy also looks at sales differences between households of different wealth (see Table A3.13). He identifies very poor, poor, average and wealthy households on the basis of the total value of their livestock in December 1983 and their agricultural equipment in 1984. These posessions indicate the possibility to use other sources to face cash needs. The majority of households sells during the dry season. The number of households selling during the harvest season is also very large for all types of households, except the weatlhy. From those households which profit from the higher prices during the July – September period, most have an average wealth. They also have the largest sales per household. Of all households selling between April and June, the wealthy households sell the largest quantity. For all wealth groups, except the very poor, at least 30% of the households which sell during the year, also sell during the rainy season between July and September. Pardy notes that the richer households sell in less periods. 56% of the poor households sells in three periods, compared to 46% of the average and 38% of the wealthy households. This strategy corresponds to the need to sell cereals to satisfy capital requirements for the poor households.

Table A3.13 Cereal sales per trimester four different different types of households, for four villages from October 1983 to September 1984.

Wealth	Oct. – Dec.	Jan. - March	April - June	July- Sept.	Yearly total
Very poor	8[1] 448[2] 20.9%[3] 56[4]	13 1,206 56.2% 93	4 85 4.0% 21	4 408 19.0% 102	 2,147 100% 74
Poor	18 3,896 39.6% 216	21 2,962 30.1% 141	16 1,224 12.4% 77	9 1,769 18.0% 197	 9,854 100% 154
Average	17 2,150 20.0% 126	21 3,583 33.3% 171	12 1,008 9.4% 84	14 4,014 37.3% 287	 10,755 100% 168
Wealthy	5 1,273 14.6% 255	16 4,392 50.5% 275	8 1,646 18.9% 206	6 1,392 16.0% 232	 8,703 100% 248.7

Notes: 1) Number of households selling during that period; 2) total sales in kg; 3) percentage of yearly total; 4) sales per selling household in kg. Source: Pardy (1987).

ICRISAT surveys

Reardon et al (1987) also report on seasonal sales patterns for the period harvest 1981 to rainy season 1985 (see Table A3.14). Their data do not exhibit evidence of 'forced sales'. It does not show up that deficient households sell in the post-harvest period, and surplus households sell whenever prices are higher. It only follows, which was already reported in Table A3.7, that surplus households sell more than deficient households. The data in this table do not demonstrate a clear relation between production and sales. It has to be noted, however, that sales patterns during the survey years may have been different from normal because of the bad rainfall during these years, which were in almost all cases below average, and in some cases even dramaticaly low.[35]

Table A3.14 Seasonal sales in kg. per adult equivalent by region for the main cereal sold.

Seasons[1]	'81-'82				'82-'83				'83-'84				'84-'85			
	hr	**cl**	**ht**	**rn**	**Hr**	**cl**	**ht**	**Rn**	**hr**	**cl**	**ht**	**rn**	**hr**	**cl**	**ht**	**rn**
Soum, deficient[2]: millet	5	9			2				2	1	1		2			
surplus[2]: millet	6	5	3	3	1			14	2	9	7	2	15			
Passoré, deficient: w.sorghum	1	2	1	2	2	1	2	1	1	1	2	7				1
surplus: w.sorghum	2	1	2	1	3	2	9	4	5	5	1	3	2	1	1	
Mouhoun,deficient: w.sorghum	1				1	4	2	1		1				1	1	1
r.sorghum/rice	10	7	4		1	2	1				1			2	2	
surplus: w.sorghum	3	3	1			3	2			2	2	3	1	2	2	
r.sorghum/rice	3	3	7	3	5	5	7	7		1	2	2		2	4	

Notes: 1) Seasons: harvest (hr = sept-nov), cold (cl = dec-feb), hot (ht = mar-may), rainy (rn = june-aug); 2) Deficient refers to shortage households, surplus to the surplus households. Source: Reardon et al. (1987)

Survey by E.P. Yonli

Yonli (1997) gives monthly sales for some villages in Yatenga and Sanmatenga between October 1991 and June 1993. Monthly sales are, especially in Yatenga, very small, but data clearly show that sales are larger during the post-harvest season (from October till March). This is even more clearly seen from quarterly sales. For the survey villages in Yatenga, sales during the lean season are totally absent. The survey villages in Sanmatenga did sell during the second and third quarters, but less than in the other quarters, although sales during the second quarter of 1993 were larger than during the preceding harvest season. A reason for the higher sales during the first and second quarter of 1993 compared to the same quarters in 1992, may be caused by the average cereal production which was for the villages in Sanmatenga much larger in 1992 than in 1991 (179.7 kg per person in 1991 and 237.3 kg per person in 1992).

[35] For example, rainfall in Djibo in the CRPA Soum was in 1984 about one third lower than the 1970-1993 average rainfall. Rainfall in Dédougou in the CRPA Mouhoun was in 1981 25% below the 1970-1993 average rainfall.

Table A3.15 Montly sales in kg per person between October 1991 and June 1993, by Yonli (1997).

Month/Year	Yatenga	Sanmatenga
10-91	0	0
11-91	0.03	1.07
12-91	0.02	1.11
4th quarter 91	0.05	2.18
1-92	0	1.13
2-92	0.06	1.33
3-92	0	0.25
1st quarter 92	0.06	2.71
4-92	0	0.20
5-92	0	0.25
6-92	0	0.15
2nd quarter 92	0	0.60
7-92	0	0.11
8-92	0	0.10

Month/Year	Yatenga	Sanmatenga
9-92	0	0
3d quarter 92	0	0.21
10-92	0	0.08
11-92	0.16	0.30
12-92	0.04	0.62
4th quarter 92	0.20	1.00
1-93	0.02	1.36
2-93	0	2.15
3-93	0	1.42
1st quarter 93	0.02	4.93
4-93	0	0.60
5-93	0	1.46
6-93	0	0.91
2nd quarter 93	0	2.97

Note: In Yatenga 24 households and in Sanmatenga 21 households were surveyed. Source: Yonli (1997).

Survey by O. Pieroni

Finally, Pieroni (1990) observes that cereals are sold during the entire year. Nevertheless, in general a negative relation exists between sales volume and cereal price. Most cereals are sold during the post-harvest season. He points at a difference between richer and poorer households. Richer household have the opportunity to delay a part of their sales until prices are more favourable. For example in the province of Kossi almost 60% of cereal sales is done between July and November, wheras in Boulgou only 25% is sold in this period. In Boulgou, the largest part is sold between December and May. Differences between villages and households within a province are, however, considerable.

To summarize the above review, the different surveys indicate that only a small part of cereal production is sold on the market. Cereal production levels are the most important determinant of annual cereal sales, cereal prices are less important. Since cereal production levels differ considerably between years (see Appendix A3.2), sales levels will also fluctuate between the years. In good rainfall years, when cereal production is good, sales will be higher than in bad rainfall years. In general, surplus households sell larger quantities than shortage households. Differences between provinces but also within provinces are large. Cereal sales depend, however, also on many other factors. Seasonal studies show that most cereals are sold in the post harvest season. Data suggest, however, that wealthier households prefer to sell later in the year, when prices are higher. On the other hand poorer households are often obliged to sell earlier in the year, at low prices, in order to repay debts. The data presented in this section are used in Section 7.3 to estimate cereal supply functions.

A3.4 Purchases

Cereal purchase behaviour of Burkinabè consumers is discussed in Section 8.1.4. This discussion is based on a number of surveys performed in the past in Burkina Faso which are discussed below. To analyse cereal purchases, a distinction is made between rural and urban consumers. Urban consumers purchase all cereals on the market, wheras rural consumers purchase only a part of their cereals consumption on the market. A large part of their consumption comes from own production. In this appendix also the timing of cereal purchases and estimates of price and income elasticities of cereal demand are discussed. The data presented in this appendix are used in Section 9.1 to estimate cereal demand as a function of cereal prices. Before analysing purchase patterns of rural households, first the studies concentrating on urban cereal demand will be reviewed.

Urban studies

Reardon et al. (1988b) surveyed between October ’84 and September ’85 118 households in Ouagadougou. Their aim was to analyse the substition of traditional cereals (millet, sorghum and maize) by non-traditional cereals (rice and flour). Based on household revenues, they classified the households in three groups of equal size, called poor households (average income 5036 FCFA per adult equivalent (AE) per month), average households (average income 9082 FCFA/AE/month) and rich households (average income 15449 FCFA/AE/month). For each household they analysed the cereal consumption pattern. As expected, poor households spend a larger share of their expenses on cereals. The poor spend 31% of their expenses on cereals, the average households 23%, and the rich households only 16%. Daily purchases for the different strata were given for four seasons in grams per person per day. In Table A3.16 cereal purchases are given in percentages, since converting the daily purchases into yearly purchases resulted in absurd quantities per person.[36] They observed that all households consumed large quantities of rice, though in total more traditional cereals were used. As expected, the share of non-traditional cereals in daily meals is larger for richer households than for poor households. Differences between the quarters are not very large. Consumption of rice is almost the same in all periods. They also looked at prices. It appeared that poor households paid, in general a higher price for their cereals than richer households[37]. The main reason was that richer household had the opportunity to purchase in larger quantities and to purchase from the governmental cereal board OFNACER which mainly sold in 100 kg sacs.

The observations that rice consumption increases, is confirmed by the rice production and import data (see Table A3.17). This table shows that rice production and imports increase fast. These figures even seem to be very high if the rice consumption per person is calculated. If the total rice availability (production + imports) in 1996/97 is divided by the rural population (see Table 8.1), who consume much more rice than urban households, it is seen that the average rice consumption per urban consumer would be 100 kg. So, half their consumption would consist of rice. It is not realistic to assume that all rice is consumed by urban households, but even if rural households consume a part of the available rice, or if the data in the table are too high, the table shows that rice consumption increases fast.

[36] The rich should purchase 1682 kg of cereals per person per year. This is an absurd quantity, if you know that required consumption is approximately 190 kg per person.
[37] This difference were significant for rice and maize, but not for sorghum and millet.

Table A3.16 Cereal purchases per stratum per quarter in Ouagadougou between October 1984 and September 1985, Reardon et al. (1988b).

Purchases differences between non-traditional and traditional cereals:						
Household	**Cereal type[1]**	**Oct-dec**	**jan-march**	**apr-june**	**july-sept**	**Total**
Poor	Non-traditional	20%	18%	20%	20%	19%
	Traditional	80%	82%	80%	80%	81%
Average	Non-traditional	17%	20%	17%	19%	18%
	Traditional	83%	80%	83%	81%	82%
Rich	Non-traditional	38%	32%	28%	30%	32%
	Traditional	62%	68%	72%	70%	68%

Notes: 1) Non-traditional cereals comprise rice and flour; traditional cereals are millet, sorghum and maize.
Source: Reardon et al. (1988b).

Table A3.17 Local rice production and rice imports in tonnes.

	1993/94	**1994/95**	**1995/96**	**1996/97**	**1997/98**
Local production	53809	61009	84026	111807	89516
Imports	87087	40093	63060	97377	n.a.

Notes: n.a. = not available. Source: Ministère de l'Agriculture et CGP.

Rural Studies

Surveys by the Universities of Michigan and Wisconsin

Szarleta (1987) reports on the purchase behaviour of the five survey villages mentioned on page 284. Almost all sample household purchased cereals on the market. Differences between the villages were, however, large, as well as differences between households within a village. Table A3.18 shows that purchases in Baré, in the surplus zone, are smallest, whereas purchases in Mené, in the shortage zone, are largest. Millet and white sorghum are purchased by most households. Also red sorghum is purchased by many households in Tissi, Dankuie and Baré, a large part of which is used for *dolo* brewing. The data show that 92% of the households in Baré sold red sorghum, whereas 58% of them purchased it (on average less was purchased than sold). So, most purchasing households rebought a part of the red sorghum they sold. Furthermore, 39 of the 42 sample households in Bougouré and 30 of the 40 households in Tissi purchased white sorghum, whereas 5 households in Bougouré and

13 in Tissi sold it. So, at least some households rebought a part of their sold cereals. Unfortunately, for the other cereals it can not be retrieved whether households bought the same type of cereals as they had sold. It can, however, be expected that many households which sold cereals early in the season had to rebuy during the lean season. This phenomenon of rebuying the same type of cereals can partly be explained by the different roles and responsiblities of a chief and his wife (wives) and the division of tasks within a household. For example, if one wife of the household sells a part of the harvest from her personal fields, it is possible that another wife of the household has to purchase these cereals to feed her children. Except for Baré, household purchases are much larger than household sales (see Table A3.6). In 1983-84 Baré was the only surplus village in the sample. The other four villages were in shortage or just in equilibrium. Many households also received and offered cereals to other households as a gift. Szarleta (1987) observes large differences between the five villages. In Tissi, almost a third of total consumption was received from gifts (on average 294 kg per household). Households in this village also gave large quantities of cereals to others (on average 416 kg per household). For the other villages these gifts were much lower (on average 61 kg given and 70 kg received per household for the other four villages; Baré did not report any cereals given to others). Szarleta explains the importance of gifts in Tissi by the large muslim population in this village.

Table A3.18 Average cereal purchases per household in different regions in 1983-94, reported by Szarleta (1987).

Province	Village[2]	Households purchasing cereals[1]	Average cereal purchases per household (kg)	Average cereal purchases per consumption equivalent (kg)
Yatenga	Mené (D)	44 (46)	690	117
Passoré	Bougouré (D)	41 (42)	449	65
Sourou	Tissi (S/D)	38 (40)	636	99
Mouhoun	Dankuie (S/D)	40 (42)	389	77
Houet	Baré (S)	45 (50)	336	42

Note: 1) Number of sample households between brackets; 2) S = village in surplus during the survey period, D = village in deficit during the survey period, S/D = village with a production which is more or less equal to the cereal consumption requirements of the village. Source: Szarleta (1987).

Although this paper only considers the aggregate ‘cereals’, and not the individual crops, it is interesting to look for a moment at differences between the different crops. Table A3.19 displays production, sales, purchases and consumption disaggregated per crop for the five survey villages. Data are presented for all households of the sample villages. Data per household are not representative because an individual household does not produce or sell necessarily all types of crops. Another reason is that the number of households producing, selling or purchasing presented in Szarleta (1987) do not match. The number of selling households may be larger than the number of producing households, which is rather odd. Aggregate data probably show a more complete picture than individual data. The data also exhibit other odd patterns. First of all, sold quantities sometimes exceed production. This might be true if these crops were still in stock from the previous harvest. It is, however, unlikely that the shortage village of Mené has 700 kg of last years’ maize in stock. Secondly, consumption does not match with produced, sold and purchased quantities. These differences are influenced by the quantity of gifts received and offered, which Szarleta does not present per crop. What is striking is the large volume of red sorghum purchases and consumption in the last three villages. Probably a large part of this is consumed as *dolo*. Furthermore, in the northern village of Mené millet is consumed the most, wheras the other villages prefer white sorghum. In Baré, the richest and most fertile of the five villages, also large quantities of maize are consumed. The data reveal that in the south-western villages Tissi and Baré red sorghum is sold in large quantities. It can, however, not be concluded that it is sold to purchase other types of cereals with it, as is sometimes suggested. For the villages of Mené and Baré, maize also serves as an income generating crop, although the data for Mené can be questioned.

ICRISAT surveys

Reardon et al. (1987) showed that purchases were much larger than sales. They observed a clear difference between deficient and surplus households. The first group, naturally, purchased more per adult equivalent. The difference between the three provinces is less clear. They also looked at gifts received and offered, and concluded that on average households offered more gifts than they received, though the quantities were much smaller than those reported by Szarleta (1987), probably because of the bad harvest. Table A3.20 shows that the importance of purchases in total consumption is limited for surplus households, but more important for deficient households.

Survey by E.P. Yonli

Yonli (1997) observes that between October 1991 and September 1992, cereal purchases by far surpass cereal sales for the 8 survey villages in Yatenga and Sanmatenga (see Table A3.10). Purchases in Yatenga exceed purchases in the higher production province Sanmatenga. Purchases are in all villages a substantial part of total cereal consumption. This is even the case in the village of Noungou, for which production exceeds consumption, and for which sales are much smaller than the difference between production and consumption. Part of the excess production is stored.

Table A3.19 Production, sales and purchases for five survey villages in 1983-94, reported by Szarleta (1987).

Province	Village[1]	Crop	Production (kg)	Sales (kg)	Purchases (kg)	Consumption (kg)
Yatenga	Mené (D)	Red sorghum			432	829
		White sorghum	9,089	32	8,493	26,698
		Millet	24,724	404	12,989	58,158
		Maize	8	700	4,445	6,511
		Rice			1,033	
		Aid			2948	45
Passoré	Bougouré (D)	Red sorghum			59	281
		White sorghum	5,141	40	9,348	26,578
		Millet	2,850	28	462	16,344
		Maize	8		306	236
		Rice		7	303	
		Aid			7944	158
Sourou	Tissi (S/D)	Red sorghum	10,394	1,085	8,766	24,748
		White sorghum	2,526	458	11,802	23,947
		Millet	5,069	298	1,395	12,523
		Maize			190	184
		Rice		202	2,011	
Mouhoun	Dankuie (S/D)	Red sorghum	7,887	144	5,344	12,887
		White sorghum	4,783	362	8,597	17,913
		Millet	3,580		1,001	7,641
		Maize	497		543	1,426
		Rice	78		90	

Cont. Table A3.19

Houet	Baré (S)	Red sorghum White sorghum Millet Maize Rice	23,732 17,985 9,977 16,092	21,510 4,523 446 3,464 59	7,549 2,069 739 431 4320	8,893 17,997 9,755 15,858

Notes: 1) S = village in surplus during the survey period, D = village in deficit during the survey period, S/D = village with a production which is more or less equal to the cereal consumption requirements of the village. Source: Szarleta (1987).

Table A3.20 Cereal purchases per adult equivalent, by Reardon et al. (1987)

Province	Soum		Passoré		Mouhoun	
	Deficient	Surplus	Deficient	Surplus	Deficient	Surplus
Purchases (kg)	35	16	28	5	21	11
Purchases (as % of consumption)	18%	4%	20%	2%	14%	3%

Source: Reardon et al. (1987).

Table A3.21 Cereal purchases in some villages in Yatenga and Sanmatenga between October 1991 and September 1992, by Yonli (1997), see also Table A3.10.

Village/ province[1]	Number of households	Average household size (a)	Purchases per household (kg) (b)	Purchases per person (kg) (c)	Purchases as % of consumption (d)
Ramsa	3	14.3	798	55.8	28.1%
Séguénéga	7	14.7	307	20.9	11.1%
Kalsaka	7	15.6	713	45.7	20.9%
Kossouka	7	13.4	525	39.2	20.1%
Yatenga	**24**	**14.6**	**552**	**37.8**	**18.9%**
Nessemtenga	1	12.0	358	29.8	34.6%
Soubeira	5	14.2	382	26.9	13.2%
Noungou	8	8.8	304	34.6	18.4%
Singué	7	11.3	384	34.0	16.0%
Sanmatenga	**21**	**11.1**	**355**	**32.3**	**16.8%**

Notes: 1) The first four villages are in Yatenga, the last four villages are in Sanmatenga. (c) = (b)/(a)
Source: Yonli (1997).

Survey by Broekhuyse

Broekhuyse (1983, 1998) observes that in the province of Sanmatenga, the 'modern' households using animal traction (AT) purchase more cereals than households applying manual labour (ML). AT households purchase on average 554 kg (average household size 10.2 members), and ML households 322 kg (average household size 7.3 members). This is inconsistent with expectations. AT households produce more, and accordingly need to purchase less. The data show that AT household purchase large quantities of red sorghum for brewing *dolo*. If red sorghum purchases are distracted from the figures, AT households still purchase more cereals (331 kg for AT and 276 kg for ML households), but purchases per person are less (32 kg for AT and 38 kg for ML households).

If the different studies are compared it can be seen that preferences differ per province. Households in the northern provinces (Yatenga and Soum) prefer to purchase millet, whereas households in the other provinces (Passoré, Sanmatenga, Mouhoun, Houet, Kossi) purchase much more white sorghum. In the northern provinces maize purchases per household were also reported much higher than in the other provinces. Reardon et al. (1987) attribute this to the maize prices which were low in these regions because they were sold by official, government sellers. In those days the government sold cereals at fixed, predetermined prices.

Seasonal purchase pattern

CEDRES survey

Some studies also collected data on seasonal purchase patterns. Despite high prices, many households purchase most cereals during the lean period, when stores are almost empty. Researchers from CEDRES (Thiombiano et al. (1988)) surveyed 104 households in the provinces of Yatenga (Thiou, Nomo, Gourcy and Rom), Bam (Kongoussi and Loagha) and Sanmatenga (Barsalogho and Tamassogo). They looked, among other things, to production, consumption, sales and purchases, but presented results only in monetary terms. Consequently, sold and purchased quantities are difficult to derive. Table A3.22 shows the distribution of cereal purchases for a period of 7 months (as a % of total purchase expenditures over this period). The last period is for households normally the most difficult period of the year. Stores are almost empty, and people have to work hard on the fields. The province of Sanmatenga shows the expected purchase pattern, that more cereals are purchased the closer one approaches the new harvests (from sept-nov). In Bam, the last period is the period in which most cereals are purchased, but the first period also shows a large percentage. The only exception is the province of Yatenga, which shows the opposite pattern. This is, for an important part, caused by the village of Thiou with exceptionally high purchases in the first period. These purchases are so high (75% of total) that this might be a data error. If the village of Thiou is skipped from the data, the pattern is clearer (28% in feb-mar, 20% in apr-may and 52% in july-aug).

Table A3.22 Distribution of cereal purchases as % of total cereal sales in the period february to august, by Thiombiano et al. (1988).

Province	Febr-Mar	Apr-May	July-Aug
Yatenga	49%	17%	33%
Sanmatenga	25%	29%	46%
Bam	38%	16%	46%

Source: Thiombiano et al. (1988)

ICRISAT surveys

Reardon et al. (1987) only report on seasonal purchases for deficient households (see Table A3.23). High purchases in the season 1984-‘85 can be explained by bad production in both ’83 and ’84. High purchases in ‘82-’83 can however not be explained by low production, since production in ’81 and ’82 were reasonable. It must, however, be noted that the table only reports on shortage households. Because differences within villages and provinces are very large, it is well possible that many households have a low production, even if total regional production is high. The table clearly shows the pattern that purchases in the lean period (the hot and rainy season) were highest. In most years, purchases were highest in the northern province. Purchases in the province of Mouhoun were also rather high.

Table A3.23 Seasonal purchases of deficient households between 1981-84 per adult equivalent by region, reported by Reardon et al. (1987).

Season[1]	'81-'82				'82-'83				'83-'84				'84-'85			
	hr	**cl**	**ht**	**rn**	**hr**	**cl**	**ht**	**rn**	**hr**	**cl**	**ht**	**rn**	**hr**	**cl**	**ht**	**rn**
Soum: deficient	7	1	5	19	9	19	48	42	6	3	1	11	11	19	27	50
Passoré: deficient	2	4	3	8	2	3	7	6	4	4	6	7	3	4	16	14
Mouhoun: deficient		5	5	8	2	16	14	21	5	7	13	16	3	4	7	4

Note: 1) Seasons: harvest (hr = sept-nov), cold (cl = dec-feb), hot (ht = mar-may), rainy (rn = june-aug). Source: Reardon et al. (1987)

Surveys from the Universities from Michigan and Wisconsin

Ellsworth and Shapiro (1989) also give data on seasonal cereal purchases, but only aggregated for the five sample villages of the surveys from the University of Michigan. These data clearly show that the largest volume is purchased during the quarter July-October, the lean season. Also the number of households with their largest volume of cereal purchases, is higher in this quarter than in the other quarters. This corresponds with the expected purchase pattern for the country. Dissaggregation by village reveals that nearly half of the households in Baré, a relatively wealthy, surplus village, made their largest purchases between January and March, when prices were still low. In the other four villages, which had a chronically or occasional cereals deficit, most households purchased their largest quantity of cereals during the other two, higher priced, quarters.

Table A3.24 Cereal purchases in 1984, by Ellsworth and Shapiro (1989).

	Jan-Mar	Apr-June	July-Oct. 10	Oct. 11- Dec
Purchases[1]	25,366	35,846	57,158	12,085
Number of households[2]	45	61	64	7

Notes: 1) Sorghum, millet, maize, rice, food aid, miscellaneous foods. 2) Number of households with their largest volume of purchases in a certain quarter. Source: Ellsworth and Shapiro (1989).

Survey by E.P. Yonli

Yonli (1997) presents the monthly purchases aggregated for his survey villages in Yatenga and Sanmatenga between October 1991 and June 1993. Table A3.25 clearly shows that purchases are highest during the lean season (the second and third quarter). For Yatenga monthly purchases always exceed montly sales (see also Table A3.15). In Sanmatenga, sales exceed purchases in some months. The low purchases in Sanmatenga during the second quarter of 1993 seem logical, considering the relatively high sales during that period. The data indicate for Sanmatenga, though not very clearly, that purchases decrease if production increases (production in '92 was much higher than production in '91). For Yatenga, however, this can not be observed ('91 production was higher than '92 production).

Table A3.25 Monthly purchases (in kg per person) between October 1991 and June 1993, by Yonli (1997).

Month/ Year	Yatenga	Sanmatenga
10-91	0.75	4.11
11-91	1.30	0.82
12-91	2.83	0.50
4th quarter 91	4.88	5.43
1-92	1.03	1.52
2-92	2.78	0.08
3-92	4.34	1.23
1st quarter 92	8.15	2.83
4-92	2.63	0.94
5-92	6.70	2.08
6-92	4.46	4.05
2nd quarter 92	13.79	7.07
7-92	4.68	10.67
8-92	6.31	3.86

Month/ Year	Yatenga	Sanmatenga
9-92	0.27	2.06
3d quarter 92	11.26	16.59
10-92	1.73	0.04
11-92	2.36	0.47
12-92	0.35	0.15
4th quarter 92	4.44	0.66
1-93	3.47	2.27
2-93	2.38	0.31
3-93	3.46	0.31
1st quarter 93	9.31	2.89
4-93	3.40	0.23
5-93	3.77	0.47
6-93	4.31	1.88
2nd quarter 93	11.48	2.58

Note: In Yatenga 24 households and in Sanmatenga 21 households were surveyed. Source: Yonli (1997).

Price and income elasticities of cereal demand

Price elasticities of demand and income elasticities have been estimated in several studies.

ICRISAT surveys

Reardon et al. (1988b) analysed the sensitivity of cereal consumption to cereal price changes. Rice, maize and millet/white sorghum consumption were regressed at the prices of these cereals, monthly household expenses, household size, and the number of children. Elasticities are estimated, which indicate the percentage increase in consumption, resulting from a percentage increase in the price of one of the cereals or in the household expenses. The calculated R^2 of the regressions were in most cases low, which indicates that the purchases were only for a small part explained by the independent variables. R^2 only had reasonable values for rice purchases by poor households (62%), and rice and millet/sorghum purchases by average households (30% and 44%, respectively). For the other purchases, R^2 was between 13% and 22%. A remarkable result was that neither the rice price, nor the prices of the other cereals, did have a significant influence on rice consumption. The income elasticity of rice consumption (% increase of rice consumption relative to a 1% increase of household revenues which are supposed to be equal to total household expenses) was for all households between 0.72 and 1.01 (see Table A3.26). This indicates that rice is not a luxury good (see Section 4.2). The same holds for maize and for millet/sorghum consumption. The analysis also shows that households with less children consume more rice (they consume more often rice purchased from prepared food sellers). On the other hand maize cultivation turns out to be dependent on its own price (for the average households) and the millet/sorghum price (for the poor and average households). If the maize price increases, average households will consume much less maize, and will substitute it partly with millet/sorghum. Finally, millet/sorghum consumption did respond weakly and not significantly on cereal price changes.

Table A3.26 Price and income elasticities of cereal consumption, by Reardon et al. (1988b).

	Poor	Average	Rich
Rice purchases			
Total household expenses	0.79	0.72	1.01
Maize purchases			
Maize price		-7.00	
Millet/sorghum price	3.11	3.80	
Total household expenses	1.11	1.03	1.28
Millet/sorghum purchases			
Total household expenses	0.87	0.91	

Notes: Only elasticities with a significance level of at least 90% are shown. Source: Reardon et al. (1988b).

Survey by Roth

Roth (1986) estimated income and demand elasticities for five rural and two urban regions in Burkina Faso. For his estimates he used some empirical estimates from other studies in different countries (see Table A3.27) and observations on rural-urban consumption patterns (e.g. more rice consumption in cities, the position of maize in daily consumption). His estimates of own-price, cross-price, and income-price elasticities are presented in Table A3.28. In this table income compensated own-price and cross-price elasticities are presented, which shows that changes in demand are not only caused by a substitution effect, but also by an income effect. After all, if prices increase also the purchasing power of households decreases. His estimates show inelastic and negative own-price elasticities of demand, and very inelastic but positive cross-price elasticities. Elasticities for the staple cereals millet and sorghum are on the countryside more elastic than rice, maize and groundnuts. In the cities, sorghum demand is not influenced by prices of the other goods. Urban households mainly demand red sorghum for brewing *dolo*. Rice and white sorghum demand are more elastic than millet. Millet is more a crop for the poor. Maize turns out to be very inelastic for all households. The relatively elastic own-price elasticities for staple crops is caused by the high proportion of income spent on them. If the price of these goods increases, the purchasing power decreases, and cheaper substitutes will be sought after. For commodities on which a smaller part of income is spent, price also plays a minor role. Income elasticities for millet and sorghum are in rural areas relatively elastic, compared to urban elasticities of these commodities. For rice, the reverse is observed. Maize and groundnuts are less elastic in rural areas, but a little more elastic in Ouagadougou and Bobo Dioulasso.

Table A3.27 Estimates of own-price and income elasticities, reported in Roth (1986).

Crops[1]		**Own-price elasticity**						**Income elasticity**				
		So	**Mi**	**Mz**	**Rc**	**Wt**	**Oth**	**So**	**Mi**	**Mz**	**Rc**	**Wt**
USDA, 1981	Sahel		-.06		-.35	-.30			.15	.46	.92	.93
	West Africa			-.05	-.53	-.15			.09	.15	.87	.65
Sawadogo, 1986, Ouaga-dougou	Low[2]							1.13				.91
	Middle							.99				.97
	High							.63				1.21
	Mean							.94				1.02
Youngblood et al., 1982, Khartoum	Low 25%								.77			1.21
	Middle 50%								.50			.90
	Upper 25%								.57			.88
	Mean								.59			.97
Strauss, 1983, Sierra Leone	Low[2]	-.15			-1.26		-1.17					
	Middle	-.26			-.78		-.40					
	High	-.31			-.45		-1.05					
	Mean	-.22			-.74		-1.01					

Notes: 1) So = sorghum, Mi = millet, Mz = maize, Rc = rice, Wt = wheat, Oth = others; 2) A distinction has been made between low, middle and high income groups. Source: Roth (1986).

Table A3.28 Own-price, cross-price and income elasticities for Burkina Faso, by Roth (1986).

Regions[1]		**Income elasticity**	**Price elasticity**						
			Ws	**Rs**	**Mi**	**Mz**	**Rc**	**Gn**	**Oth**
Central Region	Ws[2]	0.95	-0.4386	0.0200	0.0501	0.0045	0.0071	0.0091	0.3452
	Rs	0.95	0.0364	-0.4550	0.0501	0.0045	0.0071	0.0091	0.3452
	Mi	0.95	0.0364	0.0200	-0.4249	0.0045	0.0071	0.0091	0.3452
	Mz	0.75	0.0287	0.0158	0.0396	-0.3715	0.0056	0.0072	0.2725
	Rc	0.80	0.0230	0.0126	0.0316	0.0028	-0.2955	0.0057	0.2180
	Gn	0.80	0.0307	0.0168	0.0422	0.0038	0.0060	-0.3923	0.2907
	Oth	1.0462	0.0402	0.0221	0.0553	0.0049	0.0078	0.0100	-0.1433

Cont. Table A3.28

Northern Region	Ws	0.95	-0.4300	0.0012	0.0898	0.0074	0.0064	0.0107	0.3123
	Rs	0.95	0.0450	-0.4738	0.0898	0.0074	0.0064	0.0107	0.3123
	Mi	0.95	0.0450	0.0012	-0.3852	0.0074	0.0064	0.0107	0.3123
	Mz	0.75	0.0355	0.0009	0.0709	-0.3691	0.0051	0.0085	0.2465
	Rc	0.60	0.0284	0.0007	0.0567	0.0047	-0.2959	0.0068	0.1972
	Gn	0.80	0.0379	0.0010	0.0757	0.0062	0.0054	-0.3910	0.2630
	Oth	1.0612	0.0503	0.0013	0.1004	0.0083	0.0072	0.0120	-0.1818
Eastern Region	Ws	0.95	-0.4268	0.0043	0.0346	0.0047	0.0037	0.0079	0.3697
	Rs	0.95	0.0482	-0.4707	0.0346	0.0047	0.0037	0.0079	0.3697
	Mi	0.95	0.0482	0.0043	-0.4404	0.0047	0.0037	0.0079	0.3697
	Mz	0.75	0.0381	0.0034	0.0273	-0.3713	0.0030	0.0063	0.2919
	Rc	0.60	0.0305	0.0027	0.0218	0.0030	-0.2976	0.0050	0.2335
	Gn	0.80	0.0406	0.0037	0.0291	0.0040	0.0031	-0.3933	0.3113
	Oth	1.0330	0.0524	0.0047	0.0376	0.0051	0.0041	0.0086	-0.1145
Western Region	Ws	0.95	-0.4787	0.0157	0.0309	0.0046	0.0029	0.0083	0.4150
	Rs	0.95	0.0438	-0.5068	0.0309	0.0046	0.0029	0.0083	0.4150
	Mi	0.95	0.0438	0.0157	-0.4916	0.0046	0.0029	0.0083	0.4150
	Mz	0.75	0.0346	0.0124	0.0244	-0.4088	0.0023	0.0065	0.3277
	Rc	0.60	0.0277	0.0099	0.0195	0.0029	-0.3282	0.0052	0.2621
	Gn	0.80	0.0369	0.0132	0.0260	0.0039	0.0024	-0.4330	0.3495
	Oth	1.0275	0.0474	0.0170	0.0334	0.0050	0.0031	0.0090	-0.1162
South-west Region	Ws	0.95	-0.4904	0.0057	0.0127	0.0147	0.0046	0.0058	0.4452
	Rs	0.95	0.0321	-0.5168	0.0127	0.0147	0.0046	0.0058	0.4452
	Mi	0.95	0.0321	0.0057	-0.5098	0.0147	0.0046	0.0058	0.4452
	Mz	0.75	0.0253	0.0045	0.0100	-0.4009	0.0036	0.0046	0.3515
	Rc	0.60	0.0203	0.0036	0.0080	0.0093	-0.3271	0.0037	0.2812
	Gn	0.80	0.0270	0.0048	0.107	0.0124	0.0038	-0.4351	0.3749
	Oth	1.0305	0.0348	0.0062	0.00138	0.0160	0.0050	0.0063	-0.0838
Ouaga-dougou	Ws	0.70	-0.4017	0	0.0049	0.0008	0.0229	0.0124	0.3490
	Rs	0.70	0.0183	-0.4200	0.0049	0.0008	0.0229	0.0124	0.3490
	Mi	0.70	0.0183	0	-0.4151	0.0008	0.0229	0.0124	0.3490
	Mz	0.80	0.0209	0	0.0057	-0.4790	0.0262	0.0142	0.3988
	Rc	0.95	0.0248	0	0.0067	0.0011	-0.5389	0.0169	0.4736
	Gn	0.85	0.0222	0	0.0060	0.0010	0.0278	-0.4949	0.4237
	Oth	1.0424	0.0272	0	0.0074	0.0013	0.0341	0.0185	-0.1058
Bobo Diou-lasso	Ws	0.70	-0.4018	0	0.0033	0.0103	0.0210	0.0138	0.3425
	Rs	0.70	0.0182	-0.4200	0.0033	0.0103	0.0210	0.0138	0.3425
	Mi	0.70	0.0182	0	-0.4167	0.0103	0.0210	0.0138	0.3425
	Mz	0.80	0.0206	0	0.0038	-0.4683	0.0240	0.0157	0.3915
	Rc	0.95	0.0247	0	0.0045	0.0139	-0.5415	0.0187	0.4649
	Gn	0.85	0.0221	0	0.0041	0.0125	0.0255	-0.4933	0.4159
	Oth	1.0487	0.0272	0	0.0050	0.0154	0.0314	0.0206	-0.1161

Notes: 1) Central region = CRPA Centre, Centre Ouest, Centre Est, Centre Sud and the province of Passoré; North region = CRPA Centre Nord, Sahel and the province of Yatenga; East region = CRPA Est; Western region = CRPA Mouhoun; Southwest region = CRPA Hauts Bassins, Sud Ouest and Comoé; 2) Ws = white sorghum, Rs = Red sorghum, Mi = millet, Mz = maize, Rc = rice, Gn = Groundnuts, Oth = other.
Source: Roth (1986).

Study by Colman and Young

Colman and Young (1989) also present some FAO estimates of income elasticities of demand for some agricultural products (see Table A3.29). These estimates are much lower than the estimates presented by Roth (1986). The most important reason for this is that Colman and Young give aggregated estimates for all cereals. These are normally lower than disaggregated elasticities for the different cereals. After all, if the price of white sorghum increases, another cereal can substitute this demand, whereas another type of commodity (not a cereal type) has to substitute for cereals, if the 'cereal' price increases.

Table A3.29 Income elasticities of demand for some agricultural products, by Colman and Young (1989).

	Egypt '74/75		India ('73/74)		Java ('78)		Colombia '87	Mexico '77
	Rural	**Urban**	**Rural**	**Urban**	**Rural**	**Urban**		
Cereals	0.15	0.61	0.21	0.48	0.15	0.23	0.58	-0.16
Total food	0.75	1.28	0.79	0.82	0.74	0.72	0.64	0.09

Source: Colman and Young (1989).

We retain the following conclusions from the above review. Rice consumption of urban households increases. Rice consumption is higher for wealthier households, and poorer households spent a larger share of their income on cereal purchases. Almost all rural households purchase cereals on the market. The quantity of cereals purchased is for many households larger than the quantity sold. Cereal purchases are a considerable part of total consumption, certainly for shortage households. Furthermore, in general most purchases take place during the lean season, when stocks are depleted, before the new harvest. Income and price elasticities of cereal demand differ a lot between the different regions. Roth has estimated in his study income elasticities of supply which we will also use in our study. We feel that his estimates are more reliable than the other elasticities presented above, because he estimated them on the basis of elasticities reported in other studies and on emprical evidence from Burkina Faso.

A3.5 Revenues and expenditures

Non-cropping income is an important determinant of cereal purchases and sales. It is one of the major elements of the cereal demand functions discussed in Section 9.1. In Section 8.1.5 it has been showed that collecting data on these issues is a difficult task. In this appendix some surveys performed in the past are reviewed.

INSD survey

In 1994 the national statistical institute of Burkina Faso, INSD, executed a large survey on household living conditions among more than 8000 households scattered over the country (INSD, 1996a,b). This was one of the first large surveys on living conditions by the institute. Based on their surveys they divided the country in 5 representative rural regions and two urban regions. These regions are not the same as the CRPA which are applied in this study. Table A3.30 shows which provinces lie in which INSD survey region and CRPA. Assuming that revenues and expenditures are the same in all provinces of the 5 regions, it is possible to estimate revenues and expenditures per CRPA. In their reports, INSD admits that they encountered many problems, and that therefore some of the results are not as reliable as required.

Table A3.31 shows for each region total and cereal expenses and revenues per household. INSD measures total revenues and expenses as the sum of monetary and non-monetary expenses and revenues. Monetary expenses are for example purchases of cereals on the market. Non-monetary expenses comprises for example consumption of self produced cereals. Consumption of self-produced crops has been valued against the going market price to determine non-monetary expenses. An average household in Burkina Faso consists on of 7.8 people (INSD, 1996a). Because of difficulties measuring directly household revenues, only the distribution of revenues over the different sources is presented. In INSD (1996b) it has been supposed that total revenues equal total expenses.

Table A3.30 Subdivision of INSD regions and CRPA's in provinces

Province	INSD region	CRPA	Province	INSD region	CRPA
Soum	Nord	Sahel	Yatenga	Centre-Nord	Nord
Oudalan	Nord	Sahel	Passoré	Centre-Nord	Nord
Seno	Nord	Sahel	Bam	Centre-Nord	Centre-Nord
Poni	Sud and Sud-Est	Sud-Ouest	Sanmatenga	Centre-Nord	Centre-Nord
Bougouriba	Sud and Sud-Est	Sud-Ouest	Namatenga	Centre-Nord	Centre-Nord
Sissili	Sud and Sud-Est	Centre-Ouest	Gngagna	Centre-Nord	Est
Nahouri	Sud and Sud-Est	Centre-Sud	Sanguié	Centre-Sud	Centre-Ouest
Gourma	Sud and Sud-Est	Est	Boulkiemdé	Centre-Sud	Centre-Ouest
Tapoa	Sud and Sud-Est	Est	Kadiogo	Centre-Sud	Centre
Comoe	Ouest	Comoe	Oubritenga	Centre-Sud	Centre
Houet	Ouest	Hauts-Bassins	Ganzourgou	Centre-Sud	Centre
Kenedougou	Ouest	Hauts-Bassins	Bazéga	Centre-Sud	Centre-Sud
Mouhoun	Ouest	Mouhoun	Zoundweogo	Centre-Sud	Centre-Sud
Kossi	Ouest	Mouhoun	Boulgou	Centre-Sud	Centre-Est
Sourou	Centre-Nord	Mouhoun	Kouritenga	Centre-Sud	Centre-Est

Inhabitants of Ouagadougou and Bobo-Dioulasso have expenses which are three times higher than expenses from rural households. Also food expenditures are much higher. It seems peculiar that the level of cereal purchases (monetary plus non-monetary) is lower in Ouagadougou than on the country side. The main reason for this is the consumption pattern of urban households, which differs from the rural consumption pattern. Citizens consume much more rice and other food products, which is reflected by their higher food expenditures. Total expenses and food expenses may differ considerably between the regions. Expenses may differ up to 20%. The part of consumption of self-produced cereals in total consumption is striking. In the rural areas only 10% to 20% of the cereals is purchased. The remainder originates from own production. It is logical that this is the reverse in the cities. The data confirm that revenues of urban households are for the largest part monetary, originating mainly from non-agricultural sources. Non-monetary income for rural households is substantial, certainly in the region 'Centre-Sud'. The table also shows that revenues from cereal sales are very limited. Income earned by selling cotton in the western regions, and by selling livestock in the other regions often exceeds cereal income. This confirms that households prefer not to sell cereals.

Table A3.31 Annual revenues and expenses per household in FCFA in 1994.

Expenses	Ouest	Sud et Sud-Est	Centre-Nord	Centre-Sud	Nord	Ouaga/ Bobo	Other cities
(a) Total expenses (FCFA)[1]	441,360	419,741	364,301	386,998	374,982	1,141,725	940,182
(b) Food expenses	232,841	230,438	191,844	198,547	239,610	372,388	363,323
(c) (as % of total expenses)	52,8%	54,9%	52,7%	51,3%	63,9%	32,6%	38,6%
(d) Cereal expenses	111,764	97,475	89,016	80,610	119,086	98,310	115,537
(e) (as % of food expenses)	48,0%	42,3%	46,4%	40,6%	49,7%	26,4%	31,8%
Suppositions:[2]							
(f) Cereal expenses non-monetary (%)	80%	80%	80%	80%	80%	21%	49%
(g) Cereal expenses monetary (%)	20%	20%	20%	20%	20%	79%	51%
(h) Cereal expenses non-mon. (FCFA)	89,411	77,980	71,212	64,488	95,269	20,645	56,613
(i) Cereal expenses monetary (FCFA)	22,353	19,495	17,803	16,122	23,817	77,665	58,924
Revenues[3]							
Monetary revenues (%)	54,3%	33,2%	35,3%	28,1%	38,8%	80,6%	69,6%
Non-monetary revenues (%)	45,7%	66,8%	64,7%	71,9%	61,2%	19,4%	30,4%
Structure of monetary revenues							
*agriculture[4]:	71,2%	40,4%	46,3%	35,2%	43,5%	1,8%	5,8%
-cereals, groundnuts[4]	26,9%	15,9%	14,6%	13,3%	2,6%	0,5%	2,2%
-cotton[4]	33,4%	8,3%	0,9%	1,6%	0,1%	0,0%	0,1%
-livestock[4]	6,4%	10,7%	27,1%	13,1%	38,5%	0,5%	1,6%
Average household size	7.8	7.6	9.0	8.0	6.7	6.1	7.5

Notes: (c) = (b)/(a)*100%, (e) = (d)/(b)*100%, (h) = (f)*(d), (i) = (g)*(d) .1) Total expenses include monetary and non-monetary expenses. Non-monetary expenses are calculated by multiplying consumption of self-produced crops with the observed market price; 2) On-farm consumption of self-produced base cereals millet and sorghum is estimated at 80% of total cereal expenses for the rural areas, against 49% for average cities and only 21% for Ouagadougou and Bobo-Dioulasso (INSD, 1996b: p. 226); 3) It has been supposed that total revenues equal total expenses, which are given in (a). 4) As % of total monetary revenues.
Source: data based on INSD (1996a,b)

Survey by Broekhuyse

Broekhuyse (1988) reports the revenues and expenditures for households using animal traction (AT) and using manual labour (ML) in two villages in the province of Sanmatenga, see Table A3.32. The average household size was 7.3 for ML housholds, and 10.2 people for AT households. AT households had much larger revenues and expenditures than ML households. The largest differences in revenues are in the sales of cash crops, processing agricultural produce, extra agricultural activities and credits obtained. AT households can obtain more credit to purchase their traction equipment. Using this equipment they produce more cash crops. Because these households are larger, more household members do extra-agricultural activities. It is striking that cereal purchases are higher for AT households than for ML households. Furthermore, because of credit repayments and maintenance of traction equipment, debts and production costs are higher for AT households.

Table A3.32 Household income and revenues between 1979 and 1985 reported by Broekhuyse (1988) in Sanmatenga.

Revenues	**Households using manual labour**		**Households using animal traction**	
	in FCFA/household	**as % of total**	**in FCFA/household**	**as % of total**
Sales of cereals[1]	1,685	3%	2,655	3%
Sales of other crops[2]	6,641	14%	14,888	18%
Livestock	11,659	24%	11,074	13%
Processing of agr. produce	1,185	2%	3,785	5%
Extra-agricultural activities	19,814	41%	33,618	40%
Donations received	5,365	11%	8,445	10%
Credits	1,794	4%	9,053	11%
Total	48,143		83,518	
Expenditures				
Purchases of cereals[1]	26,594	58%	34,381	46%
Purchases of other crops[2]	6,120	13%	8,151	11%
Production costs	6,214	14%	20,556	27%
Donations given	1,175	3%	4,373	6%
Debts	5,540	12%	7,465	10%
Total	45,643		74,926	

Notes: 1) Cereals comprise millet, red sorghum, white sorghum and maize; 2) Other crops comprise rice, groundnuts, cowpea, cotton, manioc, aubergine, gombo, etc. Source: Broekhuyse (1988).

The monetary value of gifts received and paid in kind were also a substantial part of total revenues and expenditures. Broekhuyse estimated that gifts received in kind were on average 4,310 F CFA for ML and 4,395 F CFA for TA households. Gifts paid in kind could take a value of 4,845 F CFA for ML and 12,038 F CFA for AT households, so 11% and 16% of total expenditures, respectively. A part of these gifts were payments for labour services provided.

ICRISAT survey

Reardon et al. (1988a) analysed household strategies to cope with food insecurity. Using income and consumption survey panel data collected by ICRISAT and IFPRI in the 1984-85 cropping season they estimated the level of household income. Data were used of one village in the Soum province in the Sahelian rainfall zone, and one village in the province of Passoré in the Sudanian rainfall zone. Households in the Sahelian zone were more food secure than those in the Sudanian zone. Average household size was 10 in Soum and 11 in Passoré. Table A3.33 gives some results of these studies.

Table A3.33 Household income in two rainfall zones by Reardon et al. (1988a).

		Sahelian zone	Sudanian zone
Agriculture:	Crop production	8,500	9,010
	Agricultural wages[1]	590	8,120
Livestock husbandry		8,370	1,930
Local non-farm income[2]		9,580	4,250
Non-local non-farm income[3]		8,760	5,200
Transfers[4]		3,020	2,360
Total		38,820	30,870

Notes: 1) Wages received for work on other households' plots in the immediate region. 2) Non-migratory income earned in occupations other than cropping and livestock husbandry. 3) Migratory income earned by members of the household. 4) Food aid, gifts and remittances. Source: Reardon et al. (1988a).

CEDRES survey

Thiombiano et al. (1988) also reported on household revenues and expenditure in the north of the Central Plateau in 1984 (see Table A3.34).

Survey by Roth

Finally, Roth (1986) estimated the share of budget spent on cereals (see Table A3.35). The results do not differ much from INSD (1996) data. The budget share spent on cereals is for most rural areas higher than for the larger cities, and varies between 20% and 30% of total expenditures. Table A3.35 clearly shows the difference between the countryside and the city. In Ouagadougou and Bobo-Dioulasso, much more rice is consumed than in rural areas. Furthermore, in the north, and to a lesser extent also in the central region, millet is prefered, whereas white sorghum is prefered in the other regions. It can be questioned whether the consumption of red sorghum in the cities is indeed neglible. It is true that *dolo* consumption, for which much of the red sorghum is used, is less important in larger cities, where bottled beer is consumed more often. However, *dolo* consumption is certainly not zero in these areas.

Table A3.34 Household revenues and expenditure patterns for eight villages in the north of the Central Plateau in 1984, in FCFA per household.

Revenues	**Gourcy**	**Rom**	**Thiou**	**Nomo**	**Tamas-sogo**	**Barsa-logho**	**Kongous-si**	**Loagha**	**Average**
Cowpea, Voandzou, vegetables	0	9692	346	0	12063	0	109742	8435	17535
Livestock	111934	11635	108444	33173	15469	7123	28815	32627	43653
Handicraft	91192	769	14923	9515	0	169	18300	20905	19472
Small trade	0	6346	19423	5278	2431	2980	2653	3731	5355
Retirements, pensions	131692	0	26692	0	43077	0	0	0	25183
Other revenues	3846	34038	0	0	10576	846	13846	17308	10058
Total (FCFA)	338664	62480	169828	47966	83616	11118	173356	833006	215004

Source: Thiombiano et al. (1988)

Table A3.35 Average share of budget spent on different cereals; by Roth (1986).

	Ws[2]	**Rs**	**Mi**	**Mz**	**Rc**	**Gn**	**Oth**
Centre[1]	0.081	0.044	0.111	0.013	0.025	0.024	0.703
North	0.100	0.003	0.199	0.021	0.023	0.028	0.627
East	0.107	0.010	0.077	0.013	0.013	0.021	0.760
West	0.088	0.032	0.062	0.012	0.009	0.020	0.777
South-west	0.065	0.012	0.026	0.038	0.015	0.014	0.832
Ouagadougou	0.062	0	0.017	0.003	0.057	0.035	0.826
Bobo-Dioulasso	0.062	0	0.011	0.031	0.053	0.039	0.805

Notes: 1) Central region = CRPA Centre, Centre Ouest, Centre Est, Centre Sud and the province of Passoré; North region = CRPA Centre Nord, Sahel and the province of Yatenga; East region = CRPA Est; Western region = CRPA Mouhoun; Southwest region = CRPA Hauts Bassins, Sud Ouest and Comoé; 2) Ws = white sorghum, Rs = Red sorghum, Mi = millet, Mz = maize, Rc = rice, Gn = Groundnuts, Oth = other.
Source: Roth (1986).

The above rewiew shows that income and expenditure levels reported by the different surveys differ considerably. This is caused by the unreliability of the data, but on the other hand differences between households, between regions, and also between years are known to differ a lot. In Section 9.1 we estimate income levels per person mainly on the basis of the INSD survey (1996a,b). Despite unexplained data errors in the INSD reports, it is the most recent and largest survey available.

A3.6 Agricultural prices

To estimate the different parameters of the cereal demand functions in Section 9.1, estimates are used of average cereal consumer prices per quarter. To determine cereal supply functions in Section 9.2, the probability distribution of cereal producer prices per quarter is used. In this appendix and Section 8.1.6 the price data used for these purposes are discussed. Price data in Burkina Faso are gathered weekly by SIM/SONAGESS on 37 markets in Burkina Faso for the cereals millet, red sorghum, white sorghum, yellow maize and white maize. A distinction is made between producer and consumer prices. Producer prices ensue from transactions between producers and traders, consumer prices ensue from transactions between consumers and traders or between consumers and producers. Before discussing these data for the period 1992-1999[38], we first briefly discuss the analyses of Bassolet (2000) and Hoftijzer (1998), who used the same data, but for the period 1992-1996.

In the study of Bassolet (2000) an analysis is made of: 1) changes in the cereal market structure and the behavior of actors after market liberalization in Burkina Faso, and of: 2) the economic efficiency of market transactions. He showed that the grain policies of the government constrained the functioning of the market in the past. Liberalization policies had some favorable effects on competition, mainly due to the result of the increased number of traders and the market information system (SIM)

[38] Data obtained from STATISTIKA, the statistical department from SIM/SONAGESS and from internet: www.statistika.net.

which contributed to the transparency of the market. In the meanwhile, limited access to credit, high taxes, bad infrastructure and the irregular dissemination of prices by the market information system, still constrain competition.

Bassolet also showed that the new policies also changed farmer behaviour, in particular in surplus regions. They changed their passive commercial strategies in more active behavior. Nowadays, sales are planned in a way that the farmer profits from seasonal price fluctuations, and surpluses are only sold to the highest bid. Also traders profit from the increased transparency. They are better informed about supply and demand conditions (prices) on local surplus markets and therefore they are able to purchase the grain more efficiently. Remarkably, (semi-)wholesalers did not change their storage strategies. Most of them sell their stocks within one month. Three explanations are given for this behaviour: 1) traders are constrained by a limited availability of working capital; 2) it is costly to conserve grain for a longer period; 3) the grain board SONAGESS, responsible for the management of the national food security grain stock, revolves the stock gradually and sells during the hungry season (June - August). As a consequence, seasonal price increases in the grain market are reduced, making investments in storage less attractive for traders.

Bassolet showed that producer prices increased after liberalization. He derived a seasonal price index which is relatively stable for the consumer price series, but instable for the producer price series. The collected price data show some evidence for the conclusion that the seasonal price increase is lower than the costs of storage. This result clearly explains why traders play only a minor role in the long term storage activity. Finally, Bassolet showed that price differences between markets decreased after liberalization. Net margins for traders are low, indicating that market integration improved after liberalization. The results of tests for cointegration (Johansen procedure) are in line with these observations and show that the number of cointegrated markets increased. Bassolet concludes that this can be interpreted as a positive effect of liberalization policies and, in particular, the upshot of the market information system and the removal of regional trade barriers in the country.

An analysis of the weekly SIM prices for the period 1992-1996 for 16 villages by Hoftijzer (1998) demonstrated that no evidence could be found for a seasonal pattern with regular price increases and decreases. On most markets prices decreased after the harvest and increased during the lean season. For some markets, however, this pattern could not be observed. Furthermore, the timing of price increases and decreases as well as the amplitude of seasonal changes differed per year. However, these conclusions are based on data for only 4 harvest years. Differences between years and between villages are large. A closer look to the white sorghum producer prices in Banfora show that the price difference between the first and third quarter in 1993 is only 8%, while it is 103% in 1996. Furthermore average ‘92-’96 millet consumer prices in N’dorola are reported to have increased 130% between the first and the second quarter, whereas prices in Niangoloko increased only 8%. Some of these differences may be caused by data weakness (only few observations are available for some markets), but volatility of prices is on the other hand a well known phenomenon in African agriculture.

To estimate average prices and the distribution of cereal prices, we used cereal prices for the period 1992 –1999. For the period 1992-1996 weekly price data were available. For the period 1997-1999, however, we obtained only monthly data. Therefore, we estimated for the period 1992-1996 for each crop type the average monthly prices from the available weekly prices. Many of the data were missing[39]. A reason for the large number of missing data is that some of the crops are not traded on each market day on some of the markets. The thinness of the markets causes that some of the weekly prices are based on only a few observations. Due to the large number of missing data on red sorghum and yellow maize, not much value can be adressed to these data. Red sorghum is most often used to brew *dolo*, and is not

[39] From the 3552 possible observations (37 markets, 8 years, 12 months per year) for each crop, 44% of the millet producer prices were missing, 43% of the white sorghum data, 67% of the white maize data, 90% of the red sorghum data and 97% of the yellow maize data. For the consumer prices, the share of missing data was 8% for millet, 6% for white sorghum, 31% for white maize, 76% for red sorghum and 90% for yellow maize.

regularly traded on the market. Therefore, average monthly cereal prices are based on the cereals millet, white sorghum and white maize, only.

Using these monthly cereal prices, average cereal prices per quarter can be estimated for each CRPA[40]. In Table A3.36 these average quarterly prices are shown for each year and for the entire period '92-'99. We can make the following observations from the price data:

1. The '92-'99 cereal price averages (see Table A3.36 and Table A3.37) reveal that the CRPA Mouhoun, Hauts Bassins, Sud Ouest and Nord have on average the lowest producer prices, whereas the CRPA Centre Sud, Centre Est, Centre Ouest and Est have the highest producer prices. The CRPA Hauts Bassins, Mouhoun and Est have the lowest consumer prices, and the highest consumer prices can be observed in the CRPA Centre, Sahel and Sud Ouest. It is logical that producer prices in Mouhoun and Hauts Bassins are low, since these regions are the high production regions of the country. Consumer prices in these regions are also low, because costs to transport produce from the producers to the consumers are low. The high consumer prices in the regions Centre, with the capital Ouagadougou, and Sahel, correspond to the expectations. Prices in the Sahel are affected by high transport costs, whereas prices in Ouagadougou are affected by high demand levels from the urban consumers and from traders from the rest of the country. It is striking that price differences between the region Sahel and the northern regions Nord and Centre Nord are that large. This is partly caused by data weakness. Price from the CRPA Nord are only based on prices for the city of Ouahigouya. This market is a regional transit market, from which many cereals are transported towards the sahelian regions. Prices in the other parts of the region Nord may be substantially higher. The high producer prices in the regions Centre

[40] The 37 markets on which SIM/SONAGESS collects price data are located in the following CRPA: Centre: Gounghin, Paglayiri, Sankaryaré; Centre Nord: Kaya, Kongoussi, Tougouri; Centre Ouest: Fara, Hamélé, Koudougou, Léo; Centre Sud: Guelwongo, Manga; Sahel: Djibo, Dori, Gorom-Gorom; Mouhoun: Dédougou, Djibasso, Solenzo, Tougan; Est: Bitou, Bogande, Botou, Diapaga, Fada N'gourma, Namounou; Centre Est: Pouytenga, Tenkodogo, Zabré; Nord: Ouahigouya; Sud Ouest: Diébougou, Gaoua; Hauts Bassins: Bobo Dioulasso, Dande, Faramana, N'dorola; Comoé: Banfora, Niangoloko.

Sud, Centre Est, Centre Ouest and Est, and high consumer prices in the regions Centre Sud and Centre Est, are probably caused by cross-border trade. Traders from the neighbouring countries Ivory Coast, Ghana, Togo and Benin purchase cereals on markets close to the border, selling it in their own country. No conclusions can be drawn from the prices for the region Sud Ouest and Nord, because averages are based on only a few observations.

2. The data in Table A3.36 clearly show that average cereal prices increased a lot between 1994 and 1996 (see also Figure A3.4 and A3.5).
   - The price increase between 1994 and 1996 can not only be caused by cereal production. In almost all years prices increase if production decreases, and vice versa. However, production decreases do not explain for the large price increase between October 1994 and June 1996. In Table A3.38 cereal production between 1992 and 1998 are compared with average cereal prices for each agricultural year from October till September. The price increase is more clearly seen by looking at the indexes in Table A3.38. Between the years 94/95 and 95/96, the production index decreases from 100 to 92, and the consumer price index increases from 126 to 178, an increase of 41%. On the other hand, between the years 92/93 and 93/94, the production index increased from 100 to 103, and the consumer price index decreased from 100 to 92, a decrease of only 8%.
   - Prices seem to have stabilized after 1996. Between 1996 and 1998 price changes due to production fluctuations seem not to be excessive, and are similar to price changes in the period 1992–1994. However, the price series is too short to draw final conclusions on this issue.
   - Price increases between 1994 and 1996 are presumably caused by the devalution of the Franc CFA in January 1994, which started to have effects on cereal prices in 1995. As a result, the average Oct ’96 – Sept ’99 producer and consumer prices increased with 91% and 99%, respectively, compared to the average Jan ’92 – Sept ’94 cereal prices, see Table A3.39.
   - According to Egg et al. (1997), the devaluation caused a price increase of imported consumption goods and fertilizers, an increased export of cereals to

neighbouring countries, and changes in producer behaviour, due to an increased purchasing power of the producers and an increased competition between traders.

- Furthermore, due to the cotton sector reorganisation in 1994, cotton farmers received payments for their production earlier (Egg et al., 1997). Therefore, from 1994 onwards, distress sales to repay debts were less important, resulting in lower cereal supplies in the period following the harvest. This is confirmed by the cereal price data in Table A3.39, which show that cereal prices in the cotton producing areas have increased more between the periods '92-'94 and '96-'99, than in the non-cotton producing regions.

3. Table A3.39 shows that the average '96-'99 average producer and consumer prices are almost the double of the '92-'94 average producer prices. For the region Centre, with the capital Ouagadougou, the increase is less, and for the high production areas (Mouhoun, Hauts Bassins, and Comoe), the increase is more than the double. For the region Sud Ouest the increase is small, but this is probably caused by data weakness.
4. Looking at the commercial margins (the difference between consumer and producer price) in Table A3.39, we have to conclude that the commercial margins have increased after the devaluation. Both the margin in FCFA per kg and the margin as a percentage of the producer price increased significantly. In most regions, margins almost doubled or more than doubled. High margins for the regions Sud Ouest, Centre Nord, and Sahel have to be treated with care, because they may be caused by data weakness. This differs from the observations of Egg et al. (1997), who concluded on the basis of the '92-'97 price data that margins did not change significantly. It is not strange that commercial margins double, if prices double. The increase is among other things caused by inflation and increasing transport costs. The margin increased from an average of 8 FCFA per kg between January 1992 and September 1994 to an average of 20 FCFA per kg between October 1996 and September 1999. This indicates that traders make larger profits. On the other hand, looking at the difference between the consumer price in one region and the producer price in another region, we have to conclude

that margins for transport towards Ouagadougou did increase less than transport towards the other regions. This indicates that competition on the wholesale market of Ouagadougou has become more fierce, whereas this is less the case in the other regions. A more detailed investigation of the market situation is needed to confirm this.

5. In most cases prices increase during the lean season (from April to September). Looking at the monthly cereal price average for Burkina Faso (see Figure A3.5), it follows that in most years producer and consumer prices reach their maximum in July and August. Looking at the production year from one harvest to the other (from October to September), it follows that minimum price levels are in most cases attained in November or December. However, for the production years ‘94/’95 and ‘97/’98 producer prices reach their minimum already in October. In these years cereal production was lower than in other years, see also Table A3.38. Furthermore, in the years ‘92/’93 and ‘93/’94 the average consumer price reaches its minimum in December and January. In these years cereal production was good. On average the minimum is attained in November, and the maximum in August.

Table A3.36 Average seasonal cereal producer and consumer prices for each CRPA for the period 1992-‘99 in FCFA per kg.

Producer prices		**Average Quarterly Producer Price for the period 1992 - 1999**								**Average**
	Period	**1992**	**1993**	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**‘92-‘99**
Centre Nord	**jan-mar**	56	54	55	66	94	106	127	93	76
	apr-jun	56	60	60	74	105	103	144	99	80
	jul-sept	68	64	64	76	138	119		114	76
	oct-dec	54	51	51	74	102	108	93	76	73
Centre Ouest	**jan-mar**	64	59	42	63	92	110	124	113	79
	apr-jun	75	58	49	76	112	114	149	116	88
	jul-sept	75	59	51	81	140	112	173	114	92
	oct-dec	61	40	51	78	91	104	115	88	77
Centre Sud	**jan-mar**	68	64	56	80	86	127	129	108	88
	apr-jun	71	71	60	92		138	159	109	91
	jul-sept	67	67	64	99	128	118	203	109	92
	oct-dec	63	54	65	68	122	113	113	82	88

Cont.Table A3.36

Sahel	**jan-mar**	61	65	66	65	110	107	118	92	76
	apr-jun	59	71	84	77	119		128	95	82
	jul-sept	73	75	79	90		83		96	82
	oct-dec	63	56	60	79	101	102	86	94	73
Mouhoun	**jan-mar**	51	38	36	51	82	80	101	99	67
	apr-jun	54	43	46	60	96	86	127	88	76
	jul-sept	61	48	45	68	128	81	137	87	83
	oct-dec	42	36	42	68	78	83	88	70	65
Est	**jan-mar**	65	53	45	61	75	98	119	102	75
	apr-jun	73	52	53	67	88	102	134	102	81
	jul-sept	69	55	52	79	121	110	157	93	88
	oct-dec	51	42	51	64	94	101	112	69	72
Centre Est	**jan-mar**	69	57	49	78	90	114	118	106	79
	apr-jun	69	61	57	89	107	104	139	102	84
	jul-sept	74	63	60	94	133	118	161	103	93
	oct-dec	59	52	63	81	114	113	123	79	81
Nord	**jan-mar**	62	54	46	68			123		63
	apr-jun	65	59							61
	jul-sept		63							63
	oct-dec	61	55	49	83	99				65
Sud Ouest	**jan-mar**	68	69	57	72	89	78		88	70
	apr-jun		70	59						65
	jul-sept		72	68	89				83	73
	oct-dec	64	58	64					75	63
Hauts Bassins	**jan-mar**	46	39	33	51	79	83	93	82	62
	apr-jun	52	37	41	63	90	86	107	75	66
	jul-sept	53	48	40	70	114	86	121	85	73
	oct-dec	42	32	46	71	84	88	84	69	62
Comoe	**jan-mar**	60	48	41	63	82	104	96	111	74
	apr-jun	66	49	49	75	97	117	119	113	79
	jul-sept	67	55	53	87	142	112	137	104	84
	oct-dec	49	43	53	81	113	95	168	85	75
Burkina Faso	**jan-mar**	61	53	47	63	85	99	112	97	74
	apr-jun	64	55	52	72	98	103	131	97	79
	jul-sept	66	58	54	79	128	101	146	94	84
	oct-dec	54	45	52	72	95	99	103	76	72

Consumer prices		Average Quarterly Consumer Price for the year 1992 - 1999								Average '92-'99
	Period	1992	1993	1994	1995	1996	1997	1998	1999	
Centre	jan-mar	83	82	68	85	112	130	137	117	102
	apr-jun	87	76	70	98	127	127	156	125	108
	jul-sept	92	80	76	100	159	131	171	122	116
	oct-dec	84	74	78	104	145	122	132	115	107
Centre Nord	jan-mar	61	59	60	73	100	116	139	105	89
	apr-jun	62	65	67	80	118	123	158	106	97
	jul-sept	73	67	66	87	166	129	173	109	108
	oct-dec	59	56	62	78	124	119	113	92	88
Centre Ouest	jan-mar	67	64	47	74	103	112	130	121	90
	apr-jun	75	65	55	86	119	117	152	124	99
	jul-sept	77	68	55	89	150	113	168	120	106
	oct-dec	64	51	56	88	103	108	129	100	89
Centre Sud	jan-mar	71	66	57	87	96	125	138	114	94
	apr-jun	79	74	65	105	104	139	158	119	105
	jul-sept	72	68	69	109	128	122	151	104	102
	oct-dec	66	56	71	83	123	123	122	85	92
Sahel	jan-mar	73	67	68	78	115	131	146	121	99
	apr-jun	78	69	78	85	131	131	174	126	109
	jul-sept	87	73	77	92	174	133	189	125	119
	oct-dec	69	63	69	88	133	129	129	112	99
Mouhoun	jan-mar	54	42	41	58	90	93	117	99	75
	apr-jun	61	46	52	65	105	99	140	103	84
	jul-sept	69	53	53	74	142	99	154	103	94
	oct-dec	48	41	49	76	91	98	101	84	75
Est	jan-mar	69	57	47	66	78	106	129	103	79
	apr-jun	74	59	54	73	93	115	155	111	89
	jul-sept	77	63	58	86	124	116	175	106	99
	oct-dec	58	45	56	70	96	112	116	76	78
Centre Est	jan-mar	73	61	51	81	95	126	140	110	92
	apr-jun	80	65	60	97	113	126	156	115	101
	jul-sept	76	66	62	97	136	130	161	109	104
	oct-dec	62	53	65	89	113	126	117	89	90
Nord	jan-mar	66	58	58	76	108	106	129	108	89
	apr-jun	66	62	66	83	119	111	154	109	96
	jul-sept	77	66	66	94	161	111	163	110	106
	oct-dec	64	56	65	94	106	112	114	102	89

Cont. Table A3.36

Sud Ouest	**jan-mar**	70	70	56	74	98	127	141	133	97
	apr-jun	74	70	61	85	115	137	152	138	103
	jul-sept	80	74	64	94	147	134	186	135	115
	oct-dec	69	62	60	90	124	117	148	113	101
Hauts Bassins	**jan-mar**	51	46	41	57	86	102	110	108	75
	apr-jun	57	42	48	68	98	106	136	105	82
	jul-sept	61	50	47	74	129	95	151	108	90
	oct-dec	46	41	48	80	104	96	123	93	80
Comoe	**jan-mar**	68	54	49	69	92	118	113	127	86
	apr-jun	73	55	56	81	105	136	133	133	97
	jul-sept	77	67	61	95	135	137	159	125	107
	oct-dec	64	55	61	95	109	116	139	109	93
Burkina Faso	**jan-mar**	66	59	52	72	96	115	130	112	88
	apr-jun	72	61	60	82	111	120	152	116	96
	jul-sept	76	65	61	89	144	119	167	114	104
	oct-dec	61	53	62	84	113	113	122	95	88

Notes: Cereal prices are the averages for the cereals millet, white sorghum and white maize. Averages are based on 1992-1996 weekly prices and 1997-1999 monthly prices collected by SIM/SONAGESS on 37 markets

Table A3.37 Ranking of average 1992 –1999 cereal prices, from lowest to highest cereal price per quarter for each crpa.

	Producer price[1)]					**Consumer price**				
	Jan-Mar	**Apr-Jun**	**Jul-Sept**	**Oct-Dec**	**Annual**	**Jan-Mar**	**Apr-Jun**	**Jul-Sept**	**Oct-Dec**	**Annual**
1	HB	N	N	HB	N	HB	HB	HB	M	HB
2	N	SO	SO	SO	HB	M	M	M	E	M
3	M	HB	HB	M	SO	E	E	E	HB	E
4	SO	M	CN	N	M	COM	N	CS	CN	N
5	COM	COM	S	E	CN	CN	COM	CE	CO	COM
6	E	CN	M	S	S	N	CN	CO	N	CN
7	S	E	COM	CN	COM	CO	CO	N	CE	CO
8	CN	S	E	COM	E	CE	CE	COM	CS	CE
9	CE	CE	CO	CO	CO	CS	SO	CN	COM	CS
10	CO	CO	CS	CE	CE	SO	CS	SO	S	SO
11	CS	CS	CE	CS	CS	S	C	C	SO	S
12						C	S	S	C	C

Notes: 1) No producer prices are available for the CRPA Centre. 2) Centre = C; Centre Nord = CN; Centre Ouest = CO; Centre Sud = CS; Sahel = S; Mouhoun = M; Est = E; Centre Est = CE; Nord = N; Sud Ouest = SO; Hauts Bassins = HB; Comoe = COM. Source: SIM/SONAGESS price data, see Table A3.36.

Table A3.38 Average annual producer and consumer prices (in FCFA/kg), cereal production (in 1000 tonnes), and indexes for prices and production (base year = Oct '92/Sept '93).

	Jan '92 – Sept '92	Oct '92 – Sept '93	Oct '93 – Sept '94	Oct '94 – Sept '95	Oct '95 – Sept '96	Oct '96 – Sept '97	Oct '97 – Sept '98	Oct '98 – Sept '99
Producer price	63	55	49	67	94	99	117	98
Consumer price	71	62	57	77	109	117	140	116
Production 1)	2378	2417	2480	2414	2212	2359	1913	2553
Index Producer price 2)	115	100	90	122	171	181	213	179
Index Consumer price 2)	116	100	92	126	178	190	228	189
Index Production 2)	98	100	103	100	92	98	79	106

Notes: 1) Prices for the period Oct year *t* / Sept year *t*+1 are compared with production from the harvest from October – November from year *t*. Production is given in Table A3.2. 2) Index Producer price year *t* = Producer price year *t* / Producer price Oct '92 / Sept '93, similar for Consumer price and Production.

Table A3.39 Average cereal prices for the periods January '92 to September'94 and October '96 to September '99 for each CRPA (in FCFA/kg).

	Producer price			**Consumer price**			**Margin 2)**	
	Average Jan '92 – Sept '94	**Average Oct '96 – Sept '99**	**% Increase 1)**	**Average Oct '92 – Sept '94**	**Average Oct '96 – Sept '99**	**% Increase 1)**	**Period Jan '92- Sept '94**	**Period Oct '96 – Sept '99**
Centre				79	134	69%		
Centre Nord	58	103	78%	63	126	100%	5	23
Centre Ouest	57	115	103%	63	125	99%	6	10
Centre Sud	63	123	94%	68	128	90%	4	5
Sahel 3)	66	100	50%	73	139	91%	7	39
Mouhoun	45	94	109%	51	108	113%	6	15
Est	54	109	100%	60	120	99%	6	11
Centre Est	61	117	92%	64	127	98%	3	10
Nord 3)	58	111	92%	64	119	86%	6	8
Sud Ouest 3)	65	83	28%	68	139	104%	4	56
Hauts Bassins	42	89	112%	48	113	133%	7	24
Comoe	52	113	117%	62	129	110%	10	17
Burkina Faso	55	104	91%	62	125	99%	8	20

Notes: 1) % Increase = ((average '92-'94 / average '96-'99) – 1)*100%. 2) Margin = Consumer price – Producer price. 3) The data for these regions have to be treated with care, since many data are missing for the period '96-'99. Source: SIM/SONAGESS price data, see Table A3.36.

Figure A3.4: Average cereal prices for Burkina Faso for the period 1992 - 1999.

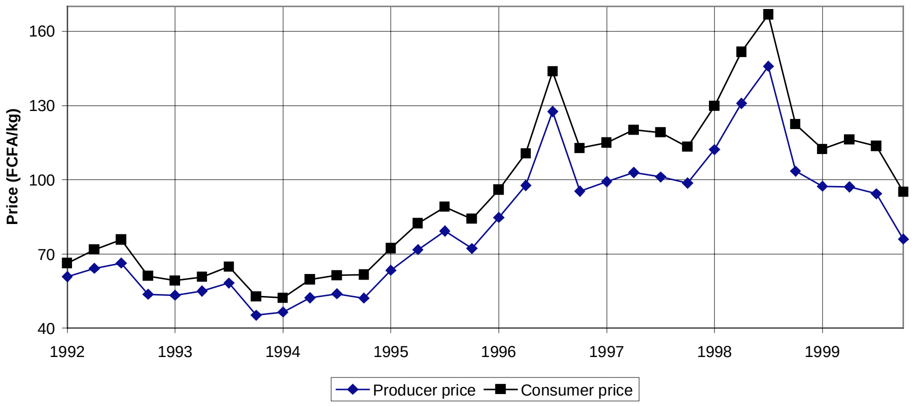
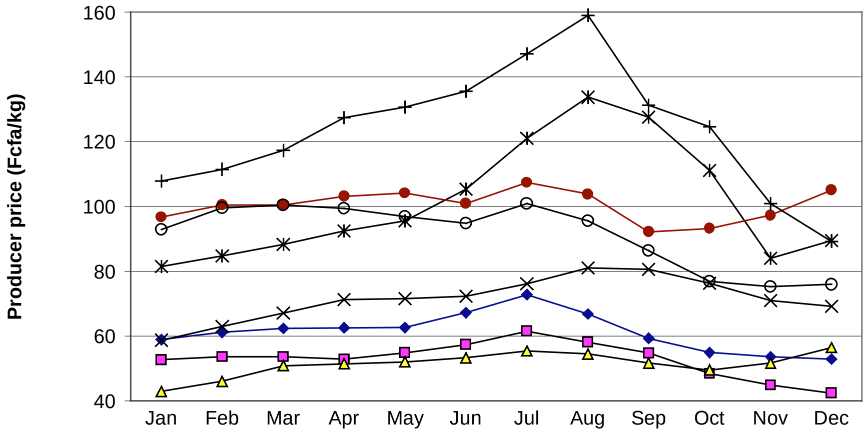


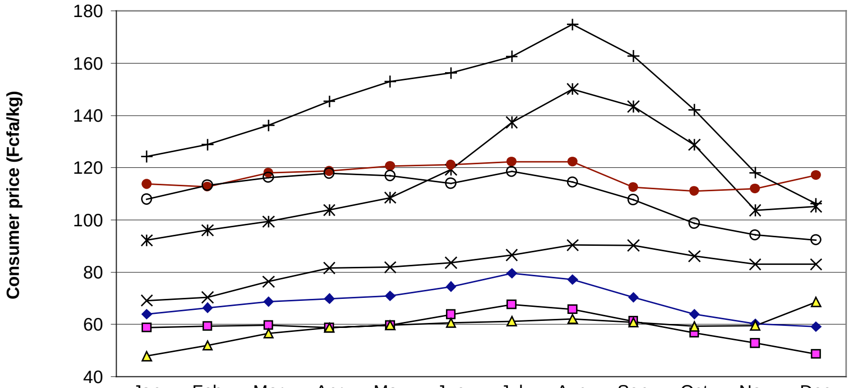
Figure A3.5: Average cereal producer and consumer price per month; 1992-1999.

A3.5a: Producer prices



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1992	59	61	62	63	63	67	73	67	59	55	54	53
1993	53	54	54	53	55	57	62	58	55	48	45	42
1994	43	46	51	52	52	53	55	55	52	50	52	57
1995	59	63	67	71	72	72	76	81	81	76	71	69
1996	82	85	88	93	96	105	121	134	128	111	84	89
1997	97	101	100	103	104	101	107	104	92	93	97	105
1998	108	112	117	127	131	136	147	159	131	125	101	89
1999	93	100	101	99	97	95	101	96	86	77	75	76

A3.5b: Consumer prices



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1992	64	66	69	70	71	74	80	77	70	64	60	59
1993	59	59	60	59	60	64	68	66	61	57	53	49
1994	48	52	57	59	60	61	61	62	61	59	60	69
1995	69	70	76	82	82	84	87	90	90	86	83	83
1996	92	96	99	104	109	119	137	150	143	129	104	105
1997	114	113	118	119	121	121	122	122	113	111	112	117
1998	124	129	136	145	153	156	163	175	163	142	118	106
1999	108	113	116	118	117	114	119	115	108	99	94	92

Appendix 4: Trading costs

In Section 8.2 estimates are made of costs made by cereal traders in Burkina Faso. Not many authors reported on the strategies of cereal traders and their trading costs. In this appendix in particular recent surveys executed by Bassolet (2000) and Sirpé (2000) and a large survey by the Universities of Michigan and Wisconsin (Sherman et al.,1987; and Déjou, 1987) are evaluated in order to be able to estimate the trading costs which are necessary for our modelling approach. Bassolet conducted a survey among 357 cereal traders at 16 markets scattered over Burkina Faso. The main objective of his inquiry was to get a picture of how cereal trade is organised and how it has been changed since the market and price liberalisation in 1992. Sirpé analysed the transport sector of Burkina Faso, with the accent on cereal transport. He interviewed in 1995 354 carriers, in order to obtain a picture of their strategies, costs and revenues. Déjou (1987) analysed the strategies of cereal traders, within the framework of a large research project on the dynamics of cereal trade in Burkina Faso, executed by the University of Michigan and the University of Wisconsin (see also Sherman et al., 1987). Between February 1984 and February 1985 a large team of Burkinabé and American researchers conducted a survey among numerous cereal traders, farmers and consumers scattered over the entire country.

A4.1 Transport costs

In Section 8.2 estimates are made of the cost of transporting a bag of cereals between a number of markets. To determine the average costs to transport cereals between the different markets, the organisation and functioning of the transport sector is of importance. Transport costs depend much on the distance travelled. If cereals are transported between two rural markets within one region, other types of trucks may be used than for transport towards an urban or a redistribution centre, and consequently prices will differ.

Survey by Sirpé

Sirpé (2000) distinguishes between three levels of transport. First, the *local* or *regional* level, where goods are transported in small quantities between rural markets, or between the rural markets and provincial towns. Roads are often not well developed, and transport takes place using pick-ups or vans. Carriers are often traders having their own means of transport. Secondly, the *national* level concerns principally transport between provincial towns or between the two main centres Ouagadougou and Bobo-Dioulasso. Distances usually do not exceed 400 km, and most often small trucks with a carrying capacity of 10 tonnes are used. Carriers are often traders. The third level concerns the *international* level, with transports between Burkina Faso (most often Ouagadougou or Bobo-Dioulasso) and the harbours of the neighbouring countries (Abidjan, Lomé, Cotonou), and to a lesser extent to the capitals of Mali and Niger. These international connections are served by large trucks with an average capacity of 32 tonnes which travel over 1000 km per journey. At this international level many goods are transported. However, cereals are less important at this level, and it is therefore not considered in our study.[41]

A survey among 354 carriers showed that 60% of them owned only one truck. Only 2% owned more than 10 trucks. Those who are also trader, use their truck to transport their own merchandise, and if possible, also from other traders. They usually do not keep any records on the costs and benefits from transport. Carriers who are not also trader, most of the times hold ties with only a few clients. They are more or less specialized in transporting only a limited number of products, although also other products may be transported occasionally. Among these carriers, the number of enterprises which go bankrupt, is considerable. The number of new enterprises which failed between 1986 and 1991 ranged between 26% and 51% (Sirpé, 2000). A reason for this is that the prices charged for their services may cover the personnel and fuel costs, but do often not cover for maintenance and depreciation. So, many

[41] The cereals imported in Burkina Faso, often arrive in smaller quantities by traders who operate close to the border. Rice, however, is imported in large quantities, and is transported with large trucks. Price formation of rice is not analysed in this paper.

entrepreneurs enter the free transport market if they can gather the starting capital to pay for the truck, the taxes and the insurance, but many of them do not earn enough to keep their business running. Competition, especially on the local segment, may be too fierce, which drives down prices below costs. This may not draw the attention of the carrier if he does not keep books of his business, or it is not a big problem if the carrier is also a trader owning his own means of transport.

A distinction is to be made between costs of transport (for the transport agent) and the prices of transport which have to be paid by the trader paying for the transport services. Evaluating all costs made by carriers, and the prices charged by them, Sirpé concludes that for pick-ups and vans the transport costs made, on average, do just not cover for the prices charged per ton per kilometer (Transport price = 178 FCFA per tonne per kilometer; Transport costs = 179 FCFA per tonne per kilometer). For 10-tonnes trucks, he concludes that the margin between the costs and the price charged is large (Transport price = 112 FCFA per tonne per kilometer; Transport costs = 60 FCFA per tonne per kilometer; Margin = 47%), while large trucks (32 tonnes) just cover the costs made (Transport price = 42 FCFA per tonne per kilometer; Transport costs = 40 FCFA per tonne per kilometer). These conclusions must, however, be treated with care, since price differences are caused by many factors, the differences between the carriers are large and since the small and medium size carriers do often not know all their costs. The transport costs are influenced considerably by the road conditions. Not only maintenance costs increase, but also the costs for fuel, lubricants and tires increases. Sirpé makes a distinction between three road types: asphalted roads, unpaved roads, and dirt roads (bad unpaved roads). He discussed how much transport costs should increase according to the 'Direction des Transports' of Burkina Faso, if merchandise was transported over unpaved roads (see Table A4.1). This shows that certainly maintenance costs should increase considerably for transports between rural markets.

Table A4.1 Increase of transport costs if unpaved roads or dirt roads are crossed.

	Increase of transport costs	
	Unpaved road	Dirt road
Fuel and lubrifiants	+25%	+65%
Tires	+45%	+75%
Maintenance	+75 to 130%	+200 to 300%

Source: Sirpé (2000), obtained from the 'Direction des Transports' of Burkina Faso.

Surveys of the Universities of Michigan and Wisconsin

Déjou (1987) also reports on cereal transport in Burkina Faso. Unlike Sirpé who analysed the transport sector from the viewpoint of the carrier, Déjou looks at the price a trader has to pay if he hires the services from a carrier or a trader owning a truck. In Table A4.2 it is shown which price a trader has to pay to transport a cereal bag of 100 kg between two markets. The difference between the price presented by Déjou (1987) and Sirpé (2000) is remarkable. If it is considered that 10-tonne trucks are used for inter-regional transport, than the estimated transport price of 11 FCFA per 100 kg bag per km observed by Sirpé, is extremely high compared to the transport costs born by the traders, which are presented in Table A4.2. In this table a distinction has been made between the dry and the rainy season. Transport during the rainy season may be more difficult, causing for more time and fuel and maintenance costs.

Table A4.2 Transport costs per 100 kg bag of cereals, reported by Déjou (1987).

From	Province	To	Province	Distance (km) (a)	Transport costs				Increase rainy season (f)
					Dry season		Rainy season		
					FCFA (b)	FCFA/km (c)	FCFA (d)	FCFA/km (e)	
Bare	Houet	Ouagadougou	Kadiogo	370[1]	600	1.6	600	1.6	0%
Bobo-Dioulasso	Houet	"	"	371[1]	500	1.4	500	1.4	0%
Dano	Bougouriba	"	"	274[2]	1,000	3.7	1,200	4.4	19%
Dedougou	Mouhoun	"	"	225[2]	1,000	4.4	1,200	5.3	20%
Djibasso	Kossi	"	"	337[2]	1,250	3.7	1,500	4.5	22%
Guelwongo	Nahouri	"	"	230[2]	750	3.3	800	3.5	6%
Koudougou	Boulkiemde	"	"	97[1]	500	5.2	500	5.2	0%
Koupela	Kouritenga	"	"	137[1]	350	2.6	350	2.6	0%
Leo	Sissili	"	"	205[3]	750	3.7	1000	4.9	32%

Cont. Table A4.2

Nouna	Kossi	"	"	281[2]	1,000	3.6	1,000	3.6	0%
Po	Nahouri	"	"	144[1]	500	3.5	500	3.5	0%
Pouytenga	Kouritenga	"	"	140[1]	350	2.5	350	2.5	0%
Solenzo	Kossi	"	"	312[2]	1,250	4	1,250	4	0%
Tenkodogo	Boulgou	"	"	183[1]	350	1.9	350	1.9	0%
Tougan	Sourou	"	"	128[2]	1,250	9.8	1,500	11.7	19%
Zabre	Boulgou	"	"	181[2]	800	4.4	900	5	14%
Ouagadougou	Kadiogo	Aribinda	Soum	380[3]	1,250	3.3	2,000	5.3	61%
"	"	Boulsa	Namentenga	176[2]	800	4.6	1,200	6.8	48%
"	"	Djibo	Soum	290[3]	600	2.1	750	2.6	24%
"	"	Dori	Seno	265[2]	1,000	3.8	1,500	5.7	50%
"	"	Gorom-Gorom	Oudalan	321[2]	2,000	6.2	3,000	9.4	52%
"	"	Kaya	Sanmatenga	98[1]	350	3.6	600	6.1	69%
"	"	Kongoussi	Bam	115[3]	400	3.5	400	3.5	0%
"	"	Markoye	Oudalan	358[2]	2,000	5.6	3,000	8.4	50%
"	"	Ouahigouya	Yatenga	181[1]	500	2.8	500	2.8	0%
"	"	Pissila	Sanmatenga	120[2]	500	4.2	750	6.3	50%
"	"	Diebougou	Bougouriba	308[2]	1,000	3.3	1,200	3.9	18%
"	"	Gaoua	Poni	381[2]	1,000	2.6	1,200	3.2	23%
"	"	Kombissiri	Bazega	40[1]	350	8.8	400	10	14%
"	"	Fada N'Gourma	Gourma	225[1]	500	2.2	500	2.2	0%
Bare	Poni	Bobo-Dioulasso	Houet	30[3]	500	16.7	500	16.7	0%
Dano	Bougouriba	"	"	175[3]	750	4.3	800	4.6	7%
Dedougou	Mouhoun	"	"	179[3]	1,000	5.6	1,000	5.6	0%
Djibasso	Kossi	"	"	290[3]	700	2.4	700	2.4	0%
N'Dorola	Kenedougou	"	"	95[2]	500	5.3	700	7.4	40%
Nouna	Kossi	"	"	235[3]	1,000	4.3	1,000	4.3	0%
Solenzo	Kossi	"	"	147[2]	800	5.4	800	5.4	0%
Tougan	Sourou	"	"	277[3]	1,250	4.5	1,250	4.5	0%
Bobo-Dioulasso	Houet	Kaya	Sanmatenga	454[1]	1,000	2.2	1,000	2.2	0%
"	"	Koudougou	Boulkiemde	288[1]	400	1.4	400	1.4	0%
"	"	Ouahigouya	Yatenga	371[3]	1,250	3.4	1,275	3.4	0%
Bobo-Dioulasso	Houet	Diebougou	Bougouriba	138[3]	1,000	7.3	1,000	7.3	0%
"	"	Gaoua	Poni	211[3]	1,250	5.9	1,375	6.5	10%
Diebougou	Bougouriba	Gaoua	Poni	73[3]	600	8.2	700	9.6	17%
Dedougou	Mouhoun	Ouahigouya	Yatenga	192[3]	1,000	5.2	1,000	5.2	0%
Djibasso	Kossi	"	"	254[3]	1,000	3.9	1,000	3.9	0%
Solenzo	Kossi	"	"	279[3]	1,250	4.5	1,250	4.5	0%
Tougan	Sourou	"	"	94[3]	700	7.5	800	8.5	13%

Cont. Table A4.2

Ouahigouya	Yatenga	Aribinda	Soum	199[3]	1,000	5.0	1,000	5.0	0%
"	"	Djibo	Soum	109[3]	500	4.6	500	4.6	0%
Nouna	Kossi	Koudougou	Boulkiemde	184[3]	800	4.4	800	4.4	0%
Djibasso	Kossi	"	"	240[3]	1,000	4.2	1,000	4.2	0%
Tougan	Sourou	"	"	131[3]	1,000	7.6	1,000	7.6	0%
Leo	Sissili	"	"	137[3]	1,500	11	1,750	12.8	16%
Dori	Seno	Gorom-Gorom	Oudalan	56[3]	1,500	26.8	2,000	35.7	33%
Pissila	Sanmatenga	Dori	Seno	137[3]	700	5.1	1,000	7.3	43%
Djibasso	Kossi	Dedougou	Mouhoun	112[3]	600	5.4	700	6.3	17%
Nouna	Kossi	"	"	56[3]	400	7.1	600	10.7	51%
Solenzo	Kossi	"	"	87[3]	700	8.1	800	9.2	14%
Tougan	Sourou	"	"	98[3]	700	7.1	700	7.1	0%
Dedougou	Mouhoun	Kaya	Sanmatenga	323[2]	1,500	4.6	1,750	5.4	17%
Koudougou	Boulkiemde	"	"	195[1]	1,000	5.1	1,000	5.1	0%
Solenzo	Kossi	"	"	410[2]	1,500	3.7	1,750	4.3	16%

Notes: 1) transport over asphalted road, 2) transport over both asphalted and unpaved roads, 3) transport over unpaved and dirt roads. (c) = (b)/(a), (e) = (d)/(a), (f) = ((d)/(b)-1)*100%. Source: Déjou (1987)

A closer look at Table A4.2 shows that the transport costs of the dry and the rainy season differ more if the villages are not connected by an asphalted road. Furthermore, transport costs are much lower for asphalted roads than for unpaved roads. For example, transporting from Ouagadougou to Fada N'Gourma is much cheaper than transporting from Dédougou to Ouagadougou, while the distances are more or less the same. The first route passes via an asphalted road, the second route is partly via a mediocre unpaved road. Some of the differences reported in the table are no longer valid today. Nowadays, the price between Ouagadougou and Kaya will no longer be much higher during the rainy season, since this road has been asphalted. The difference between dry season and rainy season transport to the Sahelian villages is large. Road conditions in the Sahel are not very favourable. Even the road to Dori and Gorom-Gorom is of a mediocre quality. Some strange observations made from the table is that transporting directly from Bobo-Dioulasso to Kaya is during the dry season more expensive than transporting from Bobo-Dioulasso to Ouagadougou and later transporting from Ouagadougou to Kaya. This is not very realistic. It is furthermore strange that transport costs from Bobo-Dioulasso to Ouagadougou are the same as transport costs from Koudougou to Ouagadougou. Going from Bobo-Dioulasso to Ouagadougou you pass by Koudougou, so you would expect transport costs to differ.

Green (1987), like Déjou (1987) also working in the research project of the Universities of Michigan and Wisconsin but concentrating on trade in the Volta Noire region (CRPA Hauts Bassins and Mouhoun), also noted the differences in transport costs between short and long distances. She concluded that for distances shorter than 15 km, transport costs per kilometer were approximately 17 FCFA for a 100 kg bag (ranging between 14 and 25 FCFA). For distances between 25 and 80 kilometers transport costs are on average 12 FCFA per bag per kilometer (between 8 and 17 FCFA). Transport costs for distances exceeding 80 kilometers amounted on average 5 FCFA per bag per kilometer (ranging between 4 and 6 FCFA).

Survey by Bassolet

Also Bassolet (2000) estimated transport prices, based on his survey among cereal traders in 1997. Table A4.3 shows that on average transport costs per kilomer are higher for transport over short distances than for transport over larger distances (compare for example transport from Solenzo to Bobo-Dioulasso or to Dédougou, or transport to Pouytenga from Fada N'Gourma, Bogande or Bobo-Dioulasso). Furthermore, transport over asphalted roads is generally cheaper than transport over unpaved roads. Differences between dry and rainy season do not show the same pattern as in Table A4.2. In general transport is more expensive during the rainy season, but the difference between asphalted and unpaved roads is less clear as in Table A4.2. For example, transport from Manné to Gorom-Gorom over an unpaved road has the same price during both seasons. On the other hand transport from Ouagadougou to Gorom-Gorom, which is partly over an asphalted road, is 1/3 more expensive during the rainy season than during the dry season. Furthermore, for the asphalted road between Bobo-Dioulasso and Ouagadougou and between Fada N'Gourma and Koupéla transport costs are the same in both seasons, but for the asphalted road Léo – Ouagadougou transport costs are twice as expensive during the rainy season. Transport costs also depend upon the possibility for the carrier to have a truck load on the return journey. If he is certain to have a return load, the price will be lower. For that reason transport costs between the large urban centres are lower, than between secondary markets. Compare for example transport costs between Pouytenga and Bobo-Dioulasso (2.1 FCFA per km) and Pouytenga and Bogandé (8.2 FCFA per km). The costs presented in this table reflect the costs paid by a trader if he hires the services of a carrier. The traders interviewed who owned their own means of transport were not asked properly to indicate the costs they made for transport and maintenance of their vehicles.

Table A4.3 Transport costs per 100 kg bag, reported by Bassolet (2000).

From	**Province**	**To**	**Province**	**Distance (km) (a)**	**Transport costs (FCFA)**				**Increase rainy season (f)**
					Mai/June (b)	**costs/km (c)**	**July/August (d)**	**costs/ km (e)**	
Kouka	Kossi	Ouahigouya	Yatenga	315[3]	1250	4,0	1575	5,0	26%
Solenzo	Kossi	"	"	279[3]	1400	5,0	1675	6,0	20%
Ouahigouya	Yatenga	Djibo	Soum	109[3]	550	5,0	650	6,0	18%
Manne	Sanmatenga	Gorom-Gorom	Oudalan	242[3]	725	3,0	725	3,0	0%
Ouagadougou	Kadiogo	"	"	321[2]	950	3,0	1300	4,0	37%
Bobo-Dioulasso	Houet	Pissila	Sanmatenga	467[2]	925	2,0	1400	3,0	51%
Ouagadougou	Kadiogo	"	"	120[2]	475	4,0	600	5,0	26%
Bobo-Dioulasso	Houet	Kaya	Sanmatenga	454[1]	900	2,0	1350	3,0	50%
Ouagadougou	Kadiogo	"	"	98[1]	500	5,1	500	5,1	0%
Solenzo	Kossi	Ouagadougou	Kadiogo	312[2]	625	2,0	625	2,0	0%
Leo	Sissili	"	"	205[3]	500	2,4	1000	4,9	100%
Bobo-Dioulasso	Houet	"	"	371[1]	700	1,9	700	1,9	0%
Pouytenga	Kouritenga	"	"	140[1]	150	1,1	350	2,5	133%
Ouagadougou	Kadiogo	Dori	Seno	285[2]	1000	3,5	1250	4,4	25%
Fada N'Gourma	Gourma	Koupela	Kouritenga	88[1]	600	6,8	600	6,8	0%
Fada N'Gourma	Gourma	Pouytenga	Kouritenga	120[1]	750	6,3	1200	10,0	60%
Bogande	Gnagna	"	"	92[3]	750	8,2	750	8,2	0%
Bobo-Dioulasso	Houet	"	"	513[1]	1075	2,1	1075	2,1	0%
Orodara	Kenedougou	N'Doroloa	Kenedougou	97[3]	500	5,2	850	8,8	70%
Koloko	Kenedougou	"	"	122[3]	800	6,6			
Koundougou	Houet	Dande	Houet	17[2]	250	14,7	350	20,6	40%
Kouka	Kossi	Bobo-Dioulasso	Houet	103[2]	600	5,8	800	7,8	33%
Banwale	Houet	"	"	75[2]	600	8,0	800	10,7	33%

Cont. Table A4.3

Solenzo	Kossi	"	"	147[2]	700	4,8	700	4,8	0%
Solenzo	Kossi	Dedougou	Mouhoun	87[3]	700	8,0	700	8,0	0%

Notes: 1) Connected by asphalted roads; 2) Transport over both asphalted and unpaved roads; 3) Connected by unpaved roads. (c) = (b)/(a), (e) = (d)/(a), (f) = ((d)/(b)-1)*100%. Source: Inquiry 1997 by Bassolet (2000).

Distances

In Section 8.2 the transport costs between the main centres in the different CRPA are estimated. The costs per kilometer are based on the above evaluation of transport surveys. The distances between the centres, as well as the distance over asphalted roads, unpaved roads and dirt roads are estimated on the basis of the road map of Burkina Faso – see Table A4.4.

A4.2 Storage costs

Storage costs determine to a certain extent the price difference between two periods. The storage costs estimated in Section 8.2 include physical and financial storage costs. Physical costs include, according to Bassolet (2000) the costs for the storehouses (rent, depreciation, maintenance), costs for conservation (insecticides, shelves), and surveillance costs. Also storage losses must be taken into account. Financial costs include, according to Bassolet, opportunity costs, which indicate the benefits the trader could earn by investing in other activities. For many costs it is difficult to estimate the costs per 100 kg bag. Rent or maintenance of a storehouse must be paid, even if it is not totally full. A storehouse will not be totally filled during the entire year. Surveillance costs must be paid, even if only one bag is stored.

Storehouses

Déjou (1987) reports that it is difficult to obtain detailed data concerning storage houses and storage costs. For producers storage costs are low. They can easily and cheaply build new cereal sheds. Storage costs for small traders are also low. They store the few bags they trade at home. On the other hand, larger traders on the semi-urban and urban markets have to rent storehouses. Déjou (1987) reports that the large storehouses may cost up to 15,000 FCFA per month. She does, however, not mention how many bags can be stored in such a storehouse. On the urban markets traders often have to rent small hangars on the market place, to store the merchandise they sell during the market day. These hangars, which are often rented with two or three traders, cost 750 to 1000 FCFA monthly during the mid 80's. Bassolet (2000) gives monthly costs to rent storehouses on market places, subdivided by type of trader (wholesaler or semi-wholesaler; see Table A4.5). Rents depend on the storage capacity of the storehouses and the availability on the market. Bassolet, however, does not mention the capacity of the storehouses.

Surveillance

For the larger store houses, also surveillance costs must be paid. According to Déjou (1987), the former cereal board OFNACER charged a daily tariff of 200 FCFA for a guard. Bassolet (2000) gives totally different surveillance costs. He mentions costs which differ between 10,000 and 50,000 FCFA per month.

Table A4.4 Distances between the most important provincial centres (in km).

Distances (km)		**Centre**	**Centre Nord**	**Centre Ouest**	**Centre Ouest**	**Centre Sud**	**Sahel**	**Sahel**	**Mouhoun**	**Mouhoun**	**Est**	**Centre Est**	**Nord**	**Sud Ouest**	**Hauts Bassins**	**Comoe**
		Ouaga-dougou	**Kaya**	**Kou-dougou**	**Leo**	**Manga**	**Dori**	**Djibo**	**Dedougou**	**Tougan**	**Fada N'Gourma**	**Koupela**	**Ouahi-gouya**	**Die-bougou**	**Bobo-Dioulasso**	**Banfora**
Centre	Ouagadougou	0	98	97	165	97	265	203	225	218	225	137	181	308	356	441
Centre Nord	Kaya	98	0	195	263	195	167	146	323	258	229	141	164	406	454	539
Centre Ouest	Koudougou	97	195	0	137	197	362	300	128	131	322	234	165	240	288	373
Centre Ouest	Leo	165	263	137	0	223	430	368	264	268	390	302	346	133	269	354
Centre Sud	Manga	97	195	194	223	0	362	300	322	315	218	130	278	356	453	538
Sahel	Dori	265	167	362	430	362	0	188	490	392	261	273	297	573	621	706
Sahel	Djibo	203	146	300	368	300	188	0	301	203	428	340	109	511	559	644
Mouhoun	Dedougou	225	323	128	264	322	490	301	0	98	450	362	192	317	179	264
Mouhoun	Tougan	218	258	131	268	315	391	203	98	0	453	365	94	415	277	362
Est	Fada N'Gourma	225	229	322	390	218	261	428	450	453	0	88	396	533	581	666
Centre Est	Koupela	137	141	234	302	130	273	340	362	365	88	0	305	445	493	578
Nord	Ouahigouya	181	164	165	346	278	297	109	192	94	396	305	0	489	371	456
Sud Ouest	Diebougou	308	406	240	133	356	573	511	317	415	533	445	489	0	138	223
Hauts Bassins	Bobo-Dioulasso	356	454	288	269	453	621	559	179	277	581	493	371	138	0	85
Comoe	Banfora	441	539	373	354	538	706	644	264	362	666	578	456	223	85	0

(Continuation Table A4.4) Distance over asphalted roads

Distances (km)		**Centre**	**Centre Nord**	**Centre Ouest**	**Centre Ouest**	**Centre Sud**	**Sahel**	**Sahel**	**Mouhoun**	**Mouhoun**	**Est**	**Centre Est**	**Nord**	**Sud Ouest**	**Hauts Bassins**	**Comoe**
		Ouaga-dougou	**Kaya**	**Kou-dougou**	**Leo**	**Manga**	**Dori**	**Djibo**	**Dedougou**	**Tougan**	**Fada N'Gourma**	**Koupela**	**Ouahi-gouya**	**Dieb-ougou**	**Bobo-Dioulasso**	**Banfora**
Centre	**Ouagadougou**	0	98	97	0	97	98	0	97	97	225	137	181	224	356	441
Centre Nord	**Kaya**	98	0	195	98	170	0	0	195	0	133	45	0	322	454	539
Centre Ouest	**Koudougou**	97	195	0	0	172	195	97	0	0	322	234	74	132	267	352
Centre Ouest	**Leo**	0	98	0	0	70	98	0	0	0	225	137	181	0	0	85
Centre Sud	**Manga**	97	170	169	70	0	170	72	169	169	133	45	253	70	428	513
Sahel	**Dori**	98	0	195	98	170	0	0	195	0	0	8	0	322	454	539
Sahel	**Djibo**	0	0	97	0	72	0	0	0	0	225	137	0	224	356	441
Mouhoun	**Dedougou**	97	195	0	0	169	195	0	0	0	322	234	0	0	0	85
Mouhoun	**Tougan**	97	0	0	0	169	0	0	0	0	322	234	0	277	277	362
Est	**Fada N'Gourma**	225	133	322	225	133	0	225	322	322	0	88	396	449	581	666
Centre Est	**Koupela**	137	45	234	137	45	8	137	234	234	88	0	305	361	493	578
Nord	**Ouahigouya**	181	0	74	181	253	0	0	0	0	396	305	0	405	371	456
Sud Ouest	**Diebougou**	224	322	132	0	70	322	224	0	277	449	361	405	0	0	85
Hauts Bassins	**Bobo-Dioulasso**	356	454	267	0	428	454	356	0	277	581	493	371	0	0	85
Comoe	**Banfora**	441	539	352	85	513	539	441	85	362	666	578	456	85	85	0

(Continuation Table A4.4) Distance over unpaved roads

Distances (km)		**Centre**	**Centre Nord**	**Centre Ouest**	**Centre Ouest**	**Centre Sud**	**Sahel**	**Sahel**	**Mouhoun**	**Mouhoun**	**Est**	**Centre Est**	**Nord**	**Sud Ouest**	**Hauts Bassins**	**Comoe**
		Ouaga-dougou	**Kaya**	**Kou-dougou**	**Leo**	**Manga**	**Dori**	**Djibo**	**Dedougou**	**Tougan**	**Fada N'Gourma**	**Koupela**	**Ouahi-gouya**	**Die-bougou**	**Bobo-Dioulasso**	**Banfora**
Centre	**Ouagadougou**	0	0	0	165	0	167	203	128	121	0	0	0	0	0	0
Centre Nord	**Kaya**	0	0	0	165	25	167	91	128	145	0	0	0	0	0	0
Centre Ouest	**Koudougou**	0	0	0	137	25	167	203	128	131	0	0	0	0	0	0
Centre Ouest	**Leo**	165	165	137	0	58	332	368	264	268	165	165	165	81	81	81
Centre Sud	**Manga**	0	25	25	58	0	192	228	153	146	85	85	25	139	25	25
Sahel	**Dori**	167	167	167	332	192	0	188	295	283	261	195	188	167	167	167
Sahel	**Djibo**	203	91	203	368	228	188	0	192	94	203	203	0	203	203	203
Mouhoun	**Dedougou**	128	128	128	264	153	295	192	0	98	128	128	192	179	179	179
Mouhoun	**Tougan**	121	145	131	268	146	282	94	98	0	131	131	94	0	0	0
Est	**Fada N'Gourma**	0	0	0	165	85	261	203	128	131	0	0	0	0	0	0
Centre Est	**Koupela**	0	0	0	165	85	195	203	128	131	0	0	0	0	0	0
Nord	**Ouahigouya**	0	0	0	165	25	188	0	192	94	0	0	0	0	0	0
Sud Ouest	**Diebougou**	0	0	0	81	139	167	203	179	0	0	0	0	0	0	0
Hauts Bassins	**Bobo-Dioulasso**	0	0	0	81	25	167	203	179	0	0	0	0	0	0	0
Comoe	**Banfora**	0	0	0	81	25	167	203	179	0	0	0	0	0	0	0

(Continuation Table A4.4) Distance over dirt roads

Distances (km)		Centre	Centre Nord	Centre Ouest	Centre Ouest	Centre Sud	Sahel	Sahel	Mouhoun	Mouhoun	Est	Centre Est	Nord	Sud Ouest	Hauts Bassins	Comoe
		Ouaga-dougou	Kaya	Kou-dougou	Leo	Manga	Dori	Djibo	Dedougou	Tougan	Fada N'Gourma	Koupela	Ouahi-gouya	Die-bougou	Bobo-Dioulasso	Banfora
Centre	**Ouagadougou**	0	0	0	0	0	0	0	0	0	0	0	0	84	0	0
Centre Nord	**Kaya**	0	0	0	0	0	0	55	0	113	96	96	164	84	0	0
Centre Ouest	**Koudougou**	0	0	0	0	0	0	0	0	0	0	0	91	108	21	21
Centre Ouest	**Leo**	0	0	0	0	95	0	0	0	0	0	0	0	52	188	188
Centre Sud	**Manga**	0	0	0	95	0	0	0	0	0	0	0	0	147	0	0
Sahel	**Dori**	0	0	0	0	0	0	0	0	109	0	70	109	84	0	0
Sahel	**Djibo**	0	55	0	0	0	0	0	109	109	0	0	109	84	0	0
Mouhoun	**Dedougou**	0	0	0	0	0	0	109	0	0	0	0	0	138	0	0
Mouhoun	**Tougan**	0	113	0	0	0	109	109	0	0	0	0	0	138	0	0
Est	**Fada N'Gourma**	0	96	0	0	0	0	0	0	0	0	0	0	84	0	0
Centre Est	**Koupela**	0	96	0	0	0	70	0	0	0	0	0	0	84	0	0
Nord	**Ouahigouya**	0	164	91	0	0	109	109	0	0	0	0	0	84	0	0
Sud Ouest	**Diebougou**	84	84	108	52	147	84	84	138	138	84	84	84	0	138	138
Hauts Bassins	**Bobo-Dioulasso**	0	0	21	188	0	0	0	0	0	0	0	0	138	0	0
Comoe	**Banfora**	0	0	21	188	0	0	0	0	0	0	0	0	138	0	0

Table A4.5 Monthly rents for storehouses (FCFA), reported by Bassolet (2000).

Market	Province	Monthly rent	
		Merchant[1]	Average trader[1]
Bobo-Dioulasso	Houet	4,000 to 6,000	1,300 to 2,000
Dandé	Houet	1,500	
Dédougou	Mouhoun	15,000	5,000 to 10,000
Djibo	Soum	4,500	1,000 to 2,500
Dori	Seno	15,000	7,000
Gorom-Gorom	Oudalan	10,000 to 20,000	1,000 to 8,000
Guelwongo	Nahouri	5,000	1,500 to 3,000
Kaya	Sanmatenga	10,000	1,000 to 5,000
Koupéla	Kouritenga	3,500	500 to 2,500
Manga	Zoundweogo	4,000	500
N'Dorola	Kenedougou	1,000	
Ouagadougou	Kadiogo	50,000 to 100,000	1,000 to 25,000
Ouahigouya	Yatenga	10,000	1,000 to 5,000
Pissila	Sanmatenga	7,500	1,000 to 5,000
Pouytenga	Kouritenga	3,000 to 6,000	500 to 2,750
Solenzo	Kossi	7,500 to 10,000	5,000

Note: 1) unfortunately Bassolet did not mention the business size in kg traded for merchants and average traders. As a consequence we can not present the rents per bag.
Source: Inquiry 1997 by Bassolet (2000)

Storage losses

Storage losses depend on the place and the way of storing. Sherman et al. (1987) report storage losses to be approximately 10% per year if the cereals are stored on the producers' farms, and 15 to 20% per year if they are stored in storehouses. Déjou (1987), however, believes that this is a little overestimated, and furthermore, this figure is less important since most traders no not stock for such a long time. Sedes et al. (1990) observed storage losses of 8% among traders in Bobo-Dioulasso, for a storage time of 5 months. This resembles the annual losses reported by Sherman et al. (1987).

Financial costs

Déjou (1987) also emphasizes the risk associated with borrowing money. Prices may suddenly fall, or the price increase may be lower than the credit costs. These credit costs may be considerable for many traders. Although official bank loans have interest rates of approximately 11 to 15% (Déjou, 1987), only the large traders do have the necessary collateral to obtain such loans (Bassolet (2000) reports that only 0.7% of the surveyed traders receive official bank loans). Other traders may obtain short term loans (for 1 to 6 months) from family members, large traders, decentralised financial institutions or other money lenders. Common interest rates for these money lenders, observed by Déjou (1987), may be up to 2 to 4% per month. Other financial institutions charge, according to Bassolet (2000) 20% per year (the cooperative savings and credit organisation COPEC) or 13% per year (the national agency for agricultural credit CNCA). Bassolet (2000) applies the interest rate of the CNCA since most surveyed traders obtain loans from this organisation. Next to these interest costs due to money borrowed, traders may have considerable costs due to loans that are not reimbursed by their customers or by intermediaries working on the account of the trader. Déjou (1987) reports that these costs appear to be significative. However, no data are available on these losses.

A4.3 Other trading costs

Other costs which have to be made by traders include personnel costs, cereal bags, and taxes. Finally, also opportunity costs must be considered. Estimates made in Section 8.2 are based on the data discussed below.

Personnel costs

Personnel costs are difficult to estimate per 100 kg bag. Personnel costs differ a lot between the different types of traders. Many traders operate alone, others have an extensive network of buying and selling agents. Some pay salaries to their middlemen, others pay them a commission per bag. Furthermore, as Déjou (1987) has shown, the profit margin which remains after all costs have been subtracted from the consumer price differs a lot between the different seasons and between the different markets. The trader's salary is not a fixed proportion of the price, and may even be negative during some market days, if the trader made some speculation errors. Déjou (1987) reports salaries of 15,000 to 20,000 FCFA per month for intermediaries with lower responsabilities up to 60,000 FCFA per month for regional coordinators. She does, however, not report how many bags are collected by these intermediaries. Intermediaries may also obtain a commission per 100 kg bag collected, which may be between 100 and 250 FCFA. Relatives sometimes only obtain an allowance for daily expenses and some 'gifts' (Déjou, 1987). Furthermore, local personnel who load and unload trucks receive 50 FCFA for each bag carried. Bassolet (2000) found other personnel costs. He observed payments to assistents of 500 FCFA per market day, and salaries of employees of 25,000 FCFA per month. Loading and unloading costs were reported to be 250 FCFA per bag on average.

Cereal bags

Next to personnel costs, also the costs for purchasing bags must be considered. Déjou (1987) reports bag prices which vary between 250 FCFA and 500 FCFA, depending on the condition of the bags. During periods of scarcity of bags, the prices may increase. Bassolet (2000) mentions prices between 275 and 300 FCFA.

Taxes

To sell on the market, traders must also pay taxes. Business taxes, which are proportional to the quantity traded, must be paid on a yearly basis. Many traders do not pay these taxes, but they pretend to work for a merchant whenever they are inspected. Many other traders often pay less than required. For importing and exporting merchants with sales exceeding 200 million FCFA annually, these taxes are approximately 10%, for smaller im/exporters it is approximately 15-20% of annual sales. Nationally operating traders pay a fixed fee and a part which is proportional to their sales. The amount of the fixed fee for the mid '80s is shown in Table A4.6. The fee proportional to their sales is between 8% and 12% (Déjou, 1987). Daily market taxes are collected from all traders who want to sell on a market by the market coordinators. These taxes may be 25 FCFA for small women retailers (*vendeuses*) but vary according to the quantity sold (Déjou, 1987).

Table A4.6 Fixed annual trade taxes (in FCFA), reported by Déjou (1987).

Amount of annual transactions (FCFA)		Fixed annual trade tax	
		Ouagadougou and Bobo-Dioulasso	Other markets
Less than	More than		
	50 millions	96,000	72,000
50 millions	25 millions	72,000	54,000
25 millions	15 millions	48,000	36,000
15 millions	10 millions	24,000	18,000
10 millions	5 millions	14,400	10,800
5 millions	3 millions	12,800	9,600
3 millions	1,500,000	6,400	4,800
1,500,000	500,000	3,200	2,400
500,000		1,600	1,200

Source: Ministère du Commerce, taken from Déjou (1987).

Bassolet (2000) distinguishes between three categories of taxes: trade taxes, market taxes and rent for stores and shops on the market places. These last are no real taxes, and have already been treated in the section on storage costs. According to him, the trade tax is an annual tax which is proportional to the business size, which is estimated by the treasury. Trade taxes mentioned by the traders on a number of markets are presented in Table A4.7. The data in this table do not show the taxes classified by the amount of transactions, like in Table A4.6. For that reason, the maximum trade tax for wholesalers may differ. Bassolet notes that taxes on the 'more dynamic' markets (like Pouytenga and Gorom-Gorom), and on the secondary markets (like Ouahigouya and Dori) are higher than on the large urban centres (Ouagadougou and Bobo-Dioulasso). This contradicts the trade taxes presented by Déjou (1987).

Table A4.7 Annual trade taxes, reported by Bassolet (2000).

Market	Province	Annual trade tax	
		Wholesaler	Retailer
Bobo-Dioulasso	Houet	20,000 to 45,000	2,000 to 15,000
Dande	Houet	25,000	7,500 to 15,000
Dedougou	Mouhoun	45,000	15,000 to 19,000
Djibo	Soum	45,000	7,000 to 40,000
Dori	Seno	35,000 to 75,000	15,000 to 30,000
Gorom-Gorom	Oudalan	50,000 to 75,000	20,000 to 25,000
Guelwongo	Nahouri	25,000 to 50,000	10,000
Kaya	Sanmatenga	50,000	8,000 to 40,000
Koupela	Kouritenga	30,000	8,000 to 20,000
Manga	Zoundweogo	25,000	15,000
N'Dorola	Kenedougou	28,000 to 50,000	8,000 to 25,000
Ouagadougou	Kadiogo	20,000 to 25,000	5,000 to 18,000
Ouahigouya	Yatenga	37,500 to 200,000	18,000 to 35,000
Pissila	Sanmatenga	30,000	4,000 to 20,000
Pouytenga	Kouritenga	30,000 to 100,000	13,000 to 24,000
Solenzo	Kossi	30,000 to 50,000	15,000 to 25,000

Source: Inquiry 1997 by Bassolet (2000).

Market taxes are paid daily by all traders, to pay for using the market infrastructure. Wholesalers who rent a storehouse or shop on the market pay monthly, other traders pay on a daily basis. The tax is proportional to the quality of the infrastructure. For example, market taxes on non-furnished, rural market places are 25 FCFA per day, while traders in Ouagadougou have to pay 50 FCFA per day and traders in Ouahigouya and Djibo even have to pay 100 FCFA daily. In Table A4.8 these market taxes are reported for a number of markets.

A4.4 Marketing margins

As becomes clear from the above discussion, it is difficult to indicate for all expenses the costs per 100 kg bag. Many costs are fixed, and are independent on the level of transactions. It is therefore complicated to estimate the marketing margins of cereal traders. The estimates of trading costs discussed in Section 8.2, must however be given in FCFA per bag of 100 kg. The surveys discussed below estimated the trading costs discussed in the Appendices A2.1 to A2.4 per bag of 100 kg of cereals.

Studies by Universities of Michigan and Wisconsin

Sherman et al. (1987) made estimates of the market margins for some well described trade routes. They argue that estimating margins is always a 'best guess' situation even with excellent data, since the variability of costs and prices between traders and between months is large. They estimated for four trade routes the costs made and the margins earned by the traders, see Table A4.9. Each of the routes presented is representative of comparable routes within the same region. For example, the costs made by a trader to purchase white sorghum in Djibasso and sell it in Ouagadougou are similar to the costs if the white sorghum would be purchased in other villages in the CRPA Mouhoun.

Table A4.8 Market taxes (in FCFA) reported by Bassolet (2000).

Market	Province	Market tax		
		Wholesaler (per month)	Semi-wholesaler (per month)	Retailer (per day)
Bobo-Dioulasso[1]	Houet	1000	750	25
Dande[3]	Houet	1000	1000	25
Dedougou[3]	Mouhoun	7500	625	
Djibo[3]	Soum	1500	400	100
Dori[3]	Seno	400	200	100
Gorom-Gorom[3]	Oudalan	400	200	100
Guelwongo[2]	Nahouri	1000	1000	100
Kaya[1]	Sanmatenga	1000	500	25
Koupela[2]	Kouritenga			15
Manga[2]	Zoundweogo	1000	200	50
N'Dorola[3]	Kenedougou		100	25
Ouagadougou[1]	Kadiogo	1000	600	50
Ouahigouya[1]	Yatenga	1000	750	100
Pissila[2]	Sanmatenga	1000	500	25
Pouytenga[2]	Kouritenga			15

Notes: 1) Daily market; 2) Market every three days; 3) Weekly market. In some cities, smaller markets are held every day, but some days are more important according to the regional schedule (once a week or once every three days).

Source: Inquiry 1997 by Bassolet (2000).

Table A4.9 Rough estimates of net margins per 100 kg bag of white sorghum for large traders, by Sherman et al., 1987.

a) Nouna (Kossi) to Ouahigouya (Yatenga)

Date	Purchase price	Transport	Storage and other trading costs					Total storage and other costs	Sales price	Estimated net margin
			Commission (a)	Personnel costs (b)	Sundry costs (c)	Bag and handling (d)	Capital costs (e)	(f)		
Jan-Feb	5700	1250	0-250	580	100-300	400	80-339	1160-1869	10500	1681 to 2390
Feb-Mar	n.a.	1250	0-250	580	100-300	400	n.a.	n.a.	n.a.	n.a.
Mar-Apr	7500	1250	0-250	580	100-300	400	98-411	1178-1941	11750	809 to 1572
Apr-Mai	9300	1250	0-250	580	100-300	400	116-483	1196-2013	11500	-1063 to –246
Mai-Jun	9300	1250	0-250	1000	100-300	400	121-500	1621-2450	11500	-1500 to –671
Jun-Jul	9300	1250	0-250	1000	100-300	400	121-500	1621-2450	11500	-1000 to –171
Jul-Aug	9300	1250	0-250	1000	100-300	400	121-500	1621-2450	12000	-500 to 330
Aug-Sep	11000	1250	0-250	1000	100-300	400	138-568	1638-2518	12500	-2268 to –1388
Sep-Oct	11000	1250	0-250	1000	100-300	400	138-568	1638-2518	13000	-1768 to -888
Oct-Nov	8500	1250	0-250	580	100-300	400	108-451	1188-1981	n.a.	n.a.
Nov-Dec	6000	1250	0-250	580	100-300	400	83-351	1163-1881	n.a.	n.a.

b) Djibasso (Kossi) to Ouagadougou (Kadiogo)

Date	Purchase price	Transport	Storage and other trading costs					Total storage and other costs	Sales price	Estimated net margin
			Commission (a)	Personnel costs (b)	Sundry costs (c)	Bag and handling (d)	Capital costs (e)	(f)		
Jan-Feb	6000	1250	0-250	400	100-300	400	82-344	982-1694	10000	1056 to 1769
Feb-Mar	n.a.	1250	0-250	400	100-300	400	n.a.	n.a.	11666	n.a.
Mar-Apr	9200	1250	0-250	400	100-300	400	114-472	1014-1822	12333	61 to 870
Apr-Mai	10000	1250	0-250	400	100-300	400	122-504	1022-1854	12200	-904 to –72
Mai-Jun	10000	1250	0-250	850	100-300	400	126-522	1476-2322	11250	-2322 to –1476
Jun-Jul	10000	1500	0-250	850	100-300	400	129-532	1479-2332	12400	-1432 to –579
Jul-Aug	11000	1500	0-250	850	100-300	400	139-572	1489-2372	13500	-1372 to –489
Aug-Sep	11500	1500	0-250	850	100-300	400	144-592	1494-2392	13812	-1580 to –682
Sep-Oct	11500	1500	0-250	850	100-300	400	144-592	1494-2392	13650	-1742 to –844
Oct-Nov	8700	1250	0-250	400	100-300	400	109-542	1009-1892	12000	248 to 1042
Nov-Dec	n.a.	1250	0-250	400	100-300	400	n.a.	n.a.	n.a.	n.a.

c) Nouna (Kossi) to Bobo-Dioulasso (Houet)

Date	Purchase price	Transport	Storage and other trading costs					Total storage and other costs	Sales price	Estimated net margin
			Commission (a)	Personnel costs (b)	Sundry costs (c)	Bag and handling (d)	Capital costs (e)	(f)		
Jan-Feb	5700	1000	0-250	500	100-300	400	77-326	1077-1776	9966	1490 to 2189
Feb-Mar	n.a.	1000	0-250	500	100-300	400	n.a.	n.a.	10800	n.a.
Mar-Apr	7500	1000	0-250	500	100-300	400	95-398	1095-1848	10800	452 to 1205
Apr-Mai	9300	1000	0-250	500	100-300	400	113-470	1113-1920	10800	-1420 to –613
Mai-Jun	9300	1000	0-250	750	100-300	400	116-480	1366-2180	11000	-1480 to –666
Jun-Jul	9300	1000	0-250	750	100-300	400	116-480	1366-2180	11000	-1480 to –666
Jul-Aug	9300	1000	0-250	750	100-300	400	116-480	1366-2180	12680	200 to 1015
Aug-Sep	11000	1000	0-250	750	100-300	400	133-548	1383-2248	13250	-998 to –133
Sep-Oct	11000	1000	0-250	750	100-300	400	133-548	1383-2248	11700	-2548 to –1683
Oct-Nov	8500	1000	0-250	500	100-300	400	105-438	1105-1888	8750	-2638 to –1855
Nov-Dec	6000	1000	0-250	500	100-300	400	80-338	1080-1788	9000	212 to 920

d) Ouagadougou (Kadiogo) to Dori (Seno)

Date	Purchase price	Transport	Storage and other trading costs					Total storage and other costs	Sales price	Estimated net margin
			Commission (a)	Personnel costs (b)	Sundry costs (c)	Bag and handling (d)	Capital costs (e)	(f)		
Jan-Feb	n.a.	1000	none	900	200-400	400	n.a.	n.a.	11000	n.a.
Feb-Mar	10000	1000	none	900	200-400	400	125-508	1625-2208	11000	-2208 to –1625
Mar-Apr	11666	1000	none	900	200-400	400	142-575	1642-2275	11000	-3941 to –3308
Apr-Mai	12333	1000	none	900	200-400	400	148-601	1648-2301	15000	-634 to 19
Mai-Jun	12200	1000	none	530	200-400	400	143-581	1273-1911	n.a.	n.a.
Jun-Jul	11250	1500	none	530	200-400	400	139-563	1269-1893	n.a.	n.a.
Jul-Aug	12400	1500	none	530	200-400	400	150-609	1280-1939	16000	161 to 820
Aug-Sep	13500	1500	none	530	200-400	400	161-653	1291-1983	16000	-983 to –291
Sep-Oct	13812	1000	none	530	200-400	400	159-646	1289-1976	n.a.	n.a.
Oct-Nov	13650	1000	none	530	200-400	400	158-639	1288-1969	n.a.	n.a.
Nov-Dec	12000	1000	none	900	200-400	400	145-588	1645-2288	n.a.	n.a.

Notes: (a) Commision to village buyers and coordinators; (b) Personnel costs are discussed in the text below; (c) Sundry costs include taxes, licence fees, bribes, warehousing, etc; (d) the costs of a bag is 200 FCFA, loading and unloading a bag from a truck is 100 FCFA each; (e) Capital costs are estimated as the return the trader could have made on his money if he had invested it in other activities, which are evaluated at 1% to 4% of the invested capital per month; (f) total storage and other trading costs = (a) + (b) + (c) + (d) + (e); (g) The net margin is the sales price minus all the other costs. Source: Sherman et al., 1987.

To estimate the personnel costs in Table A4.9 it is supposed that wholesale traders employ two apprentices and a warehouseman. The salaries of the employees are supposed to be the same as for low-level civil servants, i.e. 20,000 FCFA/month. The salary of the trader is evaluated at that of a high-level civil servant, which is supposed to compensate for his expertise and managerial capabilities. To estimate the personnel costs per bag, the salaries are divided by the number of bags traded. Since the number of bags traded differs per season, personnel costs are not the same in all periods. The warehousing costs are estimated for an estimated average storage time of one month. Estimates of storage costs are not given seperately, but are included in the sundry costs, which include taxes and license fees. Capital costs are interpreted as the returns the trader could have made on his money if he would have invested it in other activities. Sherman et al. (1987) evaluate them to be at least the commercial rate of interest, which was 12% per year.

The last column in Table A4.9 shows the estimated net margins on cereal trade. These turn out to be negative in most months. Reasons for this may be that the trader's salary is estimated too high, or that the purchase and sales prices for a 100 kg bag are not correct. The prices given in the table are the observed market prices for a bag. However, bags are normally heavier when purchased from producers then when sold to consumers. A large part of the margins will be earned from this practice, which is not taken into account in the data presented above. Another reason is that trade flows are not occuring in all months. For example, not many cereals will be transported from Ouagadougou to Dori in Februari. Traders will transport more to Dori later in the year, when local stocks are depleted.

Study by B. Bassolet

Bassolet (2000) executed some case studies to be able to calculate for some traders and for some well specified situations the total trading costs per bag. Using these estimates it is possible to get an idea of the marketing margins of these traders. Bassolet collected for a few traders detailed information on the costs they made, their strategies, the quantity they trade, and the prices they paid and received per bag. On the basis of this he calculated the costs per 100 kg bag. He measured the following elements:

1. Transport costs: the costs per bag to transport a bag from the purchase to the sales market.
2. Storage costs: renting costs, surveillance, insecticides if the traders store for at least four months. The total costs are divided by the number of bags purchased by the trader.
3. Bags: cost per bag to store cereals.
4. Annual taxes: Annual taxes are divided by the quantity traded to obtain taxes per bag.
5. Daily market taxes: It is supposed that only one bag is traded per market day. So, costs per bag are equal to the daily market tax.
6. Travel costs of the trader to travel to the markets, including costs for food, drinks, etc.
7. Personnel costs: agents working on a commission basis receive a fixed amount per bag. The personnel costs per bag for montly paid personnel is calculated as their salary divided by the number of bags traded.
8. Loading costs: costs to load and unload the trucks.
9. Gifts, etc.
10. Opportunity costs: foregone profits during the storage period.

For three different types of transactions Bassolet (2000) estimated for five different traders their trading costs. The first type of transaction considers intra-regional cereal trade by a merchant in Solenzo in the province of Kossi, who purchases in neighbouring villages, and who stores his merchandise for one month, before selling it in Solenzo. He sells approximately 600 bags per month, and he sells in retail to consumers and in bulk to merchants from the central and Sahelian regions of the country. The second type of transaction considers interregional trade with a storage period not exceeding one month. Two cases are distinguished here: a semi-merchant and a merchant from Ouagadougou. The semi-merchant, who sells approximately 200 bags per month, purchases in the south-western regions of the country. His average storage length is 15 days, and he sells to retailers and consumers. The merchant sells approximately 1200 bags per month which are stored for an average length of one month before selling it to the merchants of Dori and Ouahigouya. The third type of transaction considers interregional cereal trade with a storage length of four months. The costs of a merchant in Dori and one in Ouahigouya are evaluated. The merchant in Dori purchases on average approximately 700 bags per month from traders in Ouagadougou, Manné and Bobo-Dioulasso. The merchant from Ouahigouya purchases in Bobo-Dioulasso, Solenzo and Mali. He sells approximately 750 bags per month to consumers and to other traders. He stocks for about 4 months before selling. The trading costs of these traders are presented in Table A4.10 and Table A4.11.

Table A4.10 Trading costs in FCFA per 100 kg bag for a storage time not exceeding one month.

	Merchant from Solenzo (intra-regional trade)	**Merchant from Ouagadougou (inter-regional trade)**	**Semi-merchant from Ouagadougou (inter-regional trade)**
Transport costs	158	713	616
Storage costs	8	65	2
Bags	33	281	256
Annual taxes	6	4	4
Daily taxes	-	-	3
Travel costs	10.3	14	67
Personnel costs	21	22	-
Loading costs	28	100	93
Other costs	39	58	68
Opportunity costs	90	66	131
Total transport, storage and marketing costs:	393.3	1323	1240
Purchase price	8250	11938	11993
Sales price	9300	12417	12528
Net margin[1]	657	-844	-705

Note: 1) Net margin = Sales price – Purchase price – Total costs
Source: Bassolet (2000).

Table A4.11 Trading costs in FCFA per 100 kg bag for a storage time of approximately four months.

	Merchant from Dori (intra-regional trade)	**Merchant from Ouahigouya (inter-regional trade)**
Transport costs	750	1000
Storage costs	11	100
Bags	38	31
Annual taxes	9	1
Daily taxes	1	1
Travel costs	-	7
Personnel costs	-	-
Loading costs	50	79
Other costs	7	-
Opportunity costs	434	391
Total transport, storage and marketing costs:	1300	1610
Purchase price	9975	8950
Sales price	13333	11278
Net margin[1]	2058	718

Note: 1) Net margin = Sales price – Purchase price – Total costs
Source: Bassolet (2000).

Some of the costs reported by Bassolet seem very low. For example, no personnel costs are reported for three of the 5 traders interviewed. This is strange if you consider the quantity traded, which can impossibly be handled by the merchant alone. Salaries and commissions paid to personnel which are reported by Déjou (1987) and Bassolet himself, amount at least 250 FCFA per bag (see Appendix A2.3). Differences between storage costs are also large. It may be well possible that the traders interviewed forgot some of the costs they have to make, either because it are sunk costs or because activities are carried out by relatives who are paid in kind. The opportunity costs are non-negligible. Bassolet estimates them on the basis of the annual interest rate charged by the CNCA, which is 13% per year. The net margins which can be calculated on the basis of the results of the two tables show large differences. For two traders the net margins turn out to be negative, for the other three they are positive and large. Unlike Sherman et al. (1987) above, Bassolet does not include the salaries of the traders in the traders' costs. This is one of the reason why the estimated margins are high for some of the trade routes. Although it is normal to exclude the traders' salary if the traders' margins are calculated, we have to include them in our study. After all, the trade costs considered in our approach have to account for the entire difference between consumer and producer prices. This difference includes the margins earned by the traders (i.e. his salary).

The storage and trade costs estimated by Sherman et al. (1987) are considerably higher than those estimated by Bassolet (2000). Reasons are that Bassolet neglected some of the costs (e.g. salary of the trader himself), and that some of his estimates are rather unreliable. We therefore apply in Section 8.2 above all the study of Sherman et al. (1987) to estimate the storage and other trading costs.